Instability of magnetic fields in relativistic plasmas driven by neutrino asymmetries

M. S. Dvornikov[a,b] and V. B. Semikoz[a*];

[a] *Pushkov Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation of the Russian Academy of Sciences (IZMIRAN)*
142190, Moscow, Troitsk, Kaluzhskoe high way 4, IZMIRAN,
[b] *Institute of Physics, University of São Paulo,*
CP 66318, CEP 05315-970 São Paulo, SP, Brazil

Abstract

The antisymmetric part of the photon polarization operator ($\sim e_{ijk}k_k\Pi_2$) originated from the parity violation in electroweak plasmas causes the appearance of the Chern-Simons (CS) term in the effective lagrangian for electromagnetic fields $L_{CS} = \Pi_2\mathbf{AB}$ where $\mathbf{B} = \nabla\times\mathbf{A}$ is the magnetic field, $\mathbf{A}$ is the vector potential. The factor Π_2 calculated at the one loop level in an equilibrium medium determines the α-helicity parameter in the Faraday equation, $\alpha = \Pi_2/\sigma_{cond}$, which governs the evolution of a large-scale magnetic field in plasma with the electric conductivity σ_{cond}. In our work the CS term $\Pi_2^{(\nu l)}$ based on the neutrino interactions with charged leptons was calculated. Basing on this calculations, the magnetic field instability driven by neutrino asymmetries was revealed. This instability is implemented in different media such as the hot plasma of the early universe and a supernova (SN) with a seed magnetic field. The causal growth of the cosmological magnetic field governed by this CS term allowed us to establish the *lower* bound on the neutrino asymmetry in the hot plasma of early universe which is consistent with the well-known *upper* bound for relic neutrinos coming from the Big Bang nucleosynthesis constraints on the helium abundance. We suggest also a novel mechanism for generation of strongest magnetic fields in astrophysics such as magnetic fields in magnetars ($B \geq 10^{15}$ G) based on the presence of a non-zero electron neutrino asymmetry during a SN burst, $\Delta n_{\nu_e} \neq 0$.

1 Introduction

The generation of the cosmological magnetic field (CMF) as a seed of observable galactic magnetic fields is still an open problem [1]. The two facts enhanced a new interest to such a problem. The first observational indications of the presence of CMF in intergalactic medium which may survive till the present epoch [2, 3] were as a new incitement to the conception of CMF and its helicity. Secondly, there appeared some new models of the magnetic field instability leading to the generation of CMF. In particular, in the hot universe plasma ($T > 10\,\mathrm{MeV}$) the generation of CMF having a maximum magnetic helicity was based on the quantum chiral (Adler) anomaly in relativistic QED plasma for which the difference of right- and left-chiral electron chemical potentials $\Delta\mu = \mu_\mathrm{R} - \mu_\mathrm{L}$ is not equal to zero, $\Delta\mu \neq 0$ [4].

However, in Ref. [4] it was showed that such an asymmetry diminishes, $\Delta\mu \to 0$, due to the growth of the chirality flip rate in the cooling universe through electron-electron (ee) collisions, $\Gamma_f \sim \alpha_\mathrm{em}^2\,(m_e/3T)^2$, where $\alpha_\mathrm{em} = (137)^{-1}$ is the fine structure constant, m_e is the electron mass, and T is the plasma temperature.

*e-mail: semikoz@yandex.ru

This negative result encouraged the appearance of Ref. [5], where another mechanism for the generation of magnetic fields was proposed. It is based on the parity violation in electroweak plasma resulting in the nonzero Chern-Simons (CS) term Π_2 that enters the antisymmetric part of the photon polarization operator in plasma of massless particles. Such a term , in turn, determines the α -helicity parameter, $\alpha = \Pi_2/\sigma_{cond}$, which governs the evolution of magnetic fields through the Faraday equation for a plasma with electric conductivity σ_{cond}. Here we adopt the notation for the CS term from Ref. [5]. The similar CS term $\Pi_2^{(\nu l)}$ entering the photon self-energy (PSE) given by the neutrino interaction with charged leptons l in plasma was calculated in paper [6] in frames of the real time perturbation theory. While in the present work [7] we calculate $\Pi_2^{(\nu l)}$ using the Matsubara technics and taking into account the radiative corrections to the electron mass in a hot and dense plasmas. Contrary to calculations in Refs. [5] implied for massless particles only we obtain here results for a rare non-relativistic plasma too, $n_e \ll m_e^3$, $T \ll m_e$. This allows us to control our approach versus the limiting case of a $\nu\bar{\nu}$ -gas embedded into vacuum, $n_e \to 0$, for which the Gell-Mann theorem [8] with vanishing PSE in the one loop approximation, $\lim_{k\to 0} \Pi_2^{(\nu)}(k^2) = 0$, is fulfilled.

Another motivation to calculate $\Pi_2^{(\nu e)}$ concerns the problem of strong magnetic fields existing in neutron stars as remnants of supernovae (SN). In particular, we have been interesting here how the strongest magnetic fields observed in magnetars [9] can be generated. To solve this problem, it was recently suggested to use the chiral plasma instability [10, 11] caused by an imbalance between right- and left-handed electrons $\mu_{\mathrm{R}} - \mu_{\mathrm{L}} = 2\mu_5 \neq 0$ arising via the left-handed electron capture by protons inside the SN core (Urca process). Obviously this mechanism is similar to the generation of helical magnetic fields in a hot plasma [4]. The chirality flip in both dense media (cases of a hot plasma $T \gg [m_e, \Delta\mu]$ and a degenerate ultrarelativistic electron gas $\mu_5 \gg [T, m_e]$) leads to the damping $\Delta\mu \to 0$, $\mu_5 \to 0$ due to collisions that should be taken into account for estimates of the magnetic field generation efficiency.

We shall describe the interaction between neutrinos and charged leptons in frames of the Fermi theory which is a good approximation at low energies. Since we study a $\nu\bar{\nu}$ gas embedded into lepton plasma we can treat neutrinos (antineutrinos) as proper combinations of the external neutrino hydrodynamic currents coming from the effective SM Lagrangian for the νl interaction that is linear in the Fermi constant ($\sim G_{\mathrm{F}}$) being averaged over the neutrino ensemble. Thus, our approach is analogous to the generalized Furry representation in quantum electrodynamics.

2 Photon polarization operator in a $\nu\bar{\nu}$ gas

In this section we calculate the parity violating term in the polarization tensor in the presence of a $\nu\bar{\nu}$ gas. It should be noted that photons do not interact directly with neutrinos since latter particles are neutral. Thus the $\nu\gamma$ interaction should be mediated by charged leptons, denoted as l, which are taken to be virtual particles in this section. We shall take into account the νl interaction in propagators of l's as the external mean fields $f^{\mu}_{\mathrm{L,R}} = (f^0_{\mathrm{L,R}}, \mathbf{f}_{\mathrm{L,R}})$.

We shall be mainly interested in the case of *an isotropic* $\nu\bar{\nu}$ gas when $\mathbf{f}_{\mathrm{L}} = \mathbf{f}_{\mathrm{R}} = 0$ and the nonzero $f^0_{\mathrm{L,R}}$ are given in eq. (4). In this situation the most general expression for the polarization tensor reads [12]

$$\Pi_{\mu\nu}(k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\Pi_{\mathrm{T}} + \frac{k_\mu k_\nu}{k^2}\Pi_{\mathrm{L}} + \mathrm{i}\varepsilon_{\mu\nu\alpha\beta}k^\alpha (f^\beta_{\mathrm{L}} - f^\beta_{\mathrm{R}})\Pi_{\mathrm{P}}, \tag{1}$$

where $k^\mu = (k^0, \mathbf{k})$ is the photon momentum, $g_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$ is the Minkowski metric tensor, $\varepsilon_{\mu\nu\alpha\beta}$ is the absolute antisymmetric tensor having $\varepsilon^{0123} = +1$, and $\Pi_{\mathrm{T,L,P}}$ are the form factors of a photon. Since we study parity violating effects, we should analyze the form factor $\Pi_2 = (f^0_{\mathrm{L}} - f^0_{\mathrm{R}})\Pi_{\mathrm{P}}$. Since only real particles, considered in this section, are neutrinos, we add the superscript "ν" to photon form factors, e.g., $\Pi_2 \to \Pi_2^{(\nu)}$ etc.

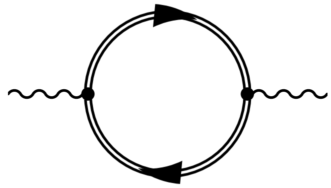

Figure 1: The Feynman diagram for the one loop contribution to PSE given in eq. (2). The lepton's propagators are shown as broad straight lines and correspond to eq. (3).

The one loop contribution to PSE is schematically shown in figure 1.

The lepton propagators are represented as broad lines since we take into account $f^{\mu}_{\mathrm{L,R}}$ in our calculations. Note that we shall consider only the contribution to $\Pi^{(\nu)}_{\mu\nu}$ linear in the external fields $f^{\mu}_{\mathrm{L,R}}$ and proportional to γ_5 -matrix to find the parity violation term $\Pi^{(\nu)}_{2}$. The expression for $\Pi^{(\nu)}_{\mu\nu}$, which leads to the nonzero $\Pi^{(\nu)}_{\mathrm{P}}$ in eq. (1), reads

$$\Pi^{(\nu)}_{\mu\nu} = \mathrm{i}e^2 \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \mathrm{tr}\left\{\gamma_\mu S_0(p+k)\gamma_\nu S_1(p) + \gamma_\nu S_0(p)\gamma_\mu S_1(p+k)\right\}, \tag{2}$$

where e is the electric charge of l and the propagators $S_{0,1}(p)$ are given by

$$\begin{aligned} S_0 =& \frac{\gamma^\mu P_\mu + m}{P^2 - m^2}, \\ S_1 =& \frac{1}{P^2 - m^2}\left[\frac{\mathrm{i}\sigma_{\alpha\beta}\gamma^5 P^\alpha (f^\beta_{\mathrm{L}} - f^\beta_{\mathrm{R}})(\gamma^\mu P_\mu + m)}{P^2 - m^2} + \frac{1}{2}\gamma_\mu \gamma^5 (f^\mu_{\mathrm{L}} - f^\mu_{\mathrm{R}})\right]. \end{aligned} \tag{3}$$

Here we should take into account that $m^2 \to m^2 - \mathrm{i}0$ in the denominators. Then for a $\nu\bar{\nu}$ -gas at rest, $\mathbf{f}_{L,R} = 0$, and temporal components given by

$$f^0_{\mathrm{L}} = 2\sqrt{2}G_{\mathrm{F}}\left[\Delta n_{\nu_e} + (\sin^2\theta_{\mathrm{W}} - 1/2)\sum_\alpha \Delta n_{\nu_\alpha}\right], \quad f^0_{\mathrm{R}} = 2\sqrt{2}G_{\mathrm{F}}\sin^2\theta_{\mathrm{W}}\sum_\alpha \Delta n_{\nu_\alpha}. \tag{4}$$

one finds from Eq. (2) after separation of the factor $\varepsilon_{\mu\nu\alpha\beta}k^\alpha$ the CS term for $\nu\bar{nu}$ gas in vacuum:

$$\Pi^{(\nu)}_{2} = -(f^0_{\mathrm{L}} - f^0_{\mathrm{R}})\frac{e^2}{4\pi^2}\frac{k^2}{m^2}\int_0^1 \mathrm{d}x \frac{x(1-x)}{1 - \frac{k^2}{m^2}x(1-x)}. \tag{5}$$

It should be noted that eq. (5) does not contain ultraviolet divergencies.

Basing on eq. (5) we find that $\Pi^{(\nu)}_{2} = 0$ at $k^2 = 0$. We also note that $\Pi^{(\nu)}_{2}$ in eq. (5) coincides with the result of ref. [12], where the more fundamental Weinberg-Salam theory was used. Moreover, the fact that $\Pi^{(\nu)}_{2}$ vanishes at $k^2 = 0$ also agrees with the finding of ref. [8], where it was shown that the neutrino-photon interaction is absent in the lowest order in the Fermi constant. Nevertheless, as demonstrated in ref. [13], the amplitude for $\nu\gamma \to \nu\gamma$ has the nonzero value in the two loops approximation.

3 Plasma contribution to polarization tensor

In this section we obtain the general expression for Π_2 taking into account both the temperature and the chemical potential of the charged leptons. It means that these leptons now are not virtual particles. On the basis of the general results we discuss the cases of low temperature and low density classical plasma, as well as hot relativistic and degenerate relativistic plasmas.

If we study the photon propagation in a plasma of charged leptons with nonzero temperature and density, the photon's dispersion relation differs from the vacuum one, $k^2 = (k_0^2 - \mathbf{k}^2) \neq 0$. As seen in eq. (5), in this case $\Pi_2^{(\nu)} \neq 0$. We can define it as $\Pi_2^{(\nu l)}$ analogously to the previous section. The presence of ν's and $\bar{\nu}$'s is essential since it is these particles which provide the nonzero contribution to the parity violating form factor based on the νl interaction.

The expression for the contribution to PSE from the plasma consisting of real leptons can be obtained if we make the following replacement in eq. (2):

$$\mathrm{i} \int \frac{\mathrm{d}p^0}{2\pi} \to T \sum_n, \quad p^0 = (2n+1)\pi T \mathrm{i} + \mu, \quad n = 0, \pm 1, \pm 2, \dots, \tag{6}$$

where T and μ are the temperature and the chemical potential of the l's plasma. In principle, we can discuss a general situation when T and μ are different from T_{ν_α} and μ_{ν_α}. However, in section , where we study the application of our calculations, the system in the thermodynamic equilibrium is considered. Thus, in the following we shall suppose that $T = T_\nu$, where T_ν is the $\nu\bar{\nu}$ gas temperature equal for all neutrino flavors. However, we shall keep different μ and μ_{ν_α}.

Let us first discuss the case of a low density plasma of l's, that corresponds to $k^2 \ll m^2$.

Using the general (cumbersome) eqs. (3.2) and (3.3) from our [7] in the limit $\max(k_0^2, \mathbf{k}^2) \ll m^2$ we obtain that

$$\begin{aligned}
\Pi_2^{(\nu l)} = & -\frac{7}{6} e^2 (f_\mathrm{L}^0 - f_\mathrm{R}^0) \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{1}{\mathcal{E}_\mathbf{p}^3} \\
& \times \left\{ \frac{m^2}{\mathcal{E}_\mathbf{p}^2} \left[\frac{1}{\exp[\beta(\mathcal{E}_\mathbf{p} - \mu)] + 1} + \frac{1}{\exp[\beta(\mathcal{E}_\mathbf{p} + \mu)] + 1} \right] \right. \\
& + \frac{m^2 \beta}{2\mathcal{E}_\mathbf{p}} \left[\frac{1}{\cosh[\beta(\mathcal{E}_\mathbf{p} - \mu)] + 1} + \frac{1}{\cosh[\beta(\mathcal{E}_\mathbf{p} + \mu)] + 1} \right] \\
& \left. - \frac{\beta^2 \mathbf{p}^2}{6} \left[\frac{\tanh[\beta(\mathcal{E}_\mathbf{p} - \mu)/2]}{\cosh[\beta(\mathcal{E}_\mathbf{p} - \mu)] + 1} + \frac{\tanh[\beta(\mathcal{E}_\mathbf{p} + \mu)/2]}{\cosh[\beta(\mathcal{E}_\mathbf{p} + \mu)] + 1} \right] \right\},
\end{aligned} \tag{7}$$

where $\mathcal{E}_\mathbf{p} = \sqrt{\mathbf{p}^2 + m^2}$, $\beta = 1/T$ is the reciprocal temperature . Note that $\Pi_2^{(\nu l)}$ in eq. (7) exactly accounts for T and μ.

To estimate the values of $\Pi_2^{(\nu)}$ and $\Pi_2^{(\nu l)}$, we shall consider the low temperature limit: $T \ll m$. We will identify l with an electron and assume that the electron gas has a classical Maxwell distribution. For this medium we get that $k^2 = 4\pi\alpha_\mathrm{em} n_e / m$, where $\alpha_\mathrm{em} = e^2/4\pi$ is the fine structure constant and n_e is the background electron density. Moreover for a classical electron gas one has that $\mu = m + T \ln \left[\frac{n_e}{g_s} \left(\frac{2\pi}{mT} \right)^{3/2} \right]$, where $g_s = 2$ is the number of spin degrees of freedom of an electron. Using eqs. (5) and (7) , we get that

$$\tilde{\Pi}_2^{(\nu)} = -\frac{2\alpha_\mathrm{em}^2}{3} (f_\mathrm{L}^0 - f_\mathrm{R}^0) \frac{n_e}{m^3}, \quad \tilde{\Pi}_2^{(\nu l)} = -\frac{7\pi\alpha_\mathrm{em}}{3} (f_\mathrm{L}^0 - f_\mathrm{R}^0) \frac{n_e}{m^3}, \tag{8}$$

where we add a tilde over $\Pi_2^{(\nu, \nu l)}$ to stress that these quantities correspond to real rather than virtual photons. In the following we shall omit the tilde in order not to encumber notations. One can see that $\Pi_2^{(\nu l)}$ in eq. (8) is $\frac{7\pi}{2\alpha_\mathrm{em}} \sim 10^3$ times greater than $\Pi_2^{(\nu)}$. Note that for a classical nonrelativistic plasma, corresponding to $m \gg \max(|\mathbf{p}|, T)$, the integrals in last two lines in eq. (7) cancel each other while the integral in the first line leads to the term $\Pi_2^{(\nu l)}$ in eq. (8).

Now let us turn to the cases of ultrarelativistic plasmas with a plenty neutrinos (antineutrinos) such as the hot plasma of early universe and a supernova. It should be noted that the electron's mass in such plasmas can significantly differ from its vacuum value. The radiative corrections to the electron's mass were studied in ref. [14]. Thus, if we consider a dense and hot

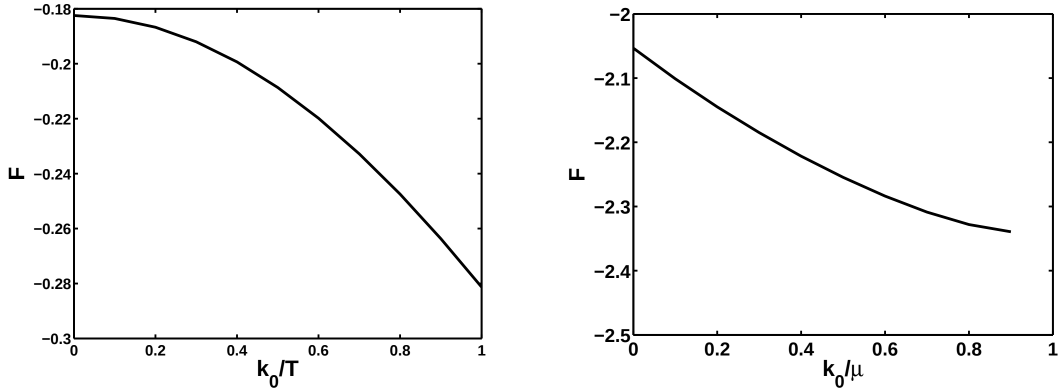

Figure 2: The function F versus k_0. (a) Hot relativistic plasma. (b) Degenerate relativistic plasma.

plasma, we should replace [1]

$$m^2 \to m^2_{\rm eff} = \frac{e^2}{8\pi^2}(\mu^2 + \pi^2 T^2). \tag{9}$$

Note that eq. (9) is valid for both $T \gg \mu$ and $\mu \gg T$. Accounting for the dispersion relation for the transverse long waves $k^2 = \omega_p^2$, $\mid \mathbf{k} \mid \ll k_0$, and the expression for the plasma frequency $\omega_p = \sqrt{4\pi\alpha_{em}}T/3$, we get the important inequality $k^2 < 4m^2_{\rm eff}$ in a hot relativistic plasma that saves us against a peculiarity in the denominator within the integral in eq. (5) when we substitute the effective mass (9). Omitting very cumbersome calculations in the Matsubara technics presented in our paper [7] we give here only the answer for the formfactor $\Pi_2(k_0, 0)$

$$\Pi_2 = \frac{\alpha_{\rm em}}{\pi}(f^0_{\rm L} - f^0_{\rm R})F, \tag{10}$$

where F is the dimensionless function which depends on k_0/T. Note that Π_2 in eq. (10) includes the contribution from eq. (5). Accounting for eq. (9), we present the behaviour of F versus we k_0/T in figure 2(a).

One can see in figure 2(a) that for a hot relativistic plasma the CS term Π_2 is nonvanishing in the static limit: $F(k_0 \to 0) \approx -0.18$. We should substitute such a limit to apply Π_2 for large-scale magnetic fields penetrating a plasma deeply, since the skin layer (penetration depth) rises as $\lambda_{skin} \sim (k_0)^{-1/2} \to \infty$. This limit results in the constant α -helicity parameter we apply below, $\alpha = \Pi_2(0)/\sigma_{cond}$.

4 Instability of magnetic fields in relativistic plasmas driven by neutrino asymmetries

We consider below the two cases for which the CS term Π_2 in the photon polarization operator $\Pi_{\mu\nu}$ plays a crucial role. A nonzero Π_2 leads to the α-dynamo amplification (instability) of a seed magnetic field even without fluid vortices or any rotation Ω in plasma which are usually exploited in the standard MHD approach for $\alpha\Omega$-dynamo [16]. The first case considered here concerns the magnetic field growth in a degenerate ultrarelativistic electron plasma, $\mu \gg \max(T, m_e)$, during the collapse and deleptonization phases of a supernova burst. In the second case we consider below a hot plasma of the early universe with the temperatures $T \gg \max(m_e, \mu)$ before the

[1] Under intermediate conditions $m_e \sim m_{\rm eff}$ in plasma with $\mu \neq 0$ or $T \neq 0$ (or both) the effective mass of an electron should be $m_e/2 + (m_e^2/4 + m^2_{\rm eff})^{1/2}$, see in ref. [15].

neutrino decoupling at $T > T_{\rm dec} \simeq 2 \div 3\,{\rm MeV}$. In both cases neutrinos are in equilibrium with a plasma environment. For these applications we use our result in eq. (10).

First, we derive in subsection 4.1 the Faraday equation generalized in SM to find the key parameters leading to the **B**-field instability. An excess of electron neutrinos during a first second of a supernova explosion[2] allow us to put $n_{\nu_e} - n_{\bar{\nu}_e} \neq 0$ in the problem of the magnetic field amplification considered in subsection 4.2. In subsection 4.3 we find the lower bound on the neutrino asymmetry providing the growth of CMF field in our causal scenario. It would be interesting to compare such limit with the upper bound on the electron neutrino-antineutrino asymmetry $|\xi_{\nu_e}| \leq 0.07$ given by the Big Bang nucleosynthesis (BBN) constraint [17]. Thus, we shall consider magnetic fields in media with a plenty of neutrinos (antineutrinos) where a nonzero neutrino asymmetry exists. Finally, in section 5 we compare our issues with what other authors found in similar problems and give some forecasting how to explain strongest magnetic fields in magnetars.

4.1 Generalized Faraday equation in the Standard Model

The existence of a neutrino asymmetry accounting for the difference in eq. (4),

$$f_{\rm L}^0 - f_{\rm R}^0 = G_{\rm F}\sqrt{2}\left[\Delta n_{\nu_e} - \frac{1}{2}\sum_\alpha \Delta n_{\nu_\alpha}\right], \tag{11}$$

leads to a non-zero parity violation term in the photon polarization operator $\Pi_{ij}(\omega, \mathbf{k}) = {\rm i}\varepsilon_{ijn}k^n\Pi_2(\omega, k)$, where Π_2 is given by eq. (10) and $\omega \equiv k_0$.

The CS polarization term in eq. (10) corresponds to the induced pseudovector current in the Fourier representation,

$$\mathbf{j}_5(\omega, \mathbf{k}) = \Pi_2(\omega, k)\mathbf{B}(\omega, \mathbf{k}), \tag{12}$$

entering the generalized Maxwell equation in the standard model (SM)

$${\rm i}\mathbf{k} \times \mathbf{B}(\omega, \mathbf{k}) + {\rm i}\omega\mathbf{E}(\omega, \mathbf{k}) = \mathbf{j}(\omega, \mathbf{k}) + \mathbf{j}_5(\omega, \mathbf{k}). \tag{13}$$

Expressing the ohmic current as $\mathbf{j}(\omega, \mathbf{k}) = \sigma_{\rm cond}\mathbf{E}(\omega, \mathbf{k})$, then neglecting the displacement current in the l.h.s. of eq. (13), that is a standard assumption in the MHD approach for which $\omega \ll \sigma_{\rm cond}$ [3], and finally using the Bianchi identity $\mathbf{k} \times \mathbf{E} = \omega\mathbf{B}$, one gets the generalized Faraday equation in SM in the coordinate representation,

$$\frac{\partial \mathbf{B}}{\partial t} = \alpha\nabla \times \mathbf{B} + \eta\nabla^2\mathbf{B}, \tag{14}$$

where α is the magnetic helicity parameter,

$$\alpha = \left(\frac{\Pi_2}{\sigma_{\rm cond}}\right), \tag{15}$$

and $\eta = (\sigma_{\rm cond})^{-1}$ is the magnetic diffusion coefficent.

Here we use the long-wave approximation for large-scale magnetic fields where the operator $\Pi_2(k_0, k = 0)$ is at least uniform, $k \to 0$, and almost stationary since the function $F(x)$ depends on a small ratio $x = k_0/T \ll 1$ or $x = k_0/\mu \ll 1$. For instance, in the long-wave limit $k \ll \omega_t$ the transversal plasmons (photons) have the spectrum $k_0^2 \equiv \omega_t^2 = \omega_p^2 + \mathbf{k}^2 \approx \omega_p^2 = 4\pi\alpha_{\rm em}T^2/9$ in a

[2] Neutrino emission prevails over the antineutrino one during first milliseconds of a supernova burst due to the reaction $e^- + p \to n + \nu_e$ (urca-process) before its equilibrium with beta-decays $n \to p + e^- + \bar{\nu}_e$ is settled in.

[3] The conductivity $\sigma_{\rm cond} = \omega_p^2/\nu_{\rm coll} = 4\pi\alpha_{\rm em}T^2/9\nu_{\rm coll} \sim T/\alpha_{\rm em} \sim 100T$ depends on the Coulomb collision frequency $\nu_{\rm coll} = \sigma_{\rm Coul}n_e = [4\pi L\alpha_{\rm em}^2/9T^2]n_e \sim \alpha_{\rm em}^2 T$. Here we use the values for the electron density $n_e = 0.183T^3$ in a hot plasma and $L \sim 10$ for the Coulomb logarithm. Obviously the MHD condition $\omega = \omega_t \ll \sigma_{\rm cond}$ is fulfilled to obtain eq. (14).

hot plasma ($T \gg \max[\mu, m_e]$) and $k_0^2 \equiv \omega_t^2 = \omega_p^2 + \mathbf{k}^2 \approx \omega_p^2 = 4\alpha_{\rm em}\mu^2/3\pi$ in the ultrarelativistic degenerate electron gas ($\mu \gg \max[m_e, T]$) [15]. In a relativistic plasma this approximation corresponds to the negligible spatial dispersion, $k_0 \gg k\langle v\rangle \sim k$, where we put $v = 1$ both in hot and degenerate relativistic plasmas. Here $k = |\mathbf{k}|$ is the wave number. Thus, the ratio $k_0/T \sim 0.1$ or $k_0/\mu \sim 0.06$ allows us to consider $\Pi_2 \approx$ const without temporal and spatial dispersion as a function of the temperature T (a hot plasma in the early universe) or the chemical potential μ (a degenerate electron gas in a supernova) only. Moreover, the skin layer width (penetration depth) for a quasistatic magnetic field, $\lambda_{skin} \sim (k_0)^{-1/2}$, motivates us to put limit $k_0 \to 0$.

4.2 Amplification of a seed magnetic field in a supernova

During the collapse (time $t < 0.1\,$s after onset of collapse) one can neglect $\nu_{\mu,\tau}$ emission and Π_2 reads

$$\Pi_2(k_0, 0) = \left[\frac{\sqrt{2}\alpha_{\rm em}G_{\rm F}(n_{\nu_e} - n_{\bar{\nu}_e})}{2\pi}\right] F\left(k_0/\mu\right), \tag{16}$$

where the function $F(x)$ is shown in figure 2(b) for a degenerate ultrarelativistic electron gas with $\mu \gg \max(T, m_e)$.

The magnetic diffusion time $t_{\rm diff} = \Lambda^2/\eta$ seen from the Faraday eq. (14),

$$t_{\rm diff} = \frac{\sigma_{\rm cond}}{k^2} = \frac{\sigma_{\rm cond}}{\Pi_2^2}, \tag{17}$$

is given by the electrical conductivity for degenerate ultrarelativistic electrons and degenerate nonrelativistic protons, $\sigma_{\rm cond} = \omega_p^2/\nu_{\rm coll}$. Note that the combined effects of the degeneracy and the shielding reduce the collision frequency $\nu_{\rm coll} \sim T^2$. Thus collisions of charged particles are blocked due to the Pauli principle since states $p < p_{\rm F}$ are busy and $\nu_{\rm coll} \to 0$ at $T \to 0$.

The electrical conductivity was found in ref. [18],

$$\sigma_{\rm cond} = \frac{1.6 \times 10^{28}}{(T/10^8\,{\rm K})^2}\left(\frac{n_e}{10^{36}\,{\rm cm}^{-3}}\right)^{3/2}\,{\rm s}^{-1}. \tag{18}$$

For typical $p_{{\rm F}_e} = 200\,$MeV and the corresponding electron density $n_e = p_{{\rm F}_e}^3/3\pi^2 = 3.7 \times 10^{37}\,{\rm cm}^{-3}$, as well as the temperature $T = 10\,$MeV $\simeq 10^{11}\,$K in the SN core eq. (18) gives $\sigma_{\rm cond} = 2250\,$MeV. This result leads to the estimate $t_{\rm diff} = 0.023\,$s. It means that any seed magnetic field B_0 existing in plasma does not dissipate ohmically during first milliseconds after onset of collapse, $t \ll t_{\rm diff}$, and evolves for a given wave number k through the α-dynamo driven by neutrino asymmetries as

$$B(t, k) = B_0 \exp\left[\int_{t_0}^{t}(|\alpha|k - \eta k^2){\rm d}t'\right]. \tag{19}$$

If $k < |\alpha|/\eta = |\Pi_2|$, the seed magnetic field in eq. (19) will grow exponentially. The fastest growth corresponds to the α^2-dynamo with $k = |\alpha|/2\eta$ for which $B(t) = B_0 \times \exp\left\{\int_{t_0}^{t}[\alpha^2(t')/4\eta(t')]{\rm d}t'\right\}$.

Unfortunately, under the same conditions (for large $n_{\nu_e} = 1.9 \times 10^{37}\,{\rm cm}^{-3}$) the scale of the magnetic field occurs to be rather small, $\Lambda = k^{-1} \simeq \eta/|\alpha| = |\Pi_2|^{-1} \sim 0.25 \times 10^{-2}\,$cm. Here we use the fact that $|F| = 2$, see figure 2(b). However, such a scale grows when the neutrino asymmetry diminishes due to a significant involvement of antineutrinos somewhere later at $t \leq 0.03 - 0.1\,$s, $\Delta n_{\nu_e} = n_{\nu_e} - n_{\bar{\nu}_e} \to 0$. It reaches the core radius $\Lambda \to R_0 = 10\,$km, $k = |\Pi_2| = R_0^{-1}$, for the neutrino asymmetry density

$$n_{\nu_e} - n_{\bar{\nu}_e} = \frac{2\pi}{R_0\alpha_{\rm em}G_{\rm F}\sqrt{2}|F|} \simeq 10^{28}\,{\rm cm}^{-3}. \tag{20}$$

The suggested mechanism of the B-field growth in a supernova driven by the electron neutrino asymmetry could lead to an additional amplification of a strong seed magnetic field ($B_0 = 10^{10} \div 10^{12}\ G$) during the first second of a SN explosion when the asymmetry $n_{\nu_e} - n_{\bar{\nu}_e} \neq 0$ remains appreciable. Here a strong seed magnetic field can arise from a small magnetic field of protostar, e.g., $B_{\text{proto}} \sim 1 \div 10^2\,\text{G}$, due to the conservation of the magnetic field flux, $B_0 = B_{\text{proto}}(R_{\text{proto}}/R_0)^2$, during the protostar collapse. The question whether this new mechanism can explain the strongest magnetic field of observed magnetars ($B = 10^{14} \div 10^{15}$ G) deserves a separate study (see also in section 5).

4.3 Growth of primordial magnetic fields provided by the lower bound on neutrino asymmetries

In a hot plasma of the early universe the magnetic helicity parameter α in Faraday eq. (14) reads as

$$\alpha(T) = \frac{\Pi_2(T)}{\sigma_{\text{cond}}(T)} = \frac{\alpha_{\text{em}} G_{\text{F}} \sqrt{2} T^2 F(k_0/T)}{12\pi\sigma_c} \left[\xi_{\nu_e} - \xi_{\nu_\mu} - \xi_{\nu_\tau}\right], \tag{21}$$

where we substituted the dimensionless neutrino asymmetries $\xi_{\nu_\alpha} = \mu_{\nu_\alpha}/T$ for the asymmetry densities $\Delta n_{\nu_\alpha} = \xi_{\nu_\alpha} T^3/6$ and used the hot plasma conductivity $\sigma_{\text{cond}} = \sigma_c T$, with $\sigma_c \simeq 100$. The magnetic field evolution with the parameter α in eq. (21) obeys the *causal* scenario, where the magnetic field scale is less than the horizon, $\Lambda_{\text{B}} \simeq \eta/|\alpha| < l_{\text{H}} = H^{-1}$, if the sum of neutrino asymmetries $-2\sum_\alpha c_\alpha^{(\text{A})} \xi_\alpha = \xi_{\nu_e} - \xi_{\nu_\mu} - \xi_{\nu_\tau}$ satisfies the inequality

$$|\xi_{\nu_e} - \xi_{\nu_\mu} - \xi_{\nu_\tau}| > \frac{2.2 \times 10^{-6} \sqrt{g^*/106.75}}{(T/\text{MeV})}. \tag{22}$$

Here we take into account that $c^{(\text{A})} = \mp 0.5$ (upper sign stays for electron neutrinos) is the SM axial coupling constant for νe interaction corresponding to the difference $f_{\text{L}}^0 - f_{\text{R}}^0$ in eq. (11). In eq. (21) we use that $|F| \simeq 0.2$, which results from figure 2(a). Moreover we account for that $l_{\text{H}} = M_0/T^2$, with $M_0 = M_{\text{Pl}}/1.66\sqrt{g^*}$, where $M_{\text{Pl}} = 1.2 \times 10^{19}\,\text{GeV}$ is the Plank mass, $g^* = 106.75$ is the number of relativistic degrees of freedom above the QCD phase transition, $T > T_{\text{QCD}} \simeq 150\,\text{MeV}$. Let us remind that to get eq. (22) we applied the photon polarization term in eq. (10) for ultrarelativistic leptons with $T \gg \max(m_e, \mu)$.

One can see that the inequality in eq. (22) does not contradict to the well-known BBN bounds on the neutrino asymmetries at the lepton stage of the universe expansion corresponding to $g^* = 10.75$, $|\xi_{\nu_\alpha}| < 0.07$, (see ref. [17]) and gives an additive (lower) bound on the neutrino asymmetry which supports the growth of CMF in our causal scenario.. Here different flavors equilibrate due to neutrino oscillations before BBN, $\xi_{\nu_e} \sim \xi_{\nu_\mu} \sim \xi_{\nu_\tau}$, somewhere at the neutrino decoupling time $T = 2 - 3\,\text{MeV}$, accounting for all active neutrino flavors with the non-zero mixing angles (including $\sin^2\theta_{13} = 0.04$), see in ref. [19].

We also obtain that the magnetic field diffusion time t_{diff} is bigger than the expansion time $\sim H^{-1}$, $t_{\text{diff}} = \sigma_{\text{cond}}/\Pi_2^2 > M_0/T^2$, or ohmic losses are not danger, if the opposite inequality for neutrino asymmetries is valid,

$$|\xi_{\nu_e} - \xi_{\nu_\mu} - \xi_{\nu_\tau}| < \frac{25.3 \times (g^*/106.75)^{1/4}}{[T/\text{GeV}]^{3/2}}. \tag{23}$$

Here just after the electroweak phase transition $T \leq T_{\text{EW}} = 100\,\text{GeV}$ the combined asymmetry in eq. (23) seems to be resonable, $|\xi_{\nu_e} - \xi_{\nu_\mu} - \xi_{\nu_\tau}| < 0.025$, while at lower temperatures $m_e \ll T \leq \mathcal{O}(\text{GeV})$ the condition in eq. (23) is obviously fulfilled and consistent with the BBN bound obtained in ref. [17].

5 Discussion

Let us give a comparison with the chiral magnetic mechanism in refs. [4, 10, 11]. In ref. [4], the magnetic helicity coefficient analogous to that in eq. (21) in our work,

$$\alpha(T) = \frac{\alpha_{\rm em}\Delta\mu(T)}{\pi\sigma_{\rm cond}(T)}, \tag{24}$$

is proportional to the magnetic chiral parameter $\Delta\mu = \mu_{e_{\rm L}} - \mu_{e_{\rm R}}$ where $\mu_{e_{\rm L}}$ ($\mu_{e_{\rm R}}$) are the left (right) electron chemical potentials. In QED plasma such a parameter arises due to the Adler anomaly in external electromagnetic fields, $\partial(j^\mu_{\rm L} - j^\mu_{\rm R})/\partial x^\mu = (2\alpha/\pi)\mathbf{E}\cdot\mathbf{B}$, evolving in a self-consistent way with the magnetic field $\mathbf{B}$. However, it tends to a small value $\Delta\mu/T \sim 10^{-6}$–10^{-7} for a small wave number $10^{-10} \leq k/T \leq 3\times10^{-9}$ at temperatures $T \geq 10\,\mathrm{MeV}$ (see figure F.1 in ref. [4]) and vanishes later at all due to the chirality flip with the increasing rate $\Gamma_f \sim (m_e^2/T^2)$ in cooling universe, $n_{e_{\rm L}} - n_{e_{\rm R}} \to 0$. This is not the case for the helicity parameter given in eq. (21) based on neutrino asymmetries ξ_{ν_α} for which there are no triangle anomalies in Maxwellian fields contrary to charged leptons [4]. Moreover, after the neutrino decoupling and relic neutrino oscillations before BBN, there are no ways to change the equivalent asymmetries $\xi_{\nu_\alpha} = \mathrm{const} \neq 0$, $\alpha = e, \mu, \tau$.

In ref. [11] one suggests a new mechanism for the production of strong magnetic fields in magnetars based on the chiral instability for electrons with the difference of chemical potentials for right- and left-handed electrons, $\mu_5 = \mu_{\rm R} - \mu_{\rm L} \neq 0$. The chirality imbalance of electrons is produced via the same electron capture inside a core we considered above (urca-process), $p + e^-_{\rm L} \to n + \nu^e_{\rm L}$, where the subscript L stands for left-handedness.

In ref. [11] the typical scales of wave number k and vector potential A relevant to such instability were obtained (see eq. (27) there):

$$k \sim \alpha_{\rm em}\mu_5, \quad |\mathbf{A}| \sim \frac{\mu_5}{\alpha}, \quad \text{thus} \quad B \sim k|\mathbf{A}| \sim \mu_5^2, \tag{25}$$

where an estimate $\mu_5 = 200\,\mathrm{MeV}$ gives huge $B_{\rm max} \sim \mu_5^2 \sim 10^{18}\ G$. The authors also show that instability proceeds faster than the danger chirality flip, $\Gamma_{\rm inst} = \alpha^2\mu_5 \gg \Gamma_{\rm flip} \sim \alpha^2(m_e/\mu_5)^2\mu_5$, or the process $\mu_5 \to 0$ due to collisions is negligible because $\mu_5 \gg m_e$. In the proposed mechanism of the magnetic field amplification it remains unclear how a large-scale magnetic field is produced in this scenario since the magnetic field generated seems to be microscopic. Indeed, the scale $k^{-1} \sim \mathrm{MeV}^{-1}$ found in ref. [11] is much smaller than we obtained in section 4.2.

To resume we have shown here that weak interactions in SM are very important for generation of magnetic fields in dense electroweak plasmas.

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