Neutrino production of electron-positron pairs at excited Landau levels in a strong magnetic field

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Abstract

The process of neutrino production of electron positron pairs in a magnetic field of arbitrary strength, where electrons and positrons can be created in the states corresponding to excited Landau levels, is analysed. The mean value of the neutrino energy loss due to the process $\nu \to \nu e^- e^+$ is calculated. The result can be applied for calculating the efficiency of the electron-positron plasma production by neutrinos in the conditions of the Kerr black hole accretion disc considered by experts as the most possible source of a short cosmological gamma burst. The presented research can be also useful for further development of the calculation technic for an analysis of quantum processes in external active medium, and in part in the conditions of moderately strong magnetic field, when taking account of the ground Landau level appears to be insufficient.

1 Introduction

An intense electromagnetic field makes possible the processes which are forbidden in a vacuum such as the neutrino production of an electron–positron pair $\nu \to \nu e^- e^+$. The list of papers devoted to an analysis of this process and the collection of the results obtained could be found e.g. in Ref. [1]. In most cases, calculations of this kind were made either in the crossed field approximation, or in the limit of a superstrong field much greater than the critical value of $B_e = m_e^2/e \simeq 4.41 \times 10^{13}$ G, when the electrons and positrons were born in states corresponding to the ground Landau level. However, there are physical situations of the so-called moderately strong magnetic field[1], $p_\perp^2 \gtrsim eB \gg m_e^2$, when electrons and positrons mainly occupy the ground Landau level, however, a noticeable fraction may be produced at the next levels.

The indicated hierarchy of physical parameters corresponds to the conditions of the Kerr black hole accretion disk, regarded by experts as the most likely source of a short cosmological gamma-ray burst. The disc is a source of copious neutrinos and anti-neutrinos, which partially annihilate above the disc and turn into $e^\mp$ pairs, $\nu\bar{\nu} \to e^- e^+$. This process was proposed and investigated in many details, see e.g. Ref. [2] and the papers cited therein, as a possible mechanism for creating relativistic, $e^\mp$-dominated jets that could power observed gamma-ray bursts. In Ref. [3], in addition to $\nu\bar{\nu}$ annihilation, the contribution of the magnetic field-induced process $\nu \to \nu e^- e^+$ to the neutrino energy deposition rate around the black hole was also included.

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[1] We use natural units $c = \hbar = k_{\rm B} = 1$, m_e is the electron mass, and e is the elementary charge.

However, in calculations of the efficiency of the electron-positron plasma production by neutrino through the process $\nu \to \nu e^- e^+$ in those physical conditions [3] (B to 180 B_e, E to 25 MeV), it should be kept in mind that approximations of both the crossed and superstrong field have a limited applicability here. We know a limited number of papers [4–6], where the probability of neutrino-electron processes was investigated, as the sum over the Landau levels of electrons (positrons). In the papers [4, 5], only the neutrino-electron scattering channel in a dense magnetized plasma was studied, which was the crossed process to the considered here neutrino creation of electron-positron pairs. In the paper [6], also devoted to the study of the process $\nu \to \nu e^- e^+$, the analytical calculations were presented in a rather cumbersome form, caused by the choice of solutions of the Dirac equation. The final results for the process probability were obtained by numerical calculations for some set of Landau levels occupied by electrons and positrons. In astrophysical applications, there exists probably more interesting value than the process probability, namely, the mean value of the neutrino energy loss, caused by the influence of an external magnetic field.

Thus, the aim of this paper is the study of the process $\nu \to \nu e^- e^+$ in the physical conditions of the moderately strong magnetic field, where the electrons and positrons would be born in the states corresponding to the excited Landau levels, and the theoretical description would contain a relatively simple analytical formulas for the mean value of the neutrino energy loss, for a wide range of Landau levels. More details of the analysis can be found in our recent paper [7].

2 The probability of the process $\nu \to \nu e^- e^+$

We use the standard calculation technics, see e.g. Ref. [1]. The effective local Lagrangian of the process can be written in the form

$$\mathcal{L} = -\frac{G_{\mathrm{F}}}{\sqrt{2}} \left[\bar{e}\gamma_\alpha (C_V - C_A \gamma_5) e\right] \left[\bar{\nu}\gamma^\alpha (1-\gamma_5)\nu\right] , \tag{1}$$

where the electron field operators are constructed on a base of the solutions of the Dirac equation in the presence of an external magnetic field. The constants C_V and C_A for different neutrino types are:

$$\begin{aligned} C_V^{(e)} &= +\frac{1}{2} + 2\sin^2\theta_{\mathrm{W}} , \qquad & C_A^{(e)} &= +\frac{1}{2} , \\ C_V^{(\mu,\tau)} &= -\frac{1}{2} + 2\sin^2\theta_{\mathrm{W}} , \qquad & C_A^{(\mu,\tau)} &= -\frac{1}{2} , \end{aligned} \tag{2}$$

where θ_{W} is the Weinberg angle.

The total probability of the process $\nu \to \nu e^-_{(n)} e^+_{(\ell)}$ where the electron and the positron are created in the states corresponding to nth and ℓth Landau levels correspondingly, is, in a general case, the sum of the probabilities of the four polarization channels:

$$W_{n\ell} = W_{n\ell}^{--} + W_{n\ell}^{-+} + W_{n\ell}^{+-} + W_{n\ell}^{++} . \tag{3}$$

For each of the channels, the differential probability over the final neutrino momentum per unit time can be written as

$$\mathrm{d}W_{n\ell}^{ss'} = \frac{1}{\mathcal{T}} \frac{\mathrm{d}^3 P' \, V}{(2\pi)^3} \int |\mathcal{S}_{n\ell}^{ss'}|^2 \, \mathrm{d}\Gamma_{e^-} \, \mathrm{d}\Gamma_{e^+} , \tag{4}$$

where $\mathcal{T}$ is the total interaction time, $V = L_x L_y L_z$ is the total volume of the interaction region, $\mathcal{S}_{n\ell}^{ss'}$ is the S-matrix element constructed with the effective Lagrangian (1), and the elements of the phase volume are introduced for the electron and the positron (the magnetic field is directed along the z axis):

$$\mathrm{d}\Gamma_{e^-} = \frac{\mathrm{d}^2 p \, L_y L_z}{(2\pi)^2}, \quad \mathrm{d}\Gamma_{e^+} = \frac{\mathrm{d}^2 p' \, L_y L_z}{(2\pi)^2} . \tag{5}$$

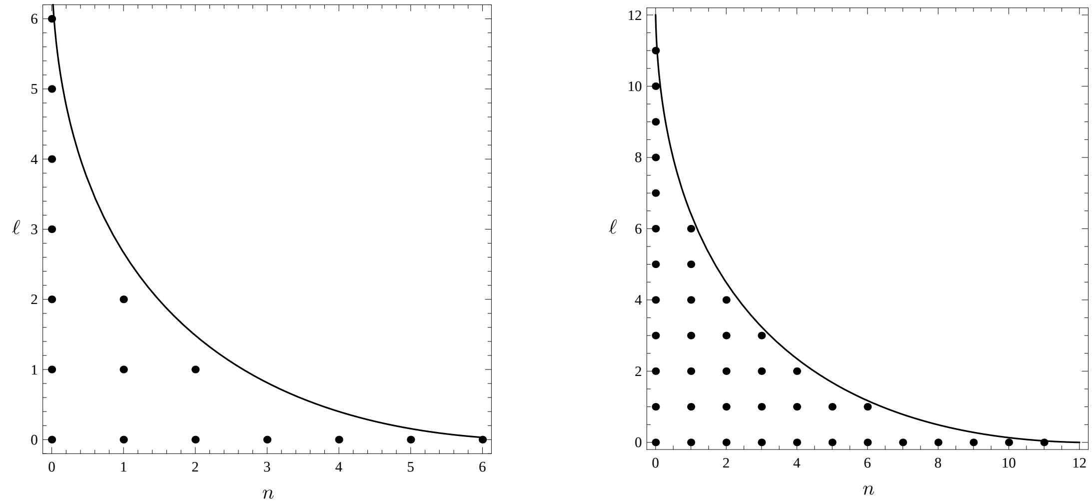

Figure 1: Landau levels of $e^-_{(n)} e^+_{(\ell)}$ to be excited when $M_n + M_\ell \leqslant E_\perp$, at $E_\perp = 25$ MeV, and at $B = 180\, B_e$ (left) and $100\, B_e$ (right).

In the integration over the momenta of the electron and the positron, a condition arises:

$$q_\parallel^2 = q_0^2 - q_z^2 \geqslant (M_n + M_\ell)^2 \,, \tag{6}$$

which determines the range of integration over the final neutrino momentum. Here, $q = P - P' = p + p'$ is the change of the four-vector of the neutrino momentum equal to the four-momentum of the $e^- e^+$ pair, $P^\alpha = (E, \boldsymbol{P})$ and $P'^\alpha = (E', \boldsymbol{P}')$ are the four-momenta of the initial and final neutrinos.

In turn, the condition (6) can be satisfied when the energy of the initial neutrino exceeds a certain threshold value. In the reference frame, hereafter called K, where the momentum of the initial neutrino directed at an angle θ to the magnetic field, the threshold energy is given by:

$$E \sin\theta \geqslant M_n + M_\ell \,. \tag{7}$$

In Fig. 1, the Landau levels of $e^-_{(n)} e^+_{(\ell)}$ are shown, similarly to Fig. 1 of Ref. [8], to be excited accoring to the condition (7), at $E \sin\theta = 25$ MeV, and at $B = 180\, B_e$ and $100\, B_e$.

It is convenient to perform further integration over the final neutrino momentum, without loss of generality, not in an arbitrary reference frame K, but in the special frame K_0, where the initial neutrino momentum is perpendicular to the magnetic field, $P_z = 0$. One can then return from K_0 to K by the Lorentz transformation along the field (recall that the field is invariant with respect to this transformation).

It is convenient to use the dimensionless cylindrical coordinates in the space of the final neutrino momentum vector $\boldsymbol{P}'$:

$$\rho = \sqrt{P'^2_x + P'^2_y}/E_\perp \,, \qquad \tan\phi = P'_y/P'_x \,, \qquad z = P'_z/E_\perp \,,$$
$$r = E'/E_\perp = \sqrt{\rho^2 + z^2} \,. \tag{8}$$

Here, $E_\perp$ is the energy of the initial neutrino in the frame K_0, which is connected with its energy E in an arbitrary frame K by the relation $E_\perp = E \sin\theta$.

We do not present here the set of expressions for the probability of the process $\nu \to \nu e^-_{(n)} e^+_{(\ell)}$, which can be found in the paper [7]. These probabilities evaluated numerically as the functions of the initial neutrino energy and of the magnetic field strength for all channels considered in Ref. [6], where the electron and positron are created in the lower Landau levels, are in a good agreement with the results of that paper.

3 Neutrino energy and momentum losses

The probability of the $\nu \to \nu e^- e^+$ process defines its partial contribution into the neutrino opacity of the medium. The estimation of the neutrino mean free path with respect to this process gives the result which is too large [1] compared with the typical size of a compact astrophysical object, e.g. the supernova remnant, where a strong magnetic field could exist. However, a mean free path does not exhaust the neutrino physics in a medium. In astrophysical applications, we could consider the values that probably are more essential, namely, the mean values of the neutrino energy and momentum losses, caused by the influence of an external magnetic field. These values can be described by the four-vector of losses Q^α,

$$Q^\alpha = E \int q^\alpha \, \mathrm{d}W = -E \, (\mathcal{I}, \boldsymbol{F}) \, . \tag{9}$$

where $\mathrm{d}W$ is the total differential probability of the process $\nu \to \nu e^- e^+$. The zeroth component of Q^α is connected with the mean energy lost by a neutrino per unit time due to the process considered, $\mathcal{I} = \mathrm{d}E/\mathrm{d}t$. The space components of the four-vector (9) are similarly connected with the mean neutrino momentum loss per unit time, $\boldsymbol{F} = \mathrm{d}\boldsymbol{P}/\mathrm{d}t$. It should be noted that the four-vector of losses Q^α can be used for evaluating the integral effect of neutrinos on plasma in the conditions of not very dense plasma, e.g. of a supernova envelope, when an one-interaction approximation of a neutrino with plasma is valid [2, 9, 10].

In Ref. [3], the formula (10) for the energy deposition rate was taken, which was calculated in the crossed field limit [10]. By the way, the value q^α defined by Eq. (10) of Ref. [3] is not the 4-vector while the value $Q^\alpha = E \, q^\alpha$ is. However, in the region of the physical parameters used in Ref. [3] (B to 180 B_e, E to 25 MeV), the approximation of a crossed field is poorly applicable, as well as the approximation of a superstrong field when $e^- e^+$ are created in the ground Landau level. The contribution of the next Landau levels which can be also excited, should be taken into account. We present here the results of our calculation of the mean neutrino energy losses caused by the process $\nu \to \nu e^- e^+$ in a moderately strong magnetic field, i.e. in the conditions of the Kerr black hole accretion disk.

We parametrize the energy deposition rate as:

$$Q_0 = (C_V^2 + C_A^2) \, \sigma_0 \, m_e^4 \, E \, f\left(\frac{E}{m_e}, \frac{B}{B_e}\right) , \tag{10}$$

where $\sigma_0 = 4 \, G_{\mathrm{F}}^2 \, m_e^2 / \pi$, and the dependence on the initial neutrino energy and the field strength is described by the function $f(E/m_e \, , B/B_e)$. This function calculated in Ref. [10] in the crossed field limit had the form

$$f^{(cr)}(y, \eta) = \frac{7 \, y^2 \, \eta^2}{1728 \, \pi^2} \, \ln(y \, \eta) \, , \tag{11}$$

On the other hand, in the strong field limit when both electron and positron are born in the ground Landau level, the function $f(y, \eta)$ was also calculated in Ref. [10] and can be presented in the form

$$f^{(00)}(y, \eta) = \frac{\eta \, y^4}{32 \, \pi^2} \int\limits_0^1 \mathrm{d}\rho \, \rho (1 - \rho^2)^2 \, \exp\left(- \frac{y^2 (1 + \rho^2)}{2 \eta}\right) I_0\left(\frac{y^2}{\eta} \rho\right) , \tag{12}$$

where $I_0(x)$ is the modified Bessel function.

In conditions of moderately strong magnetic field, when the electron and the positron are created in the process $\nu \to \nu e^-_{(n)} e^+_{(\ell)}$ in the nth and ℓth Landau levels, the result has more complicated form. It is significantly simplified when one of the particles, electron or positron, is born in the ground Landau level, and if an approximation $B \gg B_e$ is used. We obtain the contribution of the channels $\nu \to \nu e^-_{(n)} e^+_{(0)}$ and $\nu \to \nu e^-_{(0)} e^+_{(n)}$ to the function $f(y,\eta)$ as follows:

$$
\begin{aligned}
f^{(n0+0n)}(y,\eta) &= \frac{\eta\, y^4}{4\pi^2 (n-1)!} \left(\frac{y^2}{2\eta}\right)^{n-1} \int\limits_0^{1-\sqrt{b_n}} \mathrm{d}\rho\, \rho \int\limits_0^{Z_0} \frac{\mathrm{d}z (1-r)}{r(1-2r+\rho^2)^2} \\
&\times \left[(1-\rho^2)^2 + 4r^2 - 2r(1+\rho^2)\right] \int\limits_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi} (r - \rho\cos\phi) \\
&\times (1 - 2\rho\cos\phi + \rho^2)^{n-1} \exp\left(-\frac{y^2(1-2\rho\cos\phi+\rho^2)}{2\eta}\right), \qquad (13)
\end{aligned}
$$

where

$$
b_n = \frac{2neB}{E_\perp^2}, \qquad Z_0 = \frac{1}{2}\sqrt{(1+\rho^2-b_n)^2 - 4\rho^2}\,. \qquad (14)
$$

In Figs. 2–4, the function $f(y,\eta)$ obtained in different approximations is shown at $B = 180B_e$, $100B_e$, $50B_e$. It can be seen that the crossed field limit gives the overstated result which is in orders of magnitude greater than the sum of the contributions of lower excited Landau levels. On the other hand, the results with e^-e^+ created at the ground Landau level give the main contribution to the energy deposition rate, and almost exhaust it at $B = 180B_e$.

This would mean that the conclusion [3] that the contribution of the process $\nu \to \nu e^- e^+$ to the efficiency of the electron-positron plasma production by neutrino exceeds the contribution of the annihilation channel $\nu\bar{\nu} \to e^-e^+$, and that the first process dominates the energy deposition rate, does not have a sufficient basis. A new analysis of the efficiency of energy deposition by neutrinos through both processes, $\nu\bar{\nu} \to e^-e^+$ and $\nu \to \nu e^-e^+$, in a hyper-accretion disc around a black hole should be performed, with taking account of our results for the process $\nu \to \nu e^-e^+$ presented here.

4 Conclusions

In the paper, a calculation is performed of the mean value of the neutrino energy loss due to the process of electron-positron pair production, $\nu \to \nu e^-e^+$, in the magnetic field of an arbitrary strength at which the electrons and positrons can be produced in the states corresponding to the excited Landau levels, which could be essential in astrophysical applications. The results obtained should be used for calculations of the efficiency of the electron-positron plasma production by neutrinos in the conditions of the Kerr black hole accretion disk, regarded by experts as the most likely source of a short cosmological gamma-ray burst. In those conditions, the crossed field limit used in the previous calculations led to the overstated result which was in orders of magnitude greater than the sum of the lower Landau levels. The study may be also useful for further development of computational techniques for the analysis of quantum processes in an external active environment, particularly in conditions of moderately strong magnetic field, when the allowance for the contribution of only the ground Landau level is insufficient.

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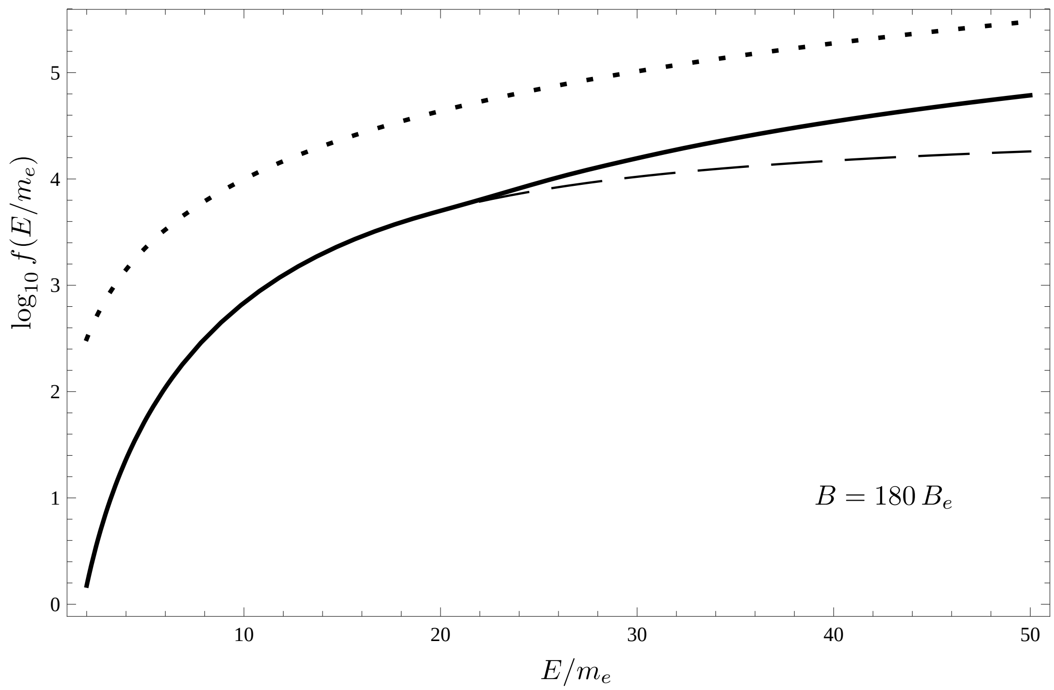

Figure 2: The function $f(E/m_e)$ for $B = 180\,B_e$ obtained in the crossed field limit (dotted line), with e^-e^+ created at the ground (0,0) Landau level (dashed line), and for the sum of all lower Landau levels which are excited in this energy interval according to the condition (7)(solid line).

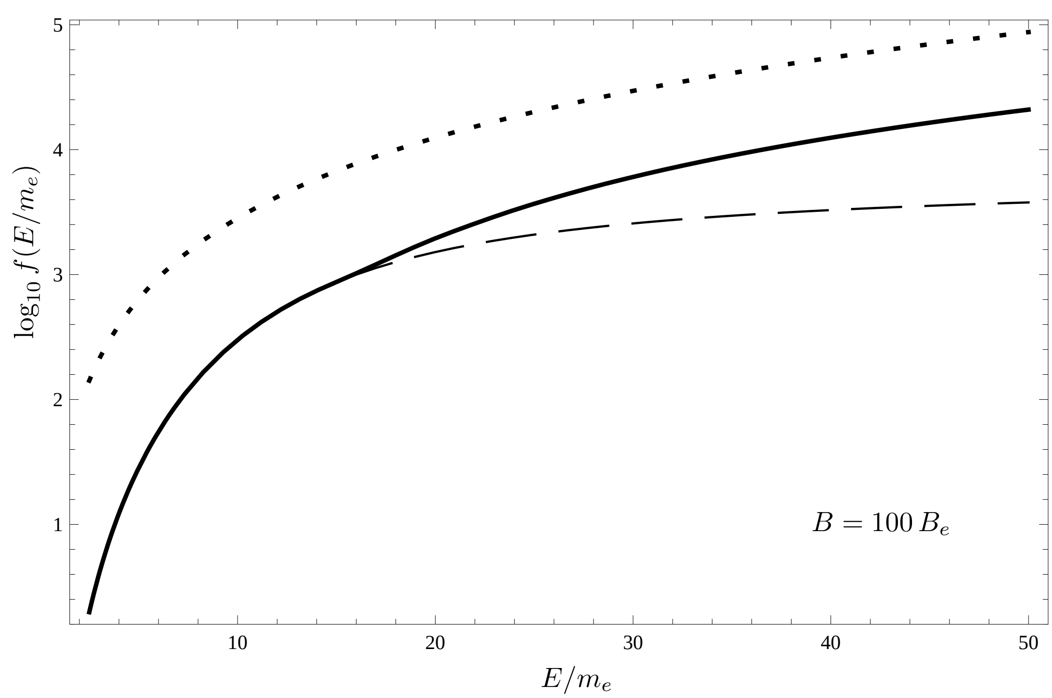

Figure 3: The same as in Fig. 2, for $B = 100\,B_e$.

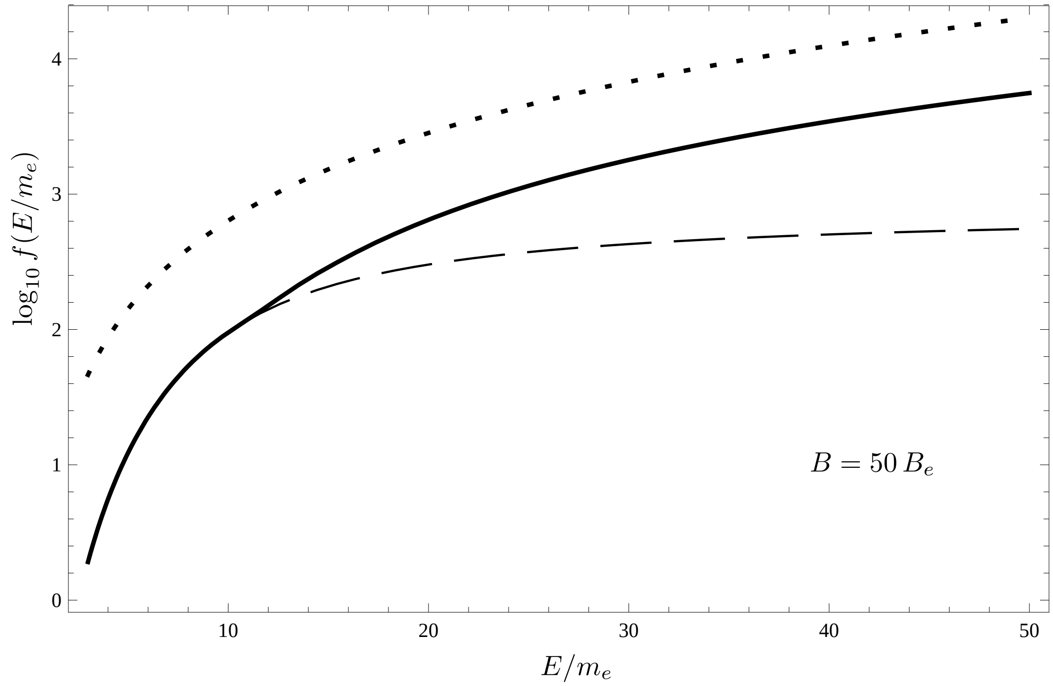

Figure 4: The same as in Fig. 2, for $B = 50\,B_e$.

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