4D CP violation from pure gauge in six dimensions

J.-M. Frere[a*], M. Libanov[b†], S. Mollet[a‡]
[a] *Service de Physique Théorique, Université Libre de Bruxelles,*
Campus de la Plane CP225, Bd du Triomphe, 1050 Brussels, Belgium
[b] *Institute for Nuclear Research of the Russian Academy of Sciences,*
60th October Anniversary Prospect 7a, 117312, Moscow, Russia

Abstract

In extra-dimensional models, extra components of gauge fields play a role of scalars in 4D and can acquire effective vev's through quantum effects. These components can be used to break CP in 4D Standard Model where pure gauge interactions respect CP and the latter is only broken through the Yukawa couplings in the poorly understood scalar sector. We present a *toy model* with 2 extra-dimensions compactified on a flat torus $\mathbb{T}^2$, where a SU(2) gauge symmetry is broken to U(1) and CP violation (in 4D) is expected.

1 Introduction

As was pointed out by Sakharov, the emergence of a matter-antimatter asymmetry from an initially symmetrical early universe requires in 4D both C and CP violation. A more general statement would be that C and any symmetry involving C must be broken, CP being just one particular case. However, in the framework of the 4D Standard Model the gauge interactions are CP conserving, and the scalars only break this symmetry, but in an arbitrary manner through the phases of the Yukawa coefficients. While pure gauge interactions are fixed by the gauge symmetry, scalar sector are badly understood – there are no (obvious) symmetries fixing a bunch of arbitrary Yukawa couplings. To avoid this arbitrariness one may ask how C or CP invariance can be broken in theories containing only fermions and their gauge interactions. This is pretty much the issue we will be discussing in the present note. More specifically, we will discuss how C or CP conservation behave in the dimensional reduction (in the present case from 6D to 4D).

An idea, which we will use through the note, is attractive, though quite old. Namely, we will use the fact that scalar fields are thought as spatial components of gauge fields (A_4 and A_5) in extra dimensions (ED) [1, 2, 3]. When extra-dimensional space is not simply connected, non trivial holonomies (or Wilson lines (WL)) can appear dynamically for non contractible cycles and lead to dynamical gauge symmetry breaking. At the level of 4D space these effective scalar fields (A_4, A_5) acquire a vev, which could cause CP violation if scalar and pseudo-scalar contributions coexists. At the classical level, the WL are determined by the topology of ED and label degenerate classical vacua. The degeneracy disappears when quantum effects are taken into account, which select the physical solution. These are encoded into the effective potential for WL which depends on topology, matter content and Scherk-Schwarz (SchSch) phases.

This idea was already used in previous works in a context of 5D model [4, 5]. However, due to the lack of scalar fields (there is only one additional component of the gauge field in 5D) half of the scalars were put in by hand. An appealing extension would be to add a second ED which will provide for this. This way was used in [6] and here we will follow it. We stress, however,

*e-mail: frere@ulb.ac.be
†e-mail: ml@ms2.inr.ac.ru
‡e-mail: smollet@ulb.ac.be

that the present note deals with "proof of concept", namely the possibility of CP violation in 4D from pure gauge theory in 6D, but does not propose a realistic model. This is notably due to the difficulty of generating a "low mass scale", providing non-zero mass to the zero modes of the compactified theory: in the present note, we will deal either with a massless low-energy sector separated from the Kaluza-Klein scale, or accept small masses controlled by arbitrary phases in the boundary conditions.

The paper is organized as follows. In section 2 we review the notions of P, C and CP symmetries in 4D and in 6D and link them through compactification schemes. Section 3 is devoted to Hosotani mechanism which takes place when compactification implies non simply connected ED. We summarize it in the special case of the flat torus $\mathbb{T}^2$ and try to include Hosotani's approach in the more modern one [7, 8]. In section 4 we use explicitly Hosotani mechanism to break CP through compactification and give a simple example. Conclusions and perspectives can be found in section 5.

2 CP transformations in 4D and 6D

In what follows we use the notations of ref. [9]; in particular, we denote four-dimensional indices by Greek letters, x^μ , $\mu, \nu = 0, \dots, 3$, six-dimensional coordinates are labeled by capital Latin indices $A, B = 0, \dots, 5$, we introduce ED coordinates y^4, y^5 and use lower case Latin indices $i, j = 4, 5$ to label them. We use also the notations γ^μ (resp. Γ^A) for 4D (resp. 6D) gamma matrices.

In any even number dimensions D the parity transformation is given by $\mathcal{P}^{-1}\Psi(t,\mathbf{x})\mathcal{P} = \Gamma^0\Psi(t,-\mathbf{x})$ [10]. The charge conjugation is given by $\mathcal{C}^{-1}\Psi(x)\mathcal{C}^{-1} = C^{(D)}\Gamma^0\Psi^*(x)$ where $C^{(D)}$ is a matrix which satisfies ${C^{(D)}}^{-1}\Gamma^A C^{(D)} = \pm\Gamma^0{\Gamma^A}^*\Gamma^0$. The solution to this equation $C^{(D)}$ can always be represented as the product of $D/2$ gamma matrices[1] (recall that D is even). In addition, in even number dimensions, the spinor can be decomposed in two semi-spinors (or Weyl spinor) $\Psi_\pm = P_\pm\Psi$ with the help of the chirality projectors $P_\pm = \frac{1\pm\Gamma_{D+1}}{2}$ where Γ_{D+1} is the product (up to a phase factor) of all (D) gamma matrices and anticommutes with all Γ's. Thus, Γ_{D+1} always anticommutes with the parity transformation, and commutes with the charge conjugation $C^{(D)}\Gamma^0$ if $D/2$ is odd (e.g., $D = 6$) and anticommutes with the $C^{(D)}\Gamma^0$ if $D/2$ is even (e.g., $D = 4$). It means that the parity always connects $+$ and $-$ spinors. On the contrary, the charge conjugation links Ψ_+ and Ψ_-^* (and vise versa) at even $D/2$ while at odd $D/2$ it links Ψ_+ with Ψ_+^* and Ψ_- with Ψ_-^*[2]. Then the CP operation which is the combination of these two connects $+$ and $+$ ($-$ and $-$) spinors in even $D/2$, but $+$ and $-$ in odd $D/2$.

Now what does it mean? Since gauge interactions connect spinors of the same chirality, gauge symmetries give no reason to introduce both chiralities on an equal footing. Then, in all generality, P is not an automatic symmetry of gauge interactions in any number of dimensions. However, while C symmetry is not automatic in even D/2, this is always the case in odd D/2, and conversely for CP. For this reason we need scalar interactions in 4 dimensions to break CP (at perturbative level). In contrast if we write a theory in 6 dimensions with only (say) a $+$ spinor then we break CP. Does it mean that the resulting effective 4D theory is not CP conserving? In other words, are the notions of CP in 4 and 6 dimensions directly related to each other? The answer is no.

To realize this we need to find a relation between 4D and 6D CP transformations. Let us focus on $+$ spinor in 6D which is a Dirac spinor at the 4D level (with L and R components[3]). Indeed, in the chiral representation for the gamma matrices, from the 4D point

[1]This is true for both sign in the equation. Recall that the $+$ sign is only valid for massless spinor which is the case here.

[2]This is related to the fact that in even $D/2$ (resp. odd $D/2$) Ψ_+ and Ψ_-^* (resp. Ψ_+ and Ψ_+^*) are equivalent representations of the Lorentz group.

[3]In 4D the sign $+$ is identified with L and the $-$ sign with R.

of view an analog of γ_5, which acts on a $6D$ spinor, is $\bar{\gamma}_5 \sim \Gamma_0\Gamma_1\Gamma_2\Gamma_3 \sim \mathrm{diag}(-\gamma_5, \gamma_5)$ [9]. That is, for $\Psi = (\Psi_+, \Psi_-)^{\mathrm{T}}$ one finds $\Psi_+ = \gamma_0(\psi_L, \psi_R)^{\mathrm{T}} \sim \psi_{\mathrm{Dirac}}$. In addition, we know that C transforms $\Psi_+(x)$ into $\Psi_+^{C_6} \sim \gamma_5\gamma_2\Psi_+^*$. On the other hand, 4D CP transformation, which acts on 6D spinor Ψ_+, should looks like $\Psi_+^{CP_4}(t, \mathbf{x}, \mathbf{y}) \sim \gamma_0\gamma_2\Psi_+^*(t, -\mathbf{x}, \mathbf{y}')$, what means $\Psi_+^{P_4}(t, \mathbf{x}, \mathbf{y}) \sim \gamma_0\gamma_5\Psi_+(t, -\mathbf{x}, \mathbf{y}')$ where $\mathbf{y}'$ can be obtained from $\mathbf{y}$ by making use of any Lorentz transformations which do not touch 4D coordinates. But the matrix $\gamma_0\gamma_5$ appearing in the P_4 transformations of Ψ_+ is nothing but the Lorentz generator of a π-rotations in the (1-2) and (3-5) planes. In other words, if $\mathbf{y}' = (y_4, -y_5)$ then P_4 is nothing but 6D rotations which are, of course, a symmetry of the 6D theory. Note also, that the rotations in the (4-5) plane on an angle θ, $\mathbf{y}' = \mathcal{R}(\theta)\mathbf{y}$, is 4D chiral rotations $\Psi_+(t, \mathbf{x}, \mathbf{y}') = \exp(i\theta\gamma_5/2)\Psi_+(t, \mathbf{x}, \mathbf{y})$.

Since this combination of transformations is a symmetry of the 6D theory, the 4D effective theory will be CP violating only if the compactification is incompatible with all the symmetries:

$$\begin{cases} \Psi_+(t, \mathbf{x}, \mathbf{y}) \to \Psi_+^{CP_4}(t, \mathbf{x}, \mathbf{y}) \sim \gamma_0\gamma_2 \mathrm{e}^{-i\gamma_5\theta/2}\Psi_+^*(t, -\mathbf{x}, \mathbf{y}') \\ Y \equiv (y_4, y_5)^T \to \mathcal{R}(\theta)\sigma^3 Y = \mathcal{R}(\theta)(y_4, -y_5)^T. \end{cases}$$

for any rotation $\mathcal{R}$. In other words, the 4D theory will be CP violating if we fail to find a chiral rotation which reabsorbs the phases.

Let us take a simple example to illustrate this. Consider a flat torus $\mathbb{T}^2$ of radii $R_4 = R_5 = R$ with the following SchSch boundary conditions (BC)[4]: $\Psi(y_4 + 2\pi R, y_5) = e^{i\beta_4}\Psi(y_4, y_5)$ and $\Psi(y_4, y_5 + 2\pi R) = e^{i\beta_5}\Psi(y_4, y_5)$. Under the prescribed transformation these BC become $\Psi(y_4 + 2\pi R\cos\theta, y_5 + 2\pi R\sin\theta) = e^{-i\beta_4}\Psi(y_4, y_5)$ and $\Psi(y_4 + 2\pi R\sin\theta, y_5 - 2\pi R\cos\theta) = e^{-i\beta_5}\Psi(y_4, y_5)$. These BC are compatible iff $\beta_4, \beta_5 = 0$ or π. That is, BC break effective 4D CP_4 symmetry as soon as β_4, β_5 are both different from 0 or π.

Note by the way that we can proceed in the same way for P and C. It is straightforward to show that P invariance requires compatibility with the transformation $Y \to \mathcal{R}\sigma^3 Y$, while C requires compatibility with $\Psi \to \Psi^*$ and $Y \to \mathcal{R}Y$. In our previous example, P is broken but not C (this leads then to CP violation).

As already mentioned in the introduction, the main point of breaking CP is to get a matter-antimatter asymmetry. Thus even if C is broken, this is in general not enough to reach this goal. Indeed any other symmetries involving C (like CP, but CS in general) leads to matter-antimatter symmetry. In 6D the C symmetry is automatic for gauge interactions and the symmetry particle/antiparticle is respected. In 4D C is not automatic but CP leads to the same conclusion. Our idea to break this symmetry is precisely to introduce a compactification which breaks all these CS symmetries.

3 Hosotani mechanism

To be specific in what follows we will work on flat space-time $\mathbb{M}^4 \times \mathbb{T}^2$ where $\mathbb{T}^2$ is a flat torus characterized by two radii R_4 and R_5 (in general, $R_4 \neq R_5$).

Let's turn off for the moment the spinors and consider a non-abelian gauge field A_A. In general, two kinds of compactification exist: the "non-magnetized" and the "magnetized" one. In the first case, a non zero field strength is unstable and the only solutions are flat connections. It is this case we will consider in what follows. An example of this kind of compactification is $SU(N)$ gauge group on a flat torus $\mathbb{T}^2$. In the second case, a non zero field strength can be stable and the solution corresponds to a physical flux orthogonal to the ED. The stability is ensured by the quantization of the flux for topological reasons [7]. An example is $U(N)$ group on flat torus.

[4]For now on Ψ means Ψ_+ unless otherwise stated.

Because of the translation symmetry on the torus, gauge fields on this manifold must be periodic up to a gauge transformation [7, 8]:

$$A_A(x, y_i + 2\pi R_i) = T_i(\mathbf{y}) A_A(x, \mathbf{y}) T_i^{-1}(\mathbf{y}) + T_i(\mathbf{y}) \partial_A T_i^{-1}(\mathbf{y}), \tag{1}$$

where the transition functions $T_i(\mathbf{y})$ must satisfy $T_4(y_4, y_5{+}2\pi R_5)T_5(\mathbf{y}) = T_5(y_4{+}2\pi R_4, y_5)T_4(\mathbf{y})$.

However one must be careful, because the BC (1) do not fix the symmetry of the effective 4D theory. Indeed, the components of the gauge fields in the ED, A_4, A_5, playing the role of the scalar fields in 4D could acquire a "*vev*" $\langle A_4\rangle, \langle A_5\rangle$ through quantum effects. More precisely, some non-integrable phases become dynamical variables and can lead to effective gauge symmetry breaking in 4D. Indeed, it's worth stressing that neither *"vev"* nor BC (T_i) are gauge invariant concepts since they are transformed:

$$\langle A_i\rangle' = \Omega\langle A_i\rangle\Omega^{-1} + \Omega\partial_i\Omega^{-1}, \quad T_i' = \Omega(y_i + 2\pi R_i)T_i(\mathbf{y})\Omega(\mathbf{y})^{-1}$$

under gauge transformations Ω. The true gauge invariant quantities are the so called *Wilson lines phases* (WLP) defined by Hosotani [3] as the eigenvalues of $W_{C_i}(y)T_{C_i}$, with:

$$W_{C_i}(y) = \mathcal{P}\ \exp\left(\oint_{C_i} \mathrm{d}y_j' \langle A_j(y')\rangle\right),$$

for all the non equivalent non-contractible cycles C_i starting at y, and T_{C_i} the associated BC.

What is the general form of the WLP? To clarify this question let's briefly repeat arguments of [7, 8] that the used compactification is "non-magnetized", that is, the field strength $\langle F_{45}\rangle$ vanishes. Indeed, the 6D Lagrangian for perturbations $\tilde{A}_A$ looks like

$$\mathrm{Tr} F_{AB}^2[\langle A_C\rangle + \tilde{A}_C] \sim \mathrm{Tr}(\langle F_{AB}\rangle + F_{AB}(\tilde{A}_C) + \ldots)^2 \sim \ldots + g f^{abc}\langle F_{45}^a\rangle \tilde{A}_4^b \tilde{A}_5^c.$$

So, $m_{bc}^2 \sim g f^{abc}\langle F_{45}^a\rangle$ is the mass matrix for 4D effective scalars $\tilde{A}_4^b$, $\tilde{A}_5^c$. If $\langle F_{45}\rangle \neq 0$ and the gauge group is simple (e.g., $SU(N)$) then $m_{bc}^2 \neq 0$. But due to the antisymmetry of the group structure constants $m_{bc}^2 = -m_{cb}^2$ and, therefore, $\mathrm{Tr}\, m^2 = 0$. It means that there should be positive and negative eigenvalues of m^2 and, thus, the configuration with $\langle F_{45}\rangle \neq 0$ is unstable (Nielsen-Olesen instability). Hence $\langle F_{45}\rangle = 0$ and the "vevs" must be a pure gauge

$$\langle A_i(\mathbf{y})\rangle = S(\mathbf{y})\partial_i S(\mathbf{y})^{-1}.$$

In addition, S must be compatible with the BC, hence

$$S(y_i + 2\pi R_i) = T_i(\mathbf{y})S(\mathbf{y})V_i^{-1}$$

where V_i are *constant* elements of the gauge group such that $[V_4, V_5] = 0$. Now, we can perform the gauge transformation with $\Omega = S^\dagger$, then $S(\mathbf{y})$ transforms to the unit matrix and, hence, $\langle A_i'(\mathbf{y})\rangle = 0$ and $T_i' V_i^{-1} = 1$. Thus the WLP is nothing but V_i:

$$W_{C_i}T_{C_i} = W_{C_i}'T_{C_i}' = V_i = \exp(i\alpha_i),$$

where α_i are commuting hermitian matrices of $SU(N)$ algebra. Therefore, all possible classical vacua can be labeled by constant α_i.

In general, among other, there are two approaches. The first one is to gauge away "vevs" $\langle A_{4,5}\rangle = 0$ and end up with the non-trivial BC

$$T_i = V_i = \exp(i\alpha_i),$$

while the second way is to gauge away BC: $T_i = 1$ and deal with the non-trivial "vevs" $\langle A_i\rangle = \alpha_i/(2\pi R_i)$. In what follows we will use the first approach.

4 CP violation induced by BC

Let's turn on the fermions. BC for the spinor in the presence of a gauge field become $\Psi(y_i + 2\pi R_i) = \exp(i\beta_i)T_i\Psi(\mathbf{y})$ or, in the chosen gauge $\langle A_{4,5}\rangle = 0$,

$$\Psi(y_i + 2\pi R_i) = \exp(i\beta_i)\exp(i\alpha_i)\Psi(\mathbf{y}).$$

As it was discussed in the section 2, CP_4 is conserved if the BC are symmetric under the transformation

$$\begin{cases} \Psi \to U^*\Psi^*, \quad \text{where } U - \text{constant matrix} \\ \mathbf{y} \to \mathbf{y}' = \mathcal{R}(\theta)\sigma_3\mathbf{y} \end{cases}$$

On the other hand, under the prescribed transformations the BC become

$$\begin{cases} \Psi^{CP_4}(y_4 + 2\pi R_4\cos\theta, y_5 + 2\pi R_4\sin\theta) = \mathrm{e}^{-i\beta_4}\exp\left[-i(U\alpha_4U^{-1})^*\right]\Psi^{CP_4}(\mathbf{y}) \\ \Psi^{CP_4}(y_4 + 2\pi R_5\sin\theta, y_5 - 2\pi R_5\cos\theta) = \mathrm{e}^{-i\beta_5}\exp\left[-i(U\alpha_5U^{-1})^*\right]\Psi^{CP_4}(\mathbf{y}) \end{cases}$$

The Table 1 shows the different symmetries which might be compatible with BC. The angle θ refers to the rotation $\mathcal{R}$. The columns marked β_4 and β_5 indicate a possible constraint for these phases. The next two columns show the constraints on the U matrix introduced above[5]. Note that for adjoint fermions, insensitive to the centre of the group, we have a little bit more freedom. The k and k' factors take this into account for SU(N) groups ($T = \mathrm{diag}(1, ..., 1, 1-N)$). k and k' can take all integer values for representations which are insensitive to the centre, but must be zero in the other case.

	θ	β_4	β_5	$U\alpha_4U^{-1}$	$U\alpha_5U^{-1}$	
$R_4 \neq R_5$	0	$\{0,\pi\}$	$[0,2\pi[$	$-\alpha_4 + \frac{2\pi k}{N}T$	$\alpha_5 + \frac{2\pi k'}{N}T$	(1)
	π	$[0,2\pi[$	$\{0,\pi\}$	$\alpha_4 + \frac{2\pi k}{N}T$	$-\alpha_5 + \frac{2\pi k'}{N}T$	(2)
$R_4 = R_5$	$\pi/2$	$-\beta_5$	$-\beta_4$	$-\alpha_5 + \frac{2\pi k}{N}T$	$-\alpha_4 + \frac{2\pi k'}{N}T$	(3)
	$3\pi/2$	β_5	β_4	$\alpha_5 + \frac{2\pi k}{N}T$	$\alpha_4 + \frac{2\pi k'}{N}T$	(4)

Table 1: Hypothetical transformations that could be identified with an effective CP symmetry in 4D if compatible with boundary conditions (BC).

There are now two main questions. (1) Which patterns can be realized (and under which conditions)? To clarify this question one may use the following strategy (see for the details [6]). First, we need to compute the effective potential for WLP for each group and representation we want to study

$$V_{\text{eff}} = \text{const} \times \left(-V_{\text{eff}}^{\text{g+gh}} + \sum_{i,R} V_{\text{eff}}^{\text{ferm}_{i,R}} + \text{possible matter contributions} \right).$$

Then find the minima of this potential (α_i) which depend on many parameters: β_i, R_i, matter content, etc. And then check whether CP_4 is conserved or not. In fact, we see that this is quite tricky problem, so that, in what follows we briefly consider one particular example.

The second question is: (2) at which level does CP_4 violation manifest itself (and what could be phenomenologically promising)? Regarding the problem of phenomenology, one of the main limitations (without any new mechanism) has been mentioned and concerns the absence of gap between light and heavy sectors. A partial answer to this issue (unfortunately quite inelegant) comes from the SchSch phases. If we choose them sufficiently small, they could account for

[5] We should write $\left(U\alpha U^{-1}\right) \sim \pm\alpha^*$, but remember that $\alpha_i^\dagger = \alpha_i$, then we can use α_i^T instead of α_i^*. However, α_i's are diagonal (or can be diagonalized because of the topology), and therefore we can use α_i. Note also that these relations are not so strict. Indeed the periodicity of the exponential factor must be taken into account.

small masses of the previously massless modes. We must however remember their influence on the dynamics of WLP.

CP violation is, even in the Standard Model, a tricky issue to characterize (the Jarlskog determinants providing a partial answer). To prove that CP is violated, the safest way is to provide an "observable". Here we will deal with a single (light) fermion species and the simplest "observable" is then the electric dipole moment (EDM) of the lightest mode[6].

For the time being we focus ourselves on simple example. Let's consider $SU(2)$ gauge group with one fermion in an adjoint representation (one can find another example as well as more detailed discussion in [6]). Numerical calculations show that, at least in the interesting regime $\beta_4, \beta_5 \in [0, 0.1]$ and $0.9 < r = R_5/R_4 < 1$, $(\alpha_4, \alpha_5) = \pi\sigma_3/2$. It means that the $SU(2)$ is broken into $U(1)$, and we have a neutral fermion with mass

$$m_{\text{light}} \simeq \frac{\beta}{R}\left(1 + \frac{\triangle\beta}{\beta} + \triangle r\right),$$

where $\triangle\beta = \beta_4 - \beta_5, \quad \triangle r = 1 - \frac{R_5}{R_4}$. The EDM of this mode is

$$\left|\frac{d_E R}{e^3}\right| \simeq 0.01\left(\triangle r + 4.5\frac{\triangle\beta}{\beta}\right).$$

This empirical formula is found by making use of numerical evaluations. Our results can be found in Table 2.

β	$\Delta\beta/\beta$	Δr	$m_{\text{light}}R$	$d_E R/e^3$
$[0, 10^{-1}]$	0	0	$\sqrt{2}\beta$	0
10^{-1}	0	10^{-1}	$1.35\ 10^{-1}$	$1.09\ 10^{-3}$
10^{-1}	0	10^{-2}	$1.41\ 10^{-1}$	$0.99\ 10^{-4}$
10^{-1}	0	10^{-3}	$1.41\ 10^{-1}$	$0.98\ 10^{-5}$
10^{-1}	0	10^{-4}	$1.41\ 10^{-1}$	$0.98\ 10^{-6}$
10^{-1}	10^{-1}	0	$1.35\ 10^{-1}$	$4.66\ 10^{-3}$
10^{-1}	10^{-2}	0	$1.41\ 10^{-1}$	$4.50\ 10^{-4}$
10^{-1}	10^{-3}	0	$1.41\ 10^{-1}$	$4.48\ 10^{-5}$
10^{-1}	10^{-4}	0	$1.41\ 10^{-1}$	$4.48\ 10^{-6}$
10^{-2}	10^{-1}	0	$1.35\ 10^{-2}$	$4.28\ 10^{-3}$
10^{-3}	10^{-1}	0	$1.35\ 10^{-3}$	$4.28\ 10^{-3}$
10^{-3}	10^{-1}	10^{-1}	$1.27\ 10^{-3}$	$5.71\ 10^{-3}$
10^{-3}	10^{-1}	10^{-2}	$1.33\ 10^{-3}$	$4.41\ 10^{-3}$
10^{-3}	10^{-1}	10^{-3}	$1.34\ 10^{-3}$	$4.29\ 10^{-3}$

Table 2: Numerical evaluation of the electric dipole moment (EDM) of the light particle in a SU(2) theory with a 6D spinor in the adjoint representation for different sets of parameters. We give also its mass m_{light} which is the lightest of the fermion spectrum.

5 Conclusion and perspectives

We made use of the Hosotani mechanism to generate both gauge and CP symmetry breaking through compactification from a 6-dimensional model. Though we found examples where it

[6]We study the lightest mode since we look for an understanding of CP violation at low energy. However a zero EDM for this state doesn't mean that CP is conserved (and that our previous analysis fails), as it may manifest itself at higher energy. Remember also that an EDM violates both P and CP. It is however easy to check that, with this mechanism, the 4D P symmetry is broken as soon as the CP one is.

works, our solutions is far from being realistic, and they must be seen more as *"proof of concept"*. One of the major difficulties of the work is the high level of entanglement in the approach. Indeed, the final result depends both on matter content (representations), BC (SchSch phases) and WL phases, while the latter depend in turn on the formers and are dynamically determined through a potential which must be numerically evaluated.

The next steps in this program should be the resolution of the two main drawbacks of the present solutions. First, new compactification mechanism (like orbifold or flux compactifications) might be employed to reach a chiral theory in 4D (at this point the only difference between left and right couplings in the gauge sector comes through a phase). Moreover, we'd like to avoid the presence of two (nearly) identical fermionic sectors without introducing anomalies in the theory (see [6] for details). Secondly (but this maybe even more ambitious), a mechanism which produces a low energy sector naturally separated from the Kaluza-Klein scale would be very welcome. For instance, in more complex situations, one can hope for an effective low energy potential between the remaining scalars, what would provide the lower mass scale, but this goes beyond this "proof of concept" paper.

Acknowledgments

We are indebted to E. Nugaev and S. Troitsky for helpful discussions. This work is funded in part by IISN and by Belgian Science Policy (IAP "Fundamental Interactions"). The work of ML (elaboration of the CP in the ED) is supported by the Russian Science Foundation, grant 14-22-00161.

References

[1] Y. Hosotani, *Dynamical Mass Generation by Compact Extra Dimensions*, *Phys. Lett. B* **126** (1983) 309.

[2] Y. Hosotani, *Dynamical Gauge Symmetry Breaking as the Casimir Effect*, *Phys. Lett. B* **129** (1983) 193.

[3] Y. Hosotani, *Dynamics of Non-integrable Phases and Gauge Symmetry Breaking*, *Annals Phys.* **190** (1989) 233.

[4] N. Cosme and J.-M. Frére, *CP Violation from Dimensional Reduction: Examples in 4+1 Dimensions*, *Phys. Rev. D* **68** (2003) 096001 [arXiv:0207024 [hep-ph]].

[5] N. Cosme and J.-M. Frére, *CP Violation in Weak Interactions from Orbifold Reduction: Possible Unification Structures*, *Phys. Rev. D* **69** (2004) 036003 [arXiv:0303037 [hep-ph]].

[6] J. M. Frere, M. Libanov and S. Mollet, *CP violation from pure gauge in extra dimensions*, JHEP **1406** (2014) 103 [arXiv:1403.2521 [hep-ph]].

[7] J. Alfaro, A. Broncano, M. B. Gavela, S. Rigolin and M. Salvatori, *Phenomenology of Symmetry Breaking from Extra Dimensions*, *JHEP* **0701** (2007) 005.

[8] M. Salvatori, *Symmetry Breaking from Scherk-Schwarz Compactification*, *JHEP* **0706** (2007) 014, [arXiv:0611309 [hep-ph]].

[9] M. V. Libanov and S. V. Troitsky, *Three fermionic generations on a topological defect in extra dimensions*, Nucl. Phys. B **599** (2001) 319 [hep-ph/0011095].

[10] M. B. Gavela and R. I. Napomechie, *Discrete Symmetries in Kaluzua-Klein Theories*, *Class. Quantum Grav.* **1** (1984) L21 [arXiv:0303037 [hep-ph]].