NLO accuracy for power terms in inclusive decays of heavy flavor particles

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Abstract

I present recent results of the computation of NLO radiative corrections to the coefficient of the power suppressed term of chromomagnetic type for the inclusive semileptonic width of a heavy flavor particle.

1 Introduction

After a convincing confirmation of the Higgs boson existence at the LHC experiments the standard model has all its ingredients established and has formally been completed. It is now the theory of particle interactions at the energy range below the scale of order 1 TeV and has been thoroughly tested experimentally. It is well known that there are physics phenomena beyond the standard model. At present some of them can readily be incorporated by minor extensions of the standard model within the existing paradigm. As for finding a piece of unexpected physics it can happen that there will be no more new fundamental modes (particles) that can be registered at modern and possible future accelerators explicitly and their existence should be deciphered from the accurate comparison of the standard model theoretical predictions with precision experimental data. Therefore it seems that the motto of the day in theoretical analysis of the standard model is high precision. A benchmark of the existing accuracy is provided by the leptonic sector of the standard model, in particular by the charged muon decay that is a source for determining the Fermi constant G_F. Historically, this decay has been investigated for a long time by now [1, 2]. The present theoretical result is available with the accuracy of the second order (NNLO) of perturbation theory expansion in the fine structure constant [3]

$$\Gamma(\mu \to \nu_\mu e \bar{\nu}_e)/\Gamma^0_\mu = 1 + \left(\frac{25}{8} - \frac{\pi^2}{2}\right)\frac{\alpha}{\pi} + 6.74\left(\frac{\alpha}{\pi}\right)^2$$

with $\Gamma^0_\mu = G_F^2 m_\mu^5/(192\pi^3)$ being a tree level expression and $\alpha^{-1} = 137.04...$. This result allows for the accuracy of the theoretical prediction of the muon decay width at the level of 1ppm which is necessary for comparison with the data of modern experiment and gives for the Fermi constant the numerical value $G_F = (1.16637 \pm 0.00001) \times 10^{-5}$ GeV.

As for new physics searches beyond the standard model the quark flavor sector seems to be very promising. The relevant couplings in this sector are a Fermi constant G_F multiplied by the quark mixing parameters V_{qQ} collected in a CKM matrix $V_{\rm CKM}$. The precision study of the quark decays mediated by charged currents is of great importance for the accurate determination of the numerical values of the CKM matrix elements V_{qQ}. In experiment the weak decays of quarks reveal themselves as decays of flavored hadrons that are bound states of

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fundamental constituents. Therefore, the theoretical description is forced to use some genuinely nonperturbative computational methods. For instance, the quark level transition $s \to u$ is represented by the decay $K \to \pi e \bar{\nu}_e$ that is well studied experimentally and would be a nice place for extracting $|V_{us}|$ provided the theoretical computations would be feasible. At present there are no analytical methods that are capable for such computations and one has to rely on models (e.g. for this special case where the hadrons are Golstone modes one can apply chiral perturbation theory – ChPT) or numerical treatment of the decay on the lattice.

For heavy particles, i.e. those that contain heavy quarks, however, the theoretical analysis is somewhat easier because the large mass of the heavy quark gives an expansion parameter [4]. Top mesons decay too fast, charmed mesons are not quite heavy enough, and the application of heavy quark approximation is almost marginal, while the case the bottom meson decays is treatable and has been intensively studied both experimentally and theoretically. The technique is applicable to $b \to (c, u)$ and $c \to (s, d)$ transition and both to semileptonic and pure hadronic decays.

The method of heavy quark expansion (HQE) in inclusive semileptonic $Q \to q$ decays with $q = (c, u, s, d)$ and $Q = b, c$ with an obvious kinematical constraint $m_Q > m_q$ has been developed to a level of theoretical precision of a few percent. The non-perturbative inputs are expressed in terms of forward matrix elements of local operators, which are fitted from the measured spectra of the inclusive semileptonic decays. For a heavy flavored particle H_Q containing $\bar{q}Q$ valent quarks the theory expression based on heavy quark expansion reads

$$\Gamma(H_Q \to X_q \ell \bar{\nu}_\ell)/\Gamma_Q^0 = |V_{qQ}|^2 \left[a_0 (1 + \frac{\mu_\pi^2}{2m_Q^2}) + a_2 \frac{\mu_G^2}{m_Q^2} + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^3}{m_Q^3}\right) \right] \tag{1}$$

where $\Gamma_Q^0 = G_F^2 m_Q^5 / (192\pi^3)$, μ_π^2 (the kinetic energy parameter), and μ_G^2 (the chromo-magnetic parameter) are the nonperturbative parameters related to the confinement scale of strong interactions $\Lambda_{\rm QCD}$. The coefficients a_i are functions of the quark (and, in general, lepton) masses and have a perturbative expansion in the strong coupling constant α_s normalized at large scale m_Q. The leading coefficient a_0 is presently known to $\mathcal{O}(\alpha_s^2)$ precision [5]. The NNLO contribution has been analytically computed in the limit of vanishing light quark q mass, leptons are taken massless as well. The coefficient of the kinetic energy parameter is a_0 due to Lorentz invariance [6]. The largest unknown contribution to the width is the α_s correction to the coefficient of the chromo-magnetic parameter a_2, which has been investigated recently in [7], where a numerical result for this contribution has been obtained. This coefficient has been analytically computed in the limit $m_q = 0$ in [8]. I report on the results of that paper.

2 Results of the calculation

The rate (1) is obtained from taking the imaginary part of the forward matrix element of the transition operator T [9] which is non-local and is given by the T-product of the form

$$T = i \int dx \, T \left[H_{\rm eff}(x) H_{\rm eff}(0) \right] \tag{2}$$

where $H_{\rm eff}$ is the effective Hamiltonian for the semileptonic transition

$$H_{\rm eff} = 2\sqrt{2} G_F \, V_{qQ} \left[(\bar{Q}_L \gamma_\mu q_L)(\bar{\nu}_L \gamma^\mu \ell_L) \right] . \tag{3}$$

In order to make explicit the dependence of the width on the heavy quark mass m_Q and, therefore, to build up an expansion in $\Lambda_{\rm QCD}/m_Q$ one matches a T-product (2) of full QCD operators (3) on an expansion over the local operators built from the HQET fields

$$\mathrm{Im} T / T_0 = C_0 \mathcal{O}_0 + C_v \frac{\mathcal{O}_v}{m_Q} + C_\pi \frac{\mathcal{O}_\pi}{2m_Q^2} + C_G \frac{\mathcal{O}_G}{2m_Q^2} + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^3}{m_Q^3}\right) \tag{4}$$

where $T_0 = \pi\Gamma_Q|V_{qQ}|^2$. The local operators in the expansion are built from the HQET modes h_v and ordered by their total dimensionality in mass units $\mathcal{O}_0 = \bar{h}_v h_v$, $\mathcal{O}_v = \bar{h}_v(v\pi)h_v$, $\mathcal{O}_\pi = \bar{h}_v\pi_\perp^2 h_v$, $\mathcal{O}_G = \bar{h}_v\frac{1}{2}[\hat{\pi}_\perp, \hat{\pi}_\perp]h_v$. Here v is a velocity of a heavy quark for the HQET construction, $\pi_\mu = i\partial_\mu + g_s A_\mu$ is a covariant derivative with the decomposition in longitudinal ($\sim v$) and transverse $\pi_\perp^\mu$ parts $\pi^\mu = v^\mu v\pi + \pi_\perp^\mu$. The quantity h_v is a relevant field variable of the HQET Lagrangian [10, 11]. The expansion (4) is a matching relation from QCD to HQET with proper operators up to dimension five and corresponding coefficient functions. Note that the operator $\mathcal{O}_v$ is eliminated by the use of equations of motion when taking the matrix elements over the hadronic states.

The Lagrangian of the fields h_v reads

$$\mathcal{L} = \mathcal{O}_v + \frac{1}{2m_Q}(\mathcal{O}_\pi + \mathcal{O}_G) + O\left(\frac{\Lambda^2}{m_Q^2}\right) \tag{5}$$

where

$$C_m(\mu) = 1 + \frac{\alpha_s(\mu)}{2\pi}\left\{C_F + C_A\left(1 + \ln\frac{\mu}{m_Q}\right)\right\} \tag{6}$$

is a coefficient of chromo-magnetic operator $\mathcal{O}_G$ at NLO with $C_F = (N_c^2 - 1)/(2N_c)$, $C_A = N_c$ for $SU(N_c)$ general color group. For QCD one takes $N_c = 3$.

The modes h_v are redefined such that terms of the order $O(1/m_Q^2)$ in the Lagrangian contain no time derivatives and are not relevant for our analysis [12, 13].

As a leading term of the heavy quark expansion one can take a full QCD local operator $\bar{Q}\hat{v}Q$. The current $\bar{Q}\gamma_\mu Q$ conserves and therefore its forward matrix elements over hadronic states $H_Q(p)$ are absolutely normalized. For this to be implemented one uses an expansion of the operator $\bar{Q}\hat{v}Q$ within HQE techniques through HQET operators. The expansion reads

$$\bar{Q}\hat{v}Q = \mathcal{O}_0 - \frac{\mathcal{O}_\pi}{2m_Q^2} + \tilde{C}_G\frac{\mathcal{O}_G}{2m_Q^2} + O(1/m_Q^3) \tag{7}$$

and is valid with an account of the radiative corrections of order α_s. Thus, the leading power operator has no radiative corrections and the kinetic operator has the same coefficient due to Lorentz (reparameterization) invariance. In a sense, the operator $v_\mu\bar{h}_v h_v$ is also absolutely normalized in all orders of the perturbative α_s expansion.

Substituting eq. (7) in eq. (4) and applying the equations of motion one obtains the representation

$$\mathrm{Im}T/T_0 = C_0\left\{\bar{Q}\hat{v}Q - \frac{\mathcal{O}_\pi}{2m_Q^2}\right\} + \left\{-C_v C_m + C_G - \tilde{C}_G C_0\right\}\frac{\mathcal{O}_G}{2m_Q^2} \tag{8}$$

The numerical value for the chromo-magnetic moment parameter μ_G^2 related to the forward matrix element of the operator $\mathcal{O}_G$ over the hadronic states is usually taken from the mass splitting between the pseudo scalar and vector ground state mesons. The mass difference of bottom mesons is $m_{B^*}^2 - m_B^2 = \Delta m_B^2 = 0.49\ \mathrm{GeV}^2$ and one can also take it as a measure for any heavy state matrix element $m_{H_Q^*}^2 - m_{H_Q}^2 = \Delta m_Q^2 \approx \Delta m_B^2$. Then one finds the expression

$$\frac{C_m(\mu)}{2M_Q}\langle H(p_Q)|\mathcal{O}_G|H(p_Q)\rangle = \frac{3}{4}\Delta m_Q^2 \tag{9}$$

Finally one gets the result for the semileptonic inclusive width in the form

$$\Gamma(H_Q \to X_q\nu\ell) = \Gamma_Q|V_{qQ}|^2\left\{C_0\left(1 + \frac{\mu_\pi^2}{2m_Q^2}\right) + \left(-C_v + \frac{C_G - \tilde{C}_G C_0}{C_m}\right)\frac{3\Delta m_Q^2}{8m_Q^2}\right\} \tag{10}$$

The matching procedure is straightforward and consists in computing matrix elements over the partonic states (quarks and gluons on shell) from both sides of the expansion (4). In this way the coefficient function C_0 of the dimension three operator $\bar{h}_v h_v$ determines the total width of the heavy quark and, at the same time, the leading contribution to the width of a Q-flavored heavy hadron. Going to the order α_s, the calculation of the matching for the transition operator T requires to consider three-loop diagrams. The computation of the LO result is well known and requires the calculation of the two-loop Feynman integrals of the simplest topology – the sunset type ones [15]. At the NLO one needs the on-shell tree-loop integrals with massive lines. The computation has been performed in dimensional regularization of both ultraviolet and infrared singularities by using the systems of symbolic manipulations REDUCE [16] and Mathematica [17] with original codes written for the purpose. The reduction to master integrals has been done within the integration-by-parts technique [19]. The original codes have been used for most of the diagram and then the program LiteRed has been used for checking and further application to complicated vertex diagrams [20]. The master integrals have been computed directly and then checked with the program HypExp [21]. The renormalization is performed on-shell by the multiplication of the bare (direct from diagrams) results by the renormalization constant Z_2^{OS}

$$Z_2^{OS} = 1 - C_F \frac{\alpha_s}{4\pi}\left(\frac{3}{\epsilon} + 3\ln\left(\frac{\mu^2}{m_Q^2}\right) + 4\right) \tag{11}$$

with $D = 4 - 2\epsilon$ being the space-time dimension. The renormalization constant Z_2^{OS} depends on both μ and m_Q and it suffices to use it for $\mu = m_Q$.

By using these methods one reproduces the known result

$$C_0 = 1 + \Delta_0^{(0)}(\rho) + C_F \frac{\alpha_s}{\pi}\{\left(\frac{25}{8} - \frac{\pi^2}{2}\right) + \Delta_0^{(1)}(\rho)\} \tag{12}$$

with $C_F = 4/3$, $\rho = m_q^2/m_Q^2$. Here $\Delta_0^{(0)}(\rho)$, $\Delta_0^{(1)}(\rho)$ are corrections due to light quark mass m_q known analytically, and $\Delta_0^{(0)}(0) = \Delta_0^{(1)}(0) = 0$.

The coupling constant is defined in $\overline{\text{MS}}$-scheme $\alpha_s \equiv \alpha_s(\mu)$. At this level the normalization point μ cannot be fixed to any relevant physical scale though and there is no explicit μ dependence of the result but through the coupling constant only.

The coefficient C_v is singled out by taking the matrix element between quarks on shell and one gluon with vanishing momentum and longitudinal polarization, i.e. $A_\mu \sim v_\mu(vA)$. The coefficient C_v reads

$$C_v = 5 + C_F \frac{\alpha_s}{\pi}\left\{-\frac{25}{24} - \frac{\pi^2}{2}\right\} \tag{13}$$

It has no C_A dependence and no μ dependence. This matches also the possibility to compute this coefficient using small momentum expansion near the heavy quark mass shell, $p = m_Q v + v(kv)$. A powerful check of the result is an explicit cancellation of the contribution proportional to the color structure $C_A = N_c$ and the renormalization (cancellation of ϵ-poles) with the same renormalization constant Z_2^{OS}.

The final expression for the coefficient of the chromomagnetic operator multiplied by the Lagrangian factor C_m has the general form

$$C_{fin} = -C_v + (C_G - \tilde{C}_G C_0)/C_m \tag{14}$$

and reads explicitly

$$\begin{aligned} C_{fin} &= -3 + \Delta_G^{(0)}(\rho) + \frac{\alpha_s}{\pi}\Delta_G^{(1)}(\rho) \qquad (15) \\ &+ \frac{\alpha_s}{\pi}\left\{C_A\left(\frac{31}{18} - \frac{\pi^2}{9}\right) + C_F\left(\frac{43}{144} - \frac{19\pi^2}{36}\right)\right\} \end{aligned}$$

The phase space function $\Delta_G^{(0)}(\rho)$ is known analytically for any value of the light quark mass m_q while the function $\Delta_G^{(1)}(\rho)$ enters the numerical computation of ref. [7]. The normalization of these functions is such that $\Delta_G^{(0)}(0) = \Delta_G^{(1)}(0) = 0$. Numerically one gets

$$\begin{aligned} C_{fin} &= -3 + \frac{\alpha_s}{\pi}(0.63C_A - 4.91C_F) \\ &= -3 + \frac{\alpha_s}{\pi}(-4.67) = -3\left(1 + 1.56\frac{\alpha_s}{\pi}\right) \end{aligned} \quad (16)$$

The μ dependence of the coefficient C_G matches the leading order anomalous dimension of the chromo-magnetic operator [22]. The mass parameter of the heavy quark m_Q is chosen to be a pole mass within perturbation theory expansion that is μ independent. Whichever unpleasant features it may have [23], the pole mass is a proper formal parameter for perturbative computations in HQET. After having got the results of perturbation theory computation one is free to change this parameter to any other one finds a preferable one [24].

The coefficients C_0, C_v, C_π, C_G contain the whole information about the expansion. One can combine them to get a preferable representation for the width.

3 Implications for phenomenology

The radiative corrections to the coefficient C_G are of a reasonable magnitude and are well under control for the numerical values of the coupling constant for $\mu \sim m_Q$ for heavy quarks b and c. This provides a clean application of the results to decay into light quarks. For a bottom meson decay, the final light hadron is then the one containing the u quark with a mass of few MeV (e.g. [25]). For a charmed meson decays the final hadrons contain s and d quarks. While the d quark is really light, the mass of the strange quark is about a hundred MeV (e.g. [26]) and can be important at the level of high precision. For the application to a $b \to c$ transition the important question is the magnitude of corrections due to nonvanishing charmed quark mass m_c. It seems that mass corrections are important but still under control. Indeed, the correction $\Delta_G^{(0)}(\rho) = 8\rho + ...$ and $\Delta_G^{(1)}(\rho) = \rho(A + 32\ln(\mu/m_b)) + ...$ at small ρ. For $m_c(3\text{ GeV}) = 0.986(10)\text{GeV}$ [27] and $m_b = 4.8$ GeV (e.g. [28]) one finds the value for rho, $\rho = 0.04$, and then

$$C_{fin} = -3 + \frac{\alpha_s}{\pi}[-4.67 + 0.04(A - 15.0)] \quad (17)$$

To get a feeling of how large the quantity A can be, one can look at an expansion of the leading power term, $\Delta_0^{(1)}(\rho) = -\rho(50 + 24\ln(\mu/m_b)) + ...$ [29] (note a difference due to the C_F factor). Assuming $|A| \leq 50$ one sees that the massless approximation for the coefficient dominates the radiative corrections though the sign of the non-logarithmic constant term can be important for numerical estimates.

The most immediate application of the reported result is the use for extraction of mixing parameters V_{ub}, V_{cb}, V_{cd} from data. At present the precision for V_{ub} is not very high, $|V_{ub}| = (4.41 \pm 0.15 \pm 0.16) \times 10^{-3}$ from the inclusive determination and $|V_{ub}| = (3.23 \pm 0.31) \times 10^{-3}$ from the exclusive ones. The precision of data in inclusive channels is limited by the experiment while the exclusive ones require an accurate knowledge of hadronic form factors. The radiative correction to the chromomagnetic operator as given here is well below the present experimental uncertainties and does not radically change the output of the phenomenological analysis. The very fact that the radiative correction for the theoretical expressinos describing these decays in the standard model is known and under control is important though.

For inclusive semileptonic B meson decays to charm the precision is high enough to worry about the correction. Indeed, $|V_{cb}| = (42.4 \pm 0.9) \times 10^{-3}$ from inclusive determination and $|V_{cb}| = (39.5 \pm 0.8) \times 10^{-3}$ from exclusive ones. The results of two determinations are only marginally consistent and the accurate theoretical formulas are of crucial importance for resolving the issue. We estimate the impact of our correction in a simplified manner. As it is small we only account for charmed quark mass at the leading order approximation with a kinematic function $\Delta_0^{(0)}(\rho) = -8\rho - 12\rho^2 \ln\rho + 8\rho^3 - \rho^4$. Taking a bit different set of parameters $m_c = 1.25$ GeV and $m_b = 4.6$ GeV one finds the change in the extracted numerical value of the mixing angle due to accounting for α_s correction in the coefficient of the chromomagnetic operator

$$V_{cb}^{new} = V_{cb}^{old} \left(1 + 4.67 \frac{\alpha_s}{\pi} \frac{3\Delta m_B^2}{8m_b^2} \frac{1}{2\left[1 + \Delta_0^{(0)}(\rho)\right]} \right) \tag{18}$$

and the shift is about half of a percent $|V_{cb}^{new}/V_{cb}^{old}| - 1 = 0.4\%$ with $\alpha_s/\pi = 0.1$. Assuming the worst scenario of constructive interference and the same size of mass corrections we see that corrections are important but still rather small. The technique that has been developed for the calculation of the total width can be also applied for the computation of differential characteristics for inclusive decays. The most direct application is computation of the moments of hadronic structure function of the transition operator.

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