Neutrino photoproduction on electron in dense magnetized medium

M. V. Chistyakov*, A. V. Kuznetsov†, N. V. Mikheev‡, D. A. Rumyantsev§, D. M. Shlenev

Yaroslavl State P. G. Demidov University
Sovietskaya 14, 150000 Yaroslavl, Russian Federation

Abstract

The effect of a strongly magnetized cold plasma on the Compton-like photoproduction of a neutrino-antineutrino pair on an electron, $\gamma e \to e \nu \bar{\nu}$, has been considered. The contribution of this process to the neutrino emissivity in both the non-resonance and the resonance cases has been calculated with taking account of the photon dispersion properties in medium. Our results show that the neutrino emissivity owing to the $\gamma e \to e \nu \bar{\nu}$ reaction is significantly modified as compared to the previously reported data.

1 Introduction

The problem of a correct description of effects of an active external medium (a strong magnetic field and/or dense plasma) on quantum processes is of current interest during a long time, for a review see e.g. [1]. First, these effects are caused by the sensitivity of charged fermions (primarily, electrons as particles with the largest specific charge) to the field. Second, a strongly magnetized plasma significantly changes the dispersion properties of photons and, thereby, the kinematics of the processes.

The conditions of the strongly magnetized plasma can exist in the interiors of magnetars, i.e., isolated neutron stars with magnetic fields much higher than the critical value $B_e = m^2/e \simeq 4.41 \times 10^{13}$ G [1]. Recent observations (see [2] and the papers cited therein) make it possible, in particular, to identify some astrophysical objects (SGR and AXP) as magnetars.

At the same time, all known theoretical models of the internal structure of neutron stars give the parameters of the medium (density and temperature) at which the magnetized plasma is transparent to neutrinos. In this case, reactions with a neutrino-antineutrino pair in the final state are decisive for neutrino cooling. In this connection, the neutrino photoproduction (the so-called photoneutrino process) $\gamma e \to e \nu \bar{\nu}$ was studied by different authors [3–6]. The formulas were obtained for the neutrino emissivity, i.e., the energy carried by the neutrino from the unit volume of a star per unit time, for both the nonrelativistic and relativistic plasmas. However, in those papers the anisotropy in the dispersion of photons was disregarded, which can change the results essentially. Furthermore, the expressions for the neutrino emissivity owing to the Compton-like process $\gamma e \to e \nu \bar{\nu}$ in nonrelativistic and relativistic plasmas contain some inaccuracies [4–6].

In this work, we study in detail the Compton-like photoproduction of the neutrino-antineutrino pair $\gamma e \to e \nu \bar{\nu}$ and the neutrino emissivity owing to this process with an accurate inclusion

*e-mail: mch@uniyar.ac.ru
†e-mail: avkuzn@uniyar.ac.ru
‡Deceased
§e-mail: rda@uniyar.ac.ru

[1] We use the natural units $c = \hbar = k_{\mathrm{B}} = 1$, m is the electron mass, and $e > 0$ is the elementary charge.

of the dispersion properties of photons in dense magnetized medium in both resonance and non-resonance cases.

2 Photon dispersion in the magnetized medium

The propagation of electromagnetic radiation in any active medium is conveniently described in terms of normal modes (eigenmodes). In turn, the polarization and dispersion properties of normal modes are connected with the eigenvectors and the eigenvalues of the polarization operator $\mathcal{P}_{\alpha\beta}$, respectively. It is convenient to decompose the tensor $\mathcal{P}_{\alpha\beta}$ in terms of the basis of 4-vectors [7] constructed of the electromagnetic field tensor reduced to a dimensionless form, and the 4-momentum of a photon $q^\alpha = (\omega, \mathbf{k})$:

$$
\begin{aligned}
b^{(1)}_\mu &= (\varphi q)_\mu, \qquad b^{(2)}_\mu = (\tilde{\varphi} q)_\mu, \\
b^{(3)}_\mu &= q^2 \, (\Lambda q)_\mu - q_\mu \, q_\perp^2, \qquad b^{(4)}_\mu = q_\mu,
\end{aligned}
\tag{1}
$$

which are the eigenvectors of the polarization operator in a static uniform magnetic field. In this case, $(b^{(1)}b^{(1)}) = -q_\perp^2$, $(b^{(2)}b^{(2)}) = -q_\parallel^2$, $(b^{(3)}b^{(3)}) = -q^2 q_\parallel^2 q_\perp^2$, $(b^{(4)}b^{(4)}) = q^2$.

Hereafter we use the following notations: the 4-vectors with the indices $\perp$ and $\parallel$ belong to the Euclidean $\{1,\, 2\}$ subspace and the Minkowski $\{0,\, 3\}$ subspace correspondingly. Then for arbitrary 4-vectors A_μ, B_μ one has

$$
\begin{aligned}
& A_\perp^\mu = (0, A_1, A_2, 0), \quad A_\parallel^\mu = (A_0, 0, 0, A_3), \\
& (AB)_\perp = (A\Lambda B) = A_1 B_1 + A_2 B_2, \\
& (AB)_\parallel = (A\widetilde{\Lambda} B) = A_0 B_0 - A_3 B_3,
\end{aligned}
$$

where the matrices $\Lambda_{\mu\nu} = (\varphi\varphi)_{\mu\nu}$, $\widetilde{\Lambda}_{\mu\nu} = (\tilde{\varphi}\tilde{\varphi})_{\mu\nu}$ are constructed with the dimensionless tensor of the external magnetic field, $\varphi_{\mu\nu} = F_{\mu\nu}/B$, and the dual tensor, $\tilde{\varphi}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\varphi^{\rho\sigma}$. The matrices $\Lambda_{\mu\nu}$ and $\widetilde{\Lambda}_{\mu\nu}$ are connected by the relation $\widetilde{\Lambda}_{\mu\nu} - \Lambda_{\mu\nu} = g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$, and play the role of the metric tensors in the perpendicular ($\perp$) and the parallel ($\parallel$) subspaces respectively.

We emphasize that, in contrast to the magnetic field, the value $\mathcal{E}_e = B_e$ for the electric field is limiting because the generation of the nearly critical electric field in a macroscopic region of space would result in the intense production of electron-positron pairs from vacuum. At the same time, if the electric field is perpendicular to the magnetic field, the electric field $\mathcal{E}_e$ can exceed the critical value B_e, remaining below B. However, even in this case, it is always possible to perform a Lorentz transformation to a reference frame where only the magnetic field exists. This statement can be generalized to the case where the plasma with a temperature T and a chemical potential μ moves as a whole along the magnetic field. To this end, it is sufficient to represent the electron distribution function in an explicit Lorentz-invariant form in terms of the 4-velocity of the medium u_α:

$$
f_e(p) = \frac{1}{1 + \exp\left[((pu)_\parallel - \mu)/T\right]}\,, \quad (pu)_\parallel = E u_0 - p_z u_z\,, \quad E = \sqrt{p_z^2 + m^2}\,. \tag{2}
$$

The condition of the absence of the electric field in this frame can be written in the relativistically covariant form $(u\Lambda)_\mu = 0$. Consequently, when the plasma moves as a whole along the magnetic field, it is possible to consider the situation where only the magnetic field exists.

In this case we can obtain the following expansion of $\mathcal{P}_{\alpha\beta}$ over the eigenvectors $r^{(\lambda)}_\alpha$ with the eigenvalues $\varkappa^{(\lambda)}$ [9–12]:

$$
\mathcal{P}_{\alpha\beta} = \sum_{\lambda=1}^{3} \varkappa^{(\lambda)} \, \frac{r^{(\lambda)}_\alpha (r^{(\lambda)}_\beta)^*}{(r^{(\lambda)})^2}\,, \quad r^{(\lambda)}_\beta = \sum_{i=1}^{3} A^{(\lambda)}_i \, b^{(i)}_\beta\,, \tag{3}
$$

where $A_i^{(\lambda)}$ are some complex coefficients. As we can see from Eq. (3), it is rather difficult to determine the dispersion properties of photons for all three polarizations, and it is true even in the strongly magnetized plasma approximation, $\beta \gg m^2$, μ^2, T^2 (hereafter $\beta = eB$). However, as the analysis shows (see, e.g., [4,13]), in the case $\beta \gg m^2$, where electrons occupy the ground Landau level, only photons with the polarization corresponding to the vector $r_\alpha^{(2)} \simeq b_\alpha^{(2)}$ will provide the leading contributions to the amplitude of the $\gamma e \to e \nu \bar{\nu}$ process in the external field. In the cold plasma approximation, $\omega, T \ll \mu - m$, the expression for the eigenvalue $\varkappa^{(2)}$ can be obtained analytically and represented in the form

$$\varkappa^{(2)} \simeq \frac{\omega_p^2 \, q_\parallel^2}{\omega^2 - v_{\rm F}^2 k_z^2}\,, \qquad v_{\rm F} = \frac{p_{\rm F}}{\mu} = \sqrt{1 - \frac{m^2}{\mu^2}}\,, \tag{4}$$

where $\omega_p^2 = (2\alpha\beta/\pi)\, v_{\rm F}$ is the so-called plasma frequency squared, $p_{\rm F}$ is the Fermi momentum.

In this case, the solution of the dispersion equation

$$q^2 - \varkappa^{(2)} = 0 \tag{5}$$

for a photon of the mode 2 propagating at a nonzero angle θ to the magnetic field direction can be found analytically as the function $\omega = \omega(\mathbf{k})$ taking the form

$$\begin{aligned} \omega &= \left\{ \frac{1}{2} \left[k^2 (1 + v_{\rm F}^2 \cos^2\theta) + \omega_p^2 \right] \right. \\ &+ \left. \frac{1}{2} \sqrt{\left[k^2 (1 - v_{\rm F}^2 \cos^2\theta) + \omega_p^2 \right]^2 - 4 \omega_p^2 (1 - v_{\rm F}^2) k^2 \cos^2\theta} \right\}^{1/2} . \end{aligned} \tag{6}$$

In the case of a nonrelativistic plasma, where $v_{\rm F} \ll 1$, we obtain

$$\omega \simeq \left\{ \frac{1}{2} \left[k^2 + \omega_p^2 \right] + \frac{1}{2} \sqrt{\left[k^2 + \omega_p^2 \right]^2 - 4 \omega_p^2 k^2 \cos^2\theta} \right\}^{1/2} . \tag{7}$$

The corresponding expression for the relativistic plasma, where $v_{\rm F} \to 1$, has the form

$$\omega \simeq \sqrt{k^2 + \omega_p^2}\,. \tag{8}$$

One more question which appears to be important in some cases, is the renormalization of the wavefunction of a photon. We note that this renormalization becomes insignificant in the case of a cold plasma because the main contribution to the physically observed characteristics (e.g., emittance) comes from the photon energy range $\omega \ll m$.

3 Neutrino emissivity in the non-resonance case

Our main goal is to obtain the expression for the neutrino emissivity caused by the process $\gamma e \to e \nu \bar{\nu}$. In turn, the neutrino emissivity can be defined as the zero component of the four-vector of the energy-momentum carried away by the neutrino pair due to this process from a unit volume of plasma per unit time. Here, we neglect the inverse effect of the energy and momentum loss on the state of plasma. In the conditions of a strongly magnetized plasma the neutrino emissivity can be represented in the form [8]:

$$\begin{aligned} Q_{\gamma e \to e \nu \bar{\nu}} &= \frac{1}{L_x} \int \frac{\mathrm{d}^3 k}{(2\pi)^3\, 2\omega}\, f_\gamma(\omega)\, \frac{\mathrm{d}^2 p}{(2\pi)^2\, 2E}\, f_e(E)\, \frac{\mathrm{d}^2 p'}{(2\pi)^2\, 2E'} \left[1 - f_e(E') \right] \\ &\times \frac{\mathrm{d}^3 p_1}{(2\pi)^3\, 2\varepsilon_1} \frac{\mathrm{d}^3 p_2}{(2\pi)^3\, 2\varepsilon_2}\, q_0' \, (2\pi)^3\, \delta^3 (P - p' - q') |\mathcal{M}_{\gamma e \to e \nu \bar{\nu}}|^2 , \end{aligned} \tag{9}$$

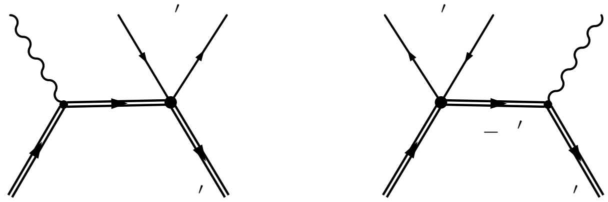

Figure 1: The Feynman diagrams for the reaction $\gamma e \to e \nu \bar{\nu}$. Double lines mean that the effects of an external field on the initial and final electron states and on the electron propagator are exactly taken into account.

where $f_\gamma(\omega) = \left[\mathrm{e}^{\omega/T} - 1\right]^{-1}$ is the equilibrium distribution function of an initial photon with the four-vector $q^\mu = (\omega, \mathbf{k})$, $f_e(E)$ and $f_e(E')$ are the equilibrium distribution functions of initial and final electrons, respectively, in the plasma rest frame $f_e(E) = \left[\mathrm{e}^{(E-\mu)/T} + 1\right]^{-1}$ (see Eq. (2)); $q_0' = \varepsilon_1 + \varepsilon_2$ is the neutrino pair energy, $\varepsilon_{1,2} = |\mathbf{p}_{1,2}|$; $\mathrm{d}^2 p = \mathrm{d}p_y \mathrm{d}p_z$; $V = L_x L_y L_z$ is the plasma volume, $P_\mu = (p+q)_\mu$.

In calculating the amplitude $\mathcal{M}_{\gamma e \to e \nu \bar{\nu}}$ of the process $\gamma e \to e \nu \bar{\nu}$, we consider the case of relatively small momentum transfers compared with the W boson mass, $|q'^2| \ll m_W^2$. Then the corresponding interaction Lagrangian can be written as follows:

$$\mathcal{L} = \frac{G_\mathrm{F}}{\sqrt{2}} \left[\bar{\Psi} \gamma^\alpha (C_V - C_A \gamma_5) \Psi\right] j'_\alpha + e(\bar{\Psi} \gamma^\alpha \Psi) \, A_\alpha \,, \tag{10}$$

where $j'_\alpha = \bar{\nu} \gamma_\alpha (1 - \gamma_5) \nu$ is the current of left-handed neutrinos, A_α is the four-potential of the photon field, $C_V = \pm 1/2 + 2 \sin^2 \theta_W$, $C_A = \pm 1/2$, θ_W is the Weinberg angle. Here, the upper sign corresponds to electron neutrinos ($\nu = \nu_e$), when there is an exchange reaction both of W and Z bosons. The lower sign corresponds to the μ and τ neutrinos, when there is only Z boson exchange.

The amplitude of the process $\gamma e \to e \nu \bar{\nu}$ in the tree approximation is described by the Feynman diagrams shown in Fig. 1 and in the non-resonance case (i.e. when the virtual electron is on the ground Landau level) has the form [4]

$$\mathcal{M}_{\gamma e \to e \nu \bar{\nu}} = 2\sqrt{2}\, e \, G_\mathrm{F} \, m \, \left[C_V \, (q' \tilde{\varphi} j') - C_A \, (q' \tilde{\varphi} \tilde{\varphi} j')\right] \frac{\sqrt{q_\parallel^2 (|Q_\parallel^2| + 4m^2)}}{(qq')_\parallel^2 - \varkappa^2 (q \tilde{\varphi} q')^2} \,, \tag{11}$$

where $q'^\mu = (p_1 + p_2)^\mu$ is the total 4-momentum of the neutrino-antineutrino pair, $\varkappa = \sqrt{1 - 4m^2/Q_\parallel^2}$, $Q^\mu = (q - q')^\mu$.

The resulting expression for the emissivity of the photoneutrino process can be significantly simplified in two limiting cases.

(i) In the case of the nonrelativistic plasma, $\mu \sim m$, and at an arbitrary relation between the plasma frequency and the temperature, the emissivity can be presented in the form

$$Q_{\gamma e \to e \nu \bar{\nu}} \simeq Q_{nr} \, F \left(\frac{\omega_p}{T}\right) \,, \tag{12}$$

where

$$Q_{nr} = \frac{8\pi^2 \alpha \, G_\mathrm{F}^2 \, \beta \, T^9}{4725 \, m \, p_\mathrm{F}} \left[\overline{C_V^2} + \overline{C_A^2}\right] \simeq 1.3 \times 10^6 B_{15}^2 \, \rho_6^{-1} \, T_8^9 \left(\frac{\mathrm{erg}}{\mathrm{cm}^3 \ \mathrm{s}}\right) \tag{13}$$

is the emissivity in the limit $\omega_p \ll T$ [3]. In Eq. (13) $B_{15} = B/(10^{15}\,\mathrm{G})$, $\rho_6 = \rho/(10^6\,\mathrm{g/cm}^3)$, $T_8 = T/(10^8\,\mathrm{K})$, and the constants $\overline{C_V^2} = 0.93$ and $\overline{C_A^2} = 0.75$ are the results of summation over all channels of the neutrino production of the types ν_e, ν_μ, ν_τ.

The function $F(\omega_p/T)$ depending on the ratio of the plasma frequency to the temperature can be represented in the form of the single integral

$$F(y) = \frac{15}{8\pi^8} \int_y^\infty \frac{\mathrm{d}x\, x^5}{\mathrm{e}^x - 1} \left(x^2 - \frac{y^2}{5} \right) . \tag{14}$$

This integral can be approximated by the formula

$$F(y) \simeq \frac{3\mathrm{e}^{-y}}{4\pi^8} \left(2y^7 + 15y^6 + 95y^5 + 495y^4 + 2040y^3 + 6240y^2 + 12600y + 12600 \right) . \tag{15}$$

The plot of the function $F(y)$ is shown in Fig. 2. We note that the numerical analysis of the integral (14) as compared to the approximation (15) gives a discrepancy no more than 0.5%.

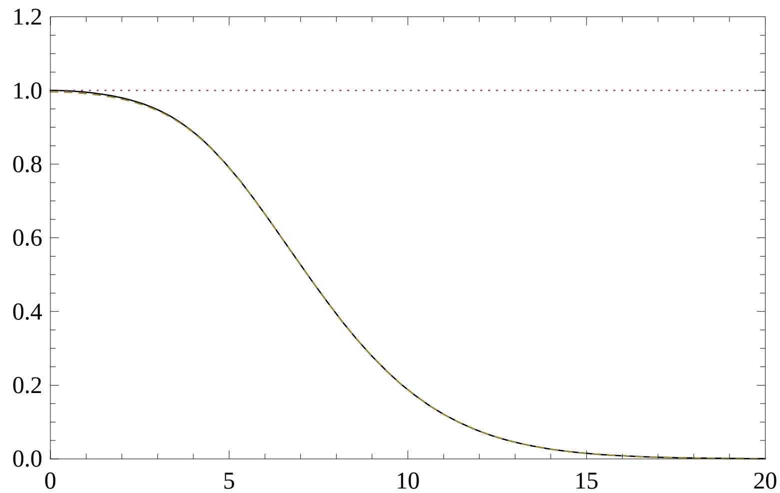

Figure 2: The function $F(y)$ defined by Eq. (14). The horizontal dotted straight line corresponds to the value $F(0) = 1$.

Our results (12)–(15) for the case of the nonrelativistic plasma should be compared with the results of the previous calculations, see Eq. (56) of Ref. [3], Eq. (32) of Ref. [4] and Eq. (32) of Ref. [6]. There was a discrepancy between those formulas in the numerical factor of $\pi/2$. Now we should confess that the factor of $\pi/2$ was lost in Eq. (32) of Ref. [4]. However, it should be noted that all those formulas, with the indicated correction, were obtained in the limit $\omega_p \ll T$, i.e. when the approximation for the function (14) was taken: $F(y) \to F(0) = 1$. As one can see from Fig. 2, this approximation is valid for the case $\omega_p \lesssim T$, but it would give an essentially overestimated result in the case $\omega_p > T$. The results (12)–(15) obtained can be used for an arbitrary relation between the plasma frequency and the temperature.

(ii) In the case of relativistic plasma, $\mu \gg m$, and at an arbitrary relation between the plasma frequency and the temperature, the emissivity can be presented in the form

$$Q_{\gamma e \to e \nu \bar{\nu}} \simeq Q_r R \left(\frac{\omega_p}{2T} \right) , \tag{16}$$

where

$$Q_r = \frac{G_\mathrm{F}^2 \, \alpha \, (\overline{C_V^2} + \overline{C_A^2})}{576 \, (2\pi)^{11/2}} \frac{B}{B_e} \left(\frac{m}{\mu} \right)^6 \omega_p^{15/2} \, T^{3/2} \, \mathrm{e}^{-\omega_p/T} \tag{17}$$

is the emissivity in the limit $\omega_p \gg T$ [5, 6].

The function $R(z)$ can be represented in the form of the double integral

$$R(z) = \frac{3z^{3/2}}{5\sqrt{\pi}}\, \mathrm{e}^{2z} \int\limits_0^\infty \mathrm{d}v v^6 \mathrm{e}^{-zv} \int\limits_0^1 \frac{\mathrm{d}t t^4 [1-(v-vt)^{-2}]}{1-\mathrm{e}^{-z[v(1-t)+(v-vt)^{-1}]}} \times \tag{18}$$
$$\times \frac{vt-(v-vt)^{-1}}{1-\mathrm{e}^{-z[vt-(v-vt)^{-1}]}}\, [vt - 5(v-vt)^{-1}]\,,$$

which can be approximated well by the formula

$$R(z) \simeq 1 + \frac{0.7627}{z^{1/2}} + \frac{66.875}{z^{3/2}} + \frac{271.654}{z^{5/2}} + \frac{2509.36}{z^{7/2}} + \frac{6754.62}{z^{9/2}} +$$
$$+ \frac{16612.9}{z^{11/2}} + \frac{19843.8}{z^{13/2}} + \frac{10188.5}{z^{15/2}}\,. \tag{19}$$

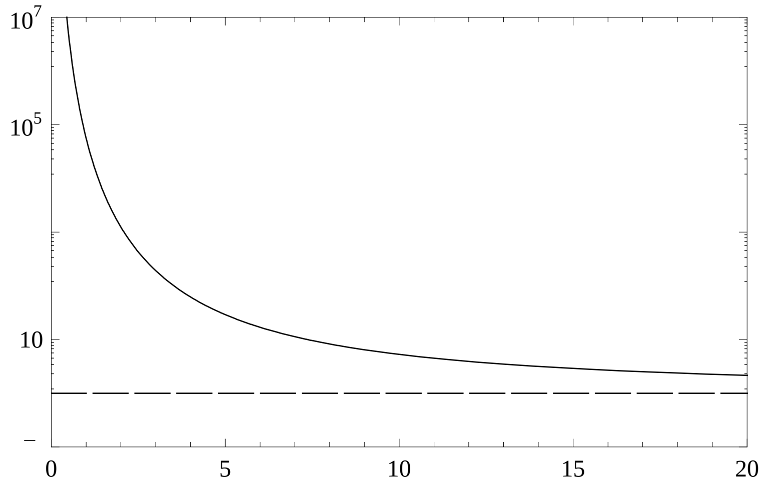

Figure 3: The function $R(z)$ defined by Eq. (18). The horizontal dashed straight line corresponds to the asymptotic value $R(\infty) = 1$.

The plot of the function $R(z)$ is shown in Fig. 3. We note that the numerical analysis of the integral (18) as compared to the approximation (19) gives a discrepancy no more than 0.8%. Therefore, the approximation (19) can be used for applications at an arbitrary ratio ω_p/T.

According to Fig. 3 and approximate formula (19), for the realistic parameters of the field and plasma of neutron stars, $B \simeq 50\, B_e$, $T \simeq 10^8$ K [8], the factor is $R(14) \simeq 3$ and the asymptotic expression $R(\infty) \simeq 1$ is inapplicable. Furthermore, it follows from Eq. (16) that the numerical estimate presented in Ref. [4] is strongly overstated. At the same time, for the parameters, used in Ref. [5]: $B \simeq 10^{16}\,$G, $\rho \simeq 10^9\,$g/cm^3, $T \simeq 10^9\,$K, we obtain from Eq. (17): $Q_r \simeq 10^{11}\,$erg/(cm$^3\,$s), but for the same parameters one has $R(2.8) \simeq 260$. Thus, the estimate obtained in Ref. [5] is understated by several orders of magnitude. However, for the realistic parameters of the inner crust of neutron stars (see above) we obtain from Eq. (16): $Q_{\gamma e \to e\nu\bar\nu} \simeq 6 \times 10^{-7}\,$erg/(cm$^3\,$s). It is reasonable to neglect the neutrino photoproduction, $\gamma e \to e\nu\bar\nu$, in the models of cooling of neutron stars at the matter density $\rho \simeq 10^9\,$g/cm^3. But at the same time, the resonance on the virtual electron in the process $\gamma e \to e\nu\bar\nu$ becomes possible at the same density. Because of this, the value $Q_{\gamma e \to e\nu\bar\nu}$ would increase (see the next section).

4 Resonance in the process $\gamma e \to e\nu\bar{\nu}$

On the boundary between the outer and the inner crust of a magnetar (where the electron density is estimated as $\rho \gtrsim 10^9\,\mathrm{g/cm^3}$), the higher Landau levels for a virtual electron begin to be excited. In this case the denominator of the electron propagator $P_\parallel^2 - m^2 - 2eBn$ can be equal to zero and the resonance on the virtual electron become possible. In addition, the virtual electron resonance occurs only in the s-channel diagram (the first diagram in Fig. 1). On the other hand, to accurately take into account the resonance behavior in the process $\gamma e \to e\nu\bar{\nu}$, it is necessary to calculate radiative corrections to the electron mass, caused by the combined action of a magnetic field and plasma. This calculation is a separate challenge. However, because of the smallness of these corrections, we can approximately replace $m^2 \to m^2 - \mathrm{i}P_0\Gamma_n$ in the denominator of the electron propagator, such that

$$\frac{1}{P_\parallel^2 - m^2 - 2\beta n} \to \frac{1}{P_\parallel^2 - m^2 - 2\beta n + \mathrm{i}P_0\Gamma_n}\,. \tag{20}$$

In this case, the main contribution to the amplitude arises from the resonance region, so that we can approximately replace the corresponding part of the amplitude (see [14]) by the δ function:

$$\begin{aligned} |\mathcal{M}_{\gamma e\to e\nu\bar{\nu}}|^2 &\simeq \sum_{n=1}^{\infty} \frac{|\mathcal{R}_n|^2}{(P_\parallel^2 - m^2 - 2\beta n)^2 + P_0^2\Gamma_n^2} \\ &\simeq \sum_{n=1}^{\infty} \frac{\pi}{P_0\Gamma_n}\,\delta(P_\parallel^2 - m^2 - 2\beta n)\,|\mathcal{R}_n|^2\,, \end{aligned} \tag{21}$$

where Γ_n is the total width of the change of the electron state. This width can be represented in the form [15]

$$\Gamma_n = \Gamma^{abs} + \Gamma^{cr} \simeq \Gamma^{cr}_{e_0\gamma\to e_n}\left[1 + \mathrm{e}^{(E_n'' - \mu)/T}\right]. \tag{22}$$

Here

$$\begin{aligned} \Gamma^{cr}_{e_0\gamma\to e_n} &= \frac{1}{2E_n''}\int \frac{\mathrm{d}^3k}{2\omega(2\pi)^3}\,f_\gamma(\omega)\,\frac{\mathrm{d}^2p}{2E(2\pi)^2}\,f_e(E) \\ &\times (2\pi)^3\,\delta^3(P - p'')\,|\mathcal{M}_{e_0\gamma\to e_n}|^2 \end{aligned} \tag{23}$$

is the width of the electron creation in the nth Landau level.

On the other hand, in the case of resonance the expression for $|R_n|^2$ being averaged over the photon polarizations can be factored in the strong field limit $\beta \gg m^2$ as follows (see, for example, [16]):

$$|\mathcal{R}_n|^2 = |\mathcal{M}_{e_0\gamma\to e_n}|^2\,|\mathcal{M}_{e_n\to e_0\nu\bar{\nu}}|^2\,, \tag{24}$$

where

$$|\mathcal{M}_{e_0\gamma\to e_n}|^2 = \frac{8\pi\alpha}{n!}\,\exp\left(-\frac{q_\perp^2}{2\beta}\right)\left(\frac{q_\perp^2}{2\beta}\right)^n M_n^2\,(p\tilde{\Lambda}q)\sum_{\lambda=1}^{3}\left[\left|A_1^{(\lambda)}\right|^2 + \left|A_2^{(\lambda)} + \sigma A_3^{(\lambda)}\right|^2\right] \tag{25}$$

is the amplitude of the absorption of a photon in the process $e_0\gamma \to e_n$, when an electron passes from the ground Landau level to a higher Landau level n. The parameter $\sigma = (p\tilde{\varphi}q)/(p\tilde{\Lambda}q) = \pm 1$ determines the direction of the photon propagation with respect to the magnetic field direction. The values $A_i^{(\lambda)}$ are introduced in Eq. (3). Finally, the amplitude squared of the electron transition from the nth Landau level to the ground level with the creation of the neutrino-antineutrino pair takes the form:

$$
\begin{aligned}
|\mathcal{M}_{e_n \to e_0 \nu\bar\nu}|^2 &= \frac{G_F^2}{n!} \exp\left(-\frac{q_\perp'^2}{2\beta}\right)\left(\frac{q_\perp'^2}{2\beta}\right)^n \frac{M_n^2}{(p'\tilde\Lambda q')}\Bigg\{ \Big| C_V (p'\tilde\Lambda j') - C_A (p'\tilde\varphi j')\Big|^2 \\
&+ \frac{(j'\tilde\Lambda j'^*)}{q_\perp'^2}\Big[C_V(p'\tilde\Lambda q') - C_A(p'\tilde\varphi q')\Big]^2 - \frac{2}{q_\perp'^2}\Big[C_V(p'\tilde\Lambda q') - C_A(p'\tilde\varphi q')\Big] \\
&\times \; Re\Big((q'\tilde\Lambda j')\Big[C_V(p'\tilde\Lambda j'^*) - C_A(p'\tilde\varphi j'^*)\Big]\Big)\Bigg\}.
\end{aligned} \tag{26}
$$

With taking account of Eq. (22), the amplitude squared of the process $\gamma e \to e\nu\bar\nu$ takes the form:

$$
\begin{aligned}
|\mathcal{M}_{\gamma e \to e\nu\bar\nu}|^2 &= \sum_{n=1}^{\infty} \int \frac{d^2 p''}{(2\pi)^2\, 2E_n''}\, (2\pi)^3\, \delta^3(P - p'')\, \frac{|\mathcal{R}_n|^2}{2E_n''\,\Gamma_n} \\
&= \sum_{n=1}^{\infty} \int \frac{d^2 p''}{(2\pi)^2\, 2E_n''}\, f_e(E_n'')\, (2\pi)^3\, \delta^3(P - p'')\, \frac{|\mathcal{R}_n|^2}{2E_n''\,\Gamma^{cr}_{e_0\gamma \to e_n}}\,.
\end{aligned} \tag{27}
$$

Here we have used the property of the δ function:

$$
\delta(P_\parallel^2 - m^2 - 2\beta n) = \frac{1}{2E_n''}\,\delta(P_0 - E_n'')\,, \tag{28}
$$

where $E_n'' = \sqrt{p_z''^2 + m^2 + 2\beta n}$.

Substituting Eq. (27) into the expression for the luminosity (9), and taking into account Eqs. (23) and (24), we obtain:

$$
Q_{\gamma e_0 \to e_0 \nu\bar\nu} = \sum_{n=1}^{\infty} Q_{e_n \to e_0 \nu\bar\nu}\,, \tag{29}
$$

where

$$
\begin{aligned}
Q_{e_n \to e_0 \nu\bar\nu} &= \frac{1}{L_x} \int \frac{d^2 p''}{(2\pi)^2\, 2E_n''}\, f_e(E_n'')\, \frac{d^2 p'}{(2\pi)^2\, 2E'}\, \big[1 - f_e(E')\big]\, \frac{d^3 p_1}{(2\pi)^3\, 2\varepsilon_1}\, \frac{d^3 p_2}{(2\pi)^3\, 2\varepsilon_2} \\
&\times \; q_0'\, (2\pi)^3\, \delta^3(p'' - p' - q')\, |\mathcal{M}_{e_n \to e_0 \nu\bar\nu}|^2\,,
\end{aligned} \tag{30}
$$

is the neutrino luminosity due to the process $e_n \to e_0\nu\bar\nu$ [8].

5 Conclusion

In conclusion, let us summarize some of our results. We have considered the neutrino photoproduction on an electron, $e\gamma \to e\nu\bar\nu$, in dense magnetized medium in both resonance and non-resonance cases. The changes of the photon dispersion properties in a magnetized medium are investigated. It has been shown, that taking into account of the photon dispersion anisotropy in the limit of non-relativistic plasma leads to substantial modification of the neutrino emissivity due to the process $\gamma e \to e\nu\bar\nu$, if compared with the previously obtained results. We have obtained the most general expression for the neutrino emissivity due to the process $\gamma e \to e\nu\bar\nu$ in relativistic and non-relativistic plasma at an arbitrary relation between the plasma frequency and the temperature. It has been shown that the result known in the literature for the contribution of the photoneutrino process to the neutrino emissivity in the limit of a relativistic plasma was understated by several orders of magnitude. It has been shown that in the case of resonance on the virtual electron, the neutrino emissivity due to the process $\gamma e_0 \to e_0\nu\bar\nu$ can be expressed in terms of the neutrino emissivity due to the process $e_n \to e_0\nu\bar\nu$.

Acknowledgements

This work was started together with Nickolay Vladimirovich Mikheev, who passed away on June 19, 2014. The paper is dedicated to the blessed memory of our teacher, colleague, and friend.

We are grateful to A. A. Gvozdev, and I. S. Ognev for useful discussions and valuable remarks. A. K. and D. R. express their deep gratitude to the organizers of the Seminar "Quarks-2014" for warm hospitality.

The study was performed with the support by the Project No. 92 within the base part of the State Assignment for the Yaroslavl University Scientific Research, and was supported in part by the Russian Foundation for Basic Research (Project No. 14-02-00233-a).

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