Primordial black hole constraints on some models of dissipative inflation

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Abstract

We consider the cosmological scenario in which the inflaton is a pseudo-Nambu-Goldstone boson (axion) which has a coupling to some gauge field. In such a model inflation is accompanied by the gauge quanta production and a strong rise of the curvature power spectrum amplitude at small scales is predicted. We show that data on primordial black hole searches can be used for a derivation of essential constraints on the model parameters in axion inflation scenarios. We compare our numerical results with the similar results published earlier.

1 Introduction

It is widely assumed that early Universe went through an inflationary phase of accelerated expansion. To drive inflation, a vacuum-like equation of state is needed, which can be provided by one or more scalar fields. One problem of inflationary paradigm is the need to have a rather flat inflaton potential, so that "slow-roll" parameters are small.

It had been shown very long ago that the simplest and most natural solution of this problem is to assume that the inflaton φ is a pseudo-Nambu-Goldstone boson (PNGB) [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12], because in this case there is the shift symmetry, $\varphi \to \varphi + \text{const}$, broken by instanton effects (or explicitly). In the limit when this symmetry is exact, the potential is flat, and the corrections to slow-roll parameters are under control due to the smallness of the symmetry breaking.

If PNGB is pseudoscalar (e.g., it is axion), it is natural to assume that there is a coupling of it with some gauge field. This coupling is not forbidden by the shift symmetry and, in general, is phenomenologically favourable (e.g., it can lead to successful reheating). This coupling is essential if the axion decay constant, f, is not too large (because the interaction term is inversely proportional to f, see Eq. (1) below). In UV-complete models of axion inflation (e.g., those based on the string theory [6]) one has $f \ll M_P$ and, at the same time, large excursion of the axion field is allowed. The inflationary potential in these models is similar with the potential in large field models.

The main feature of the axion inflation with inflaton-gauge field coupling is that such a coupling leads to a production of gauge quanta, and, through the inverse decay of these quanta into inflaton perturbations, to a rise of non-Gaussianity effects and violation of scale-invariance. In particular, a rather essential formation of primordial black holes (PBHs) becomes possible [13, 14].

In the present work we consider a process of PBH formation and PBH constraints for the axion inflation models in which the inflationary expansion is accompanied by the gauge quanta production.

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2 Axion inflation with gauge field production

2.1 Outline of the model

We consider the model of axion inflation in which there is a coupling of the pseudoscalar inflaton (axion) to gauge fields of the form

$$\mathcal{L}_{int} = -\frac{\alpha}{4f}\varphi F_{\mu\nu}\tilde{F}^{\mu\nu} , \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength corresponding to some $U(1)$ gauge field A_μ, and $\tilde{F}^{\mu\nu} = \eta^{\mu\nu\omega\theta}F_{\omega\theta}/(2\sqrt{-g})$ is the dual strength, f is the axion decay constant, α is the dimensionless parameter.

It had been shown in [8] that the evolution (rolling) of the inflaton leads to a generation of the field A_μ and to a subsequent amplification (due to tachyonic instability) of its modes. The solutions for the amplified modes are well parameterized by the formula (index $+$ means the circular polarization of quanta)

$$\tilde{A}_+(k,\tau) \cong \frac{1}{\sqrt{2k}}\left(\frac{k}{2\xi aH}\right)^{1/4} \exp\left[\pi\xi - 2\sqrt{\frac{2\xi k}{aH}}\right] , \tag{2}$$

where

$$\xi \equiv \frac{\alpha\dot{\varphi}}{2fH} , \tag{3}$$

and $\tau \cong -1/(aH)$. During inflationary expansion the value of ξ changes with time. If ξ is larger than 1, the amplification factor $e^{\pi\xi}$ is essential. The production of gauge field quanta can affect the inflationary process. In general, it prolongs inflation [8] because it sources inflation perturbations through the inverse decay: $\delta A + \delta A \to \delta\varphi$ [15].

The tachyonic amplification of gauge field modes leads to a characteristic evolution of the power spectrum of primordial curvature perturbations. The production of gauge quanta causes strong increasing of the spectrum amplitude. To put constraints on this increase from PBHs one must study the behavior of ξ-parameter as a function of time during inflationary expansion. The cosmological evolution equations for the inflaton with extra contributions from the gauge field are [8]

$$\ddot{\varphi} + 3H\dot{\varphi} + V' = \frac{\alpha}{f}\langle \vec{E}\cdot\vec{B}\rangle , \tag{4}$$

$$3H^2M_P^2 = \frac{1}{2}\dot{\varphi}^2 + V + \frac{1}{2}\langle \vec{E}^2 + \vec{B}^2\rangle. \tag{5}$$

Here,

$$\vec{B} \equiv \frac{1}{a^2}\vec{\nabla}\times\vec{A} , \qquad \vec{E} \equiv -\frac{1}{a^2}\vec{A}' . \tag{6}$$

The connection of $\langle\vec{E}\cdot\vec{B}\rangle$ and $\langle\vec{E}^2+\vec{B}^2\rangle$ with ξ is given by [8]

$$\langle\vec{E}\cdot\vec{B}\rangle \approx -2.4\times 10^{-4}\,\frac{H^4}{\xi^4}e^{2\pi\xi} , \qquad \frac{1}{2}\langle\vec{E}^2+\vec{B}^2\rangle \approx 1.4\times 10^{-4}\,\frac{H^4}{\xi^3}e^{2\pi\xi} . \tag{7}$$

For a calculation of the curvature power spectrum one needs the evolution equation for inflaton field perturbation, $\delta\varphi$. Deriving this equation one must take into account the backreaction effects [8, 16]. The approximate accounting of these effects leads to the (operator) equation [8, 16]

$$\delta\ddot{\varphi} + 3\beta H\delta\dot{\varphi} - \frac{\nabla^2}{a^2}\delta\varphi + V''\delta\varphi = \frac{\alpha}{f}\left[\vec{E}\cdot\vec{B} - \langle\vec{E}\cdot\vec{B}\rangle\right] , \tag{8}$$

where β is defined by the expression

$$\beta \equiv 1 - 2\pi\xi \frac{\alpha}{f} \frac{\langle \vec{E} \cdot \vec{B} \rangle}{3H\dot{\varphi}}. \tag{9}$$

Equations (4) and (5) are solved numerically, giving the solutions $\varphi(t)$ and $H(t)$ with initial conditions for $\varphi(0)$ and $H(0)$, where $t = 0$ corresponds, in our case, to the moment when CMB scales exit the horizon. As a byproduct one obtains the function $\xi(t)$.

2.2 Axion potential

A typical axion inflationary potential which is exploited in natural inflation models [1, 2] is given by the formula

$$V(\varphi) = \Lambda^4 \left[1 - \cos\left(\frac{\varphi}{f}\right)\right]. \tag{10}$$

In UV-complete models of axion inflation, the axion action is shift-symmetric, i.e., the shift symmetry $\varphi \to \varphi + \mathrm{const}$ is broken only non-perturbatively. In particular, in closed string models with type IIB Calabi-Yau orientifold compactifications such axions are available (see, e.g., the review paper [17]). The inflationary potential in such models is periodic, due to instanton effects, but it is flat enough for driving inflation only in the case when the axion decay constant is larger than M_P. It is well known, however, that it is difficult to obtain such large values of f in UV-complete theories [18, 19]. So, the potential of single axion, Eq. (10), cannot provide the large field inflation with long slow-roll evolution and a large value of the field excursion.

There are several groups of models in which the large field inflation is possible with sub-Planckian axion decay constants: "Racetrack inflation" models [20], N-flation models [4], assisted inflation models [21, 22], axion monodromy inflation models [6, 7, 11, 9, 10]. The latter approach looks very promising and we used it in the present paper for numerical calculations. In particular, it had been shown in [6] that, in IIB string theory, the presence of space-filling D_p-branes wrapping some two-cycles of the compact internal space leads to a breaking of the shift symmetry and to the monodromy phenomenon: the potential energy for the axion arising from integrating two-form fields over these two-cycles is not periodic and increases with an increase of the axion field. As a result, one has the additional component of the axion potential,

$$V(\varphi) = V_{sr}(\varphi) + V_{inst}(\varphi). \tag{11}$$

Here, the abbreviation "sr" means slow-roll, and "inst" means instanton. In the concrete model [6], with the C_2-axion and $NS5$-brane wrapping Σ_2 (see [17] for notations), the potential V_{sr} is given by the expression

$$V_{sr}(\varphi) = \frac{\epsilon}{g_s^2 (2\pi)^5 \alpha'^2} \sqrt{L^4 + g_s^2 \frac{\varphi^2}{f^2}}. \tag{12}$$

Here, L is the dimensionless modulus (L^2 is the size of the 2-cycle Σ_2), g_s is the string coupling constant, $1/(2\pi\alpha')$ is the string tension, ϵ is the warp-factor [6]. At large values of φ/f one has the linear potential,

$$V_{sr}(\varphi) \approx \mu^3 \varphi. \tag{13}$$

The different realization of the monodromy idea (which is not based on the string theory) had been suggested in [9, 10]. In these works, the axion potential is generated by modification of the action introducing there the coupling of the axion to a 4-form. This new coupling leads to a spontaneous breaking of the shift symmetry and to appearing (in the simplest case) the quadratic axion potential just like in the original chaotic inflation scenario [23].

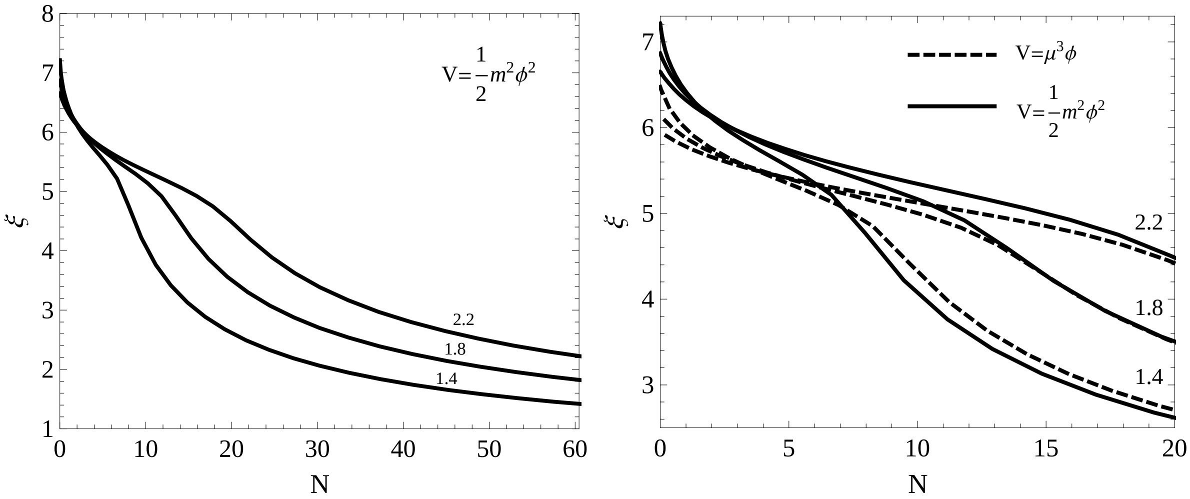

Figure 1: The value of $\xi(N)$ for different values of ξ at CMB scales and different choice of model potential. Solid curves are for the case of potential (14). Dashed curves are for potential (13). The curves are labeled with the value of $\xi(N_{CMB})$.

In this work we will consider both cases: the axion inflation with the quadratic potential

$$V(\varphi) = \frac{m^2\varphi^2}{2} \tag{14}$$

(PBH constraints for axion inflationary model with such a potential have been considered in work [14]) and the inflation with the linear potential given by Eq. (13). We assume that effects from the presence of V_{inst} are subdominant and neglect this term.

Using the expressions for axion potentials, Eqs. (4) and (5) can be solved. In Fig. 1 we show the results of our numerical calculations: the dependence of ξ on N, the number of e-folds before an end of inflation, for different values of ξ at CMB scales.

One should note, closing this subsection, that axion monodromy inflation with potentials given by Eqs. (13) and (14) predict rather large values of tensor-to-scalar ratio: $r = 0.07$ for the linear potential, and $r = 0.14$ for the quadratic one. The latter value is not excluded by the Planck [24] and BICEP2 [25] data.

2.3 Curvature power spectrum

In the limit of very small backreaction one has $\beta \to 1$. In this limit, the solution of Eq. (8) is [15, 26]

$$\mathcal{P}_\zeta(k) = \mathcal{P}_{\zeta,sr}(k)\left(1 + \mathcal{P}_{\zeta,sr}(k) f_2(\xi) e^{4\pi\xi}\right), \tag{15}$$

$$\mathcal{P}_{\zeta,sr}(k) = \left(\frac{H^2}{2\pi\dot{\varphi}}\right)^2. \tag{16}$$

The function $f_2(\xi)$ is defined in [15, 26].

Near horizon crossing one has the approximate solution of Eq. (8) (everywhere below we omit the contribution of the vacuum part, i.e., the solution of the homogenous equation):

$$\delta\varphi \approx \frac{\alpha}{f}\frac{\left(\vec{E}\cdot\vec{B} - \langle\vec{E}\cdot\vec{B}\rangle\right)}{3\beta H^2}, \tag{17}$$

and, correspondingly, one has for the curvature:

$$\zeta \approx -\frac{\alpha}{f}\frac{\left(\vec{E}\cdot\vec{B} - \langle\vec{E}\cdot\vec{B}\rangle\right)}{3\beta H\dot{\varphi}}. \tag{18}$$

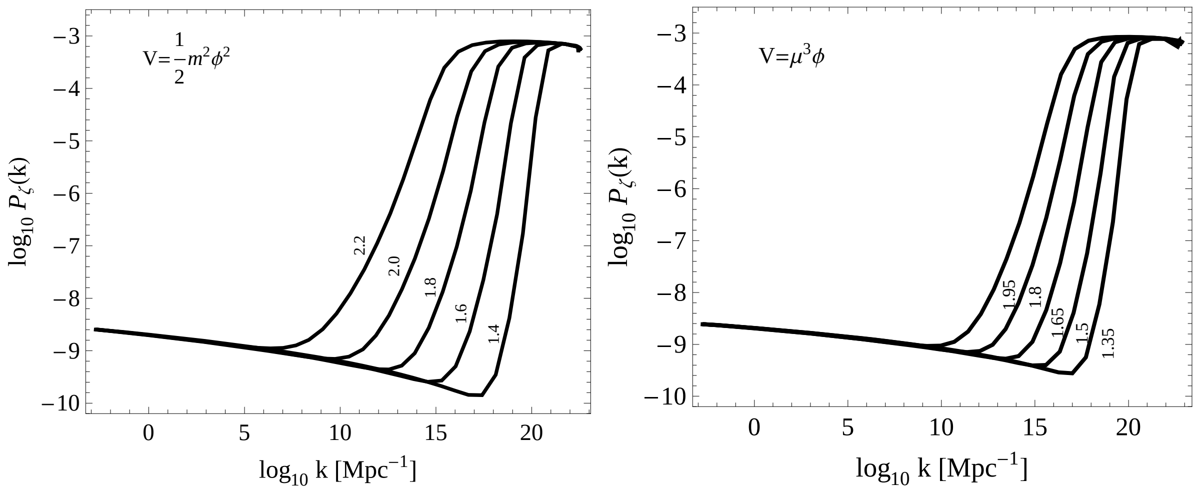

Figure 2: The curvature perturbation power spectrum $\mathcal{P}_\zeta(k)$ calculated for different values of ξ_{CMB}, for two shapes of inflaton potential. The curves are labeled with the value of $\xi(N_{CMB})$.

The variance of the curvature power spectrum is [14]

$$\langle \zeta(x)^2 \rangle = \frac{H^2}{\dot{\varphi}^2} \langle \delta\varphi^2 \rangle \approx \frac{\alpha^2}{f^2} \frac{\langle \vec{E} \cdot \vec{B} \rangle^2}{(3\beta H \dot{\varphi})^2}. \tag{19}$$

From this equation, in the limit of large backreaction, when $\beta \gg 1$, the simple approximate formula for the power spectrum is obtained [8, 16, 14]:

$$\mathcal{P}_\zeta(k) \approx \langle \zeta(x)^2 \rangle = \frac{1}{(2\pi\xi)^2}. \tag{20}$$

Some examples of the power spectrum solutions are shown in Fig. 2. For the calculations we used the approximate formula (19) which takes into account backreaction (at latest stages of inflation the backreaction effects are quite essential). Everywhere we add the contribution of the vacuum part which is dominant at small values of k. The connection of the comoving wave number k with N is given by

$$k = a_e H(N) e^{-N}, \tag{21}$$

where a_e is the scale factor at the end of inflation.

3 PDFs and non-Gaussianity

For a derivation of the PBH constraints we need an expression for the PDF of the ζ-field. Evidently, this is a technical problem in non-Gaussian case because, for a calculation of the PDF one must know, in principle, all cumulants (moments) contributing to its series expansion.

In our case, the simplest assumption which we can use is the following: ζ-field is distributed as a square of some Gaussian field χ,

$$\zeta = A(\chi^2 - \langle \chi^2 \rangle), \tag{22}$$

having in mind that non-Gaussianity of fluctuations $\delta\varphi$, described by, e.g., Eq. (8), arises just from the fact that the particular solution of this equation is bilinear in the field A_μ (the latter is assumed to be Gaussian).

If Eq. (22) holds (in this case, we have so-called χ^2-model), the PDF of ζ is given by (see, e.g., [27, 28])

$$p_\zeta(\zeta) = \frac{1}{A\sqrt{\frac{\zeta}{A} + \langle\chi^2\rangle}}\, p_\chi\left(\sqrt{\frac{\zeta}{A} + \langle\chi^2\rangle}\right), \tag{23}$$

$$p_\chi(\chi) = \frac{1}{\sigma_\chi\sqrt{2\pi}} e^{-\frac{\chi^2}{2\sigma_\chi^2}}, \qquad \sigma_\chi^2 \equiv \langle\chi^2\rangle. \tag{24}$$

Variance and skewness of the ζ-field are, respectively,

$$\langle\zeta^2\rangle = 2A^2\langle\chi^2\rangle^2, \qquad \langle\zeta^3\rangle = 8A^3\langle\chi^2\rangle^3, \tag{25}$$

so that the first non-trivial reduced cumulant is

$$D_3 = \frac{\langle\zeta^3\rangle}{\langle\zeta^2\rangle^{3/2}} = \sqrt{8}. \tag{26}$$

It is interesting also to calculate the bispectrum of the χ^2-model. It is given by the expression [29]

$$B(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3}) = \frac{8}{3}\left[\int \frac{d^3k'}{(2\pi)^3} P_G(|\mathbf{k_1} - \mathbf{k}'|) P_G(|\mathbf{k_2} + \mathbf{k}'|) P_G(k') + 2 \text{ perm.}\right], \tag{27}$$

where $P_G(k)$ is the curvature power spectrum of χ-field, $P_G(k) \sim k^n$. The shape S of the bispectrum, which is defined by the formula

$$S(k_1, k_2, k_3) = (k_1k_2k_3)^2 B(k_1, k_2, k_3) \tag{28}$$

has a characteristic "squeezed" form, shown in Fig. 3 (upper panel; $n = -2.9$).

The approximate correctness of using the χ^2-model in our case, when one has for the ζ-field instead of Eq. (18) the expression (22) can be checked by comparison of the corresponding values of cumulants and shapes of the polyspectra (e.g., shapes of bispectra). The reduced cumulant D_3 in our case is given by (in the region where backreaction is large [14])

$$D_3 = \frac{\langle\zeta^3\rangle}{\langle\zeta^2\rangle^{3/2}} \cong \frac{1/(4\pi^3\xi^3)}{(1/(2\pi\xi)^2)^{3/2}} = 2. \tag{29}$$

This value is rather close to D_3 in Eq. (26) (although it hints that the use, for our aims, of χ_2^2-model [with two degrees of freedom], rather than χ^2-model, would be, probably, more appropriate).

The bispectrum in our model is calculated using the formula

$$B(k_1, k_2, k_3) = \frac{3}{10}\mathcal{P}^3_{\zeta,sr} e^{6\pi\xi} \frac{k_1^3 + k_2^3 + k_3^3}{k_1^3k_2^3k_3^3} f_3\left(\xi, \frac{k_2}{k_1}, \frac{k_3}{k_1}\right). \tag{30}$$

Here, the function f_3 is defined in [26, 30]. The example of the calculation of the corresponding shape function (for $\xi = 6$) is shown in Fig. 3 (lower panel).

Comparing two shape functions, one can see that the shape function of our model is not too similar with the typical equilateral shape function (see, e.g., [31] for examples of equilateral shapes) as well as with the shape function of χ^2-model (although on both figures there is some concentration of points along the diagonal line).

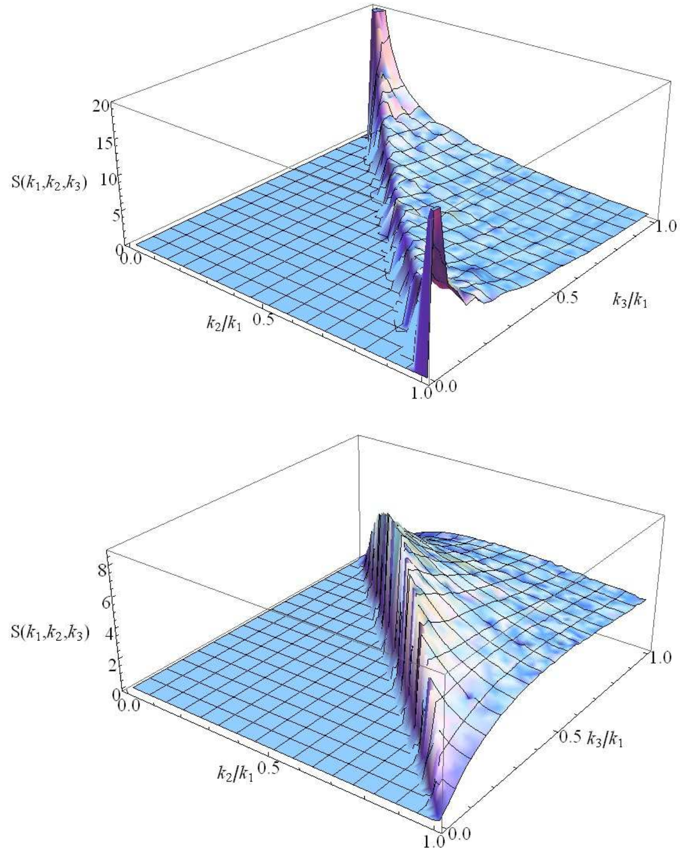

Figure 3: Shape functions $S(k_1, k_2, k_3)$ (arbitrarily normalized) for χ^2-model (upper panel) and axion inflation model (lower panel).

4 PBH constraints

Although arguments in favour of χ^2-model do not look very convincing, we nevertheless will use this model for obtaining the PBH constraints. Namely, we assume that the PDF of ζ-field is given by [27, 28]

$$p_\zeta(\zeta) = \frac{1}{\langle\zeta^2\rangle^{1/2}}\, p(\tilde{\nu}), \quad \tilde{\nu} = \frac{\zeta}{\langle\zeta^2\rangle^{1/2}}, \tag{31}$$

$$p(\tilde{\nu}) = \frac{1}{\sqrt{\pi(1+\sqrt{2}\tilde{\nu})}} e^{-\frac{1}{2}(1+\sqrt{2}\tilde{\nu})}. \tag{32}$$

This expression for PDF follows from Eqs. (23, 24) and from the expression connecting the variances,

$$\langle\zeta^2\rangle = \int\limits_{\zeta_{min}}^{\infty} \zeta^2 p_\zeta(\zeta) d\zeta = 2\zeta_{min}^2 = 2A^2\langle\chi^2\rangle^2. \tag{33}$$

Taking into account smoothing effects leads to a change in Eq. (31):

$$\langle\zeta^2\rangle \to \langle\zeta_R^2\rangle = \int\limits_0^\infty \tilde{W}^2(kR)\mathcal{P}_\zeta(k)\frac{dk}{k}. \tag{34}$$

Here, $\tilde{W}(kR)$ is the window function.

One can show that the energy density fraction of the Universe contained in PBHs which form near the time of formation, $t = t_f$ (at this time the horizon mass is equal to $M_h(t_f) = M_h^f$)

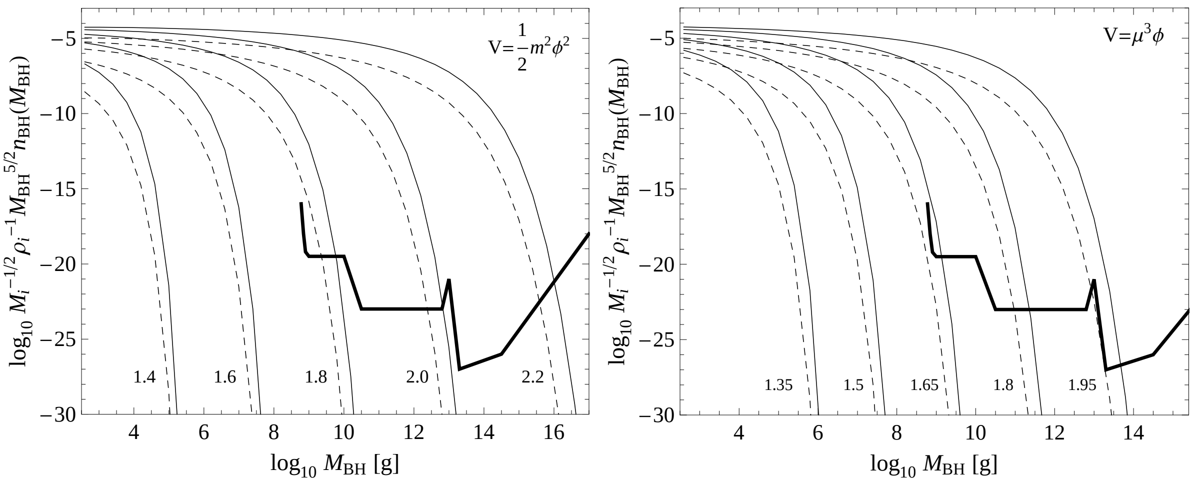

Figure 4: Primordial black hole mass spectra corresponding to curvature perturbation power spectra shown in Fig. 2. Solid curves are for the case $\zeta_c = 0.75$ while dashed curves are for $\zeta_c = 1$. The curves are labeled with the value of $\xi(N_{CMB})$. Thick line schematically shows existing constraints on PBH abundance [35, 36].

is given by [28, 32]

$$\Omega_{PBH}(M_h^f) \approx \frac{(M_h^f)^{5/2}}{\rho_i M_i^{1/2}} n_{BH}(M_{BH}) \,|_{M_{BH} \approx f_h M_h^f} . \tag{35}$$

Here, $n_{BH}(M_{BH})$ is the PBH mass spectrum, ρ_i and M_i are, correspondingly, the energy density and horizon mass at the beginning of radiation era (if the reheating is fast, it coincides with an end of inflation). f_h is the constant [equal to $(1/3)^{1/2}$] which connects the value of PBH mass forming at the moment t_f with the horizon mass at that moment (see, e.g., [33]). Approximately, one has

$$\Omega_{PBH}(M_h^f) \approx \beta_{PBH}(M_h^f), \tag{36}$$

where β_{PBH} is, by definition, the fraction of the Universe's mass in PBHs at their formation time,

$$\beta_{PBH}(M_h^f) \equiv \frac{\rho_{PBH}(t_f)}{\rho(t_f)}. \tag{37}$$

Now, having Eqs. (35, 36), one can use the experimental limits on the value of β_{PBH} [35, 36] to constrain parameters of models used for PBH production predictions. The PBH mass spectrum needed for a derivation of Ω_{PBH} in Eq. (35) depends on the amplitude of the curvature power spectrum $\mathcal{P}_\zeta$ and is calculated using the Press-Schechter [34] formalism (see [32, 28, 27] for details).

The results of PBH mass spectra calculation for the considered model are given in Fig. 4 for several values of the parameter $\xi_{CMB} \equiv \xi(N_{CMB})$ and for two choices of the parameter ζ_c, which is a model-dependant PBH formation threshold (see, e.g., [32]).

5 Results and discussion

The main results of the paper are shown in Figs. 2 and 4. Fig. 2 illustrates the fact that due to tachyonic instability of gauge field, an amplitude of the curvature power spectrum is very large (up to 10^{-3}) at small scales, $k \sim (10^{15} - 10^{20})\mathrm{Mpc}^{-1}$, for a broad range of ξ_{CMB} values. Fig. 4 shows the PBH mass spectra for definite values of the parameter ξ_{CMB}. On the vertical axis of Fig. 4 the combination $M_i^{-1/2} \rho_i^{-1} M_{BH}^{5/2} n_{BH}(M_{BH})$ is shown; just this combination is approximately equal to β_{PBH}, as it follows from Eq. (35). We compare these spectra with PBH data [35, 36], in which we consider only data for $M_{BH} > 10^9$ g, as most reliable ones. For such a comparison we drew in Fig. 4 the zigzag line representing, schematically, the well known β_{PBH}-constraint summary curve (see Fig. 9 in Ref. [36]). If some of our curves crosses this zigzag line, the corresponding ξ-value is, according to our logic, forbidden. Finally we obtain the constraint on the value of ξ_{CMB}, for quadratic potential (14),

$$\xi_{CMB} < 1.75 \; . \tag{38}$$

This constraint can be compared with the corresponding result of the work [14], $\xi_{CMB} < 1.5$. In terms of α and f constants, the limit (38) corresponds to $\alpha/f < 26 M_P^{-1}$.

We performed similar analysis for the case of linear potential (13), and in this case the constraint on ξ_{CMB} turns out to be more strong,

$$\xi_{CMB} < 1.65 \; , \tag{39}$$

corresponding to $\alpha/f < 35 M_P^{-1}$.

One should note, in conclusion, that PBH constraints are stronger than those from CMB scales [12] and forthcoming constraints from gravity wave experiments [37].

Acknowledgments

The work of P.K. was partially supported by the grant of President of RF number SS-3110.2014.2.

References

[1] K. Freese, J. A. Frieman and A. V. Olinto, Phys. Rev. Lett. **65**, 3233 (1990).

[2] F. C. Adams, J. R. Bond, K. Freese, J. A. Frieman and A. V. Olinto, Phys. Rev. D **47**, 426 (1993) [hep-ph/9207245].

[3] J. E. Kim, H. P. Nilles and M. Peloso, JCAP **0501**, 005 (2005) [hep-ph/0409138].

[4] S. Dimopoulos, S. Kachru, J. McGreevy and J. G. Wacker, JCAP **0808**, 003 (2008) [hep-th/0507205].

[5] R. Easther and L. McAllister, JCAP **0605**, 018 (2006) [hep-th/0512102].

[6] L. McAllister, E. Silverstein and A. Westphal, Phys. Rev. D **82**, 046003 (2010) [arXiv:0808.0706 [hep-th]].

[7] R. Flauger, L. McAllister, E. Pajer, A. Westphal and G. Xu, JCAP **1006**, 009 (2010) [arXiv:0907.2916 [hep-th]].

[8] M. M. Anber and L. Sorbo, Phys. Rev. D **81**, 043534 (2010) [arXiv:0908.4089 [hep-th]].

[9] N. Kaloper and L. Sorbo, Phys. Rev. Lett. **102**, 121301 (2009) [arXiv:0811.1989 [hep-th]].

[10] N. Kaloper, A. Lawrence and L. Sorbo, JCAP **1103**, 023 (2011) [arXiv:1101.0026 [hep-th]].

[11] M. Berg, E. Pajer and S. Sjors, Phys. Rev. D **81**, 103535 (2010) [arXiv:0912.1341 [hep-th]].

[12] E. Pajer and M. Peloso, Class. Quant. Grav. **30**, 214002 (2013) [arXiv:1305.3557 [hep-th]].

[13] C. -M. Lin and K. -W. Ng, Phys. Lett. B **718**, 1181 (2013) [arXiv:1206.1685 [hep-ph]].

[14] A. Linde, S. Mooij and E. Pajer, Phys. Rev. D **87**, 103506 (2013) [arXiv:1212.1693 [hep-th]].

[15] N. Barnaby and M. Peloso, Phys. Rev. Lett. **106**, 181301 (2011) [arXiv:1011.1500 [hep-ph]].

[16] N. Barnaby, E. Pajer and M. Peloso, Phys. Rev. D **85**, 023525 (2012) [arXiv:1110.3327 [astro-ph.CO]].

[17] M. Cicoli and F. Quevedo, Class. Quant. Grav. **28**, 204001 (2011) [arXiv:1108.2659 [hep-th]].

[18] T. Banks, M. Dine, P. J. Fox and E. Gorbatov, JCAP **0306**, 001 (2003) [hep-th/0303252].

[19] N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa, JHEP **0706**, 060 (2007) [hep-th/0601001].

[20] J. J. Blanco-Pillado, C. P. Burgess, J. M. Cline, C. Escoda, M. Gomez-Reino, R. Kallosh, A. D. Linde and F. Quevedo, JHEP **0411**, 063 (2004) [hep-th/0406230].

[21] A. R. Liddle, A. Mazumdar and F. E. Schunck, Phys. Rev. D **58**, 061301 (1998) [astro-ph/9804177].

[22] E. J. Copeland, A. Mazumdar and N. J. Nunes, Phys. Rev. D **60**, 083506 (1999) [astro-ph/9904309].

[23] A. D. Linde, Phys. Lett. B **129**, 177 (1983).

[24] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1303.5082 [astro-ph.CO].

[25] P. A. R. Ade *et al.* [BICEP2 Collaboration], Phys. Rev. Lett. **112**, 241101 (2014) [arXiv:1403.3985 [astro-ph.CO]].

[26] N. Barnaby, R. Namba and M. Peloso, JCAP **1104**, 009 (2011) [arXiv:1102.4333 [astro-ph.CO]].

[27] E. V. Bugaev and P. A. Klimai, Int. J. Mod. Phys. D **22**, 1350034 (2013) [arXiv:1303.3146 [astro-ph.CO]].

[28] E. Bugaev and P. Klimai, Phys. Rev. D **88**, 023521 (2013) [arXiv:1212.6529 [astro-ph.CO]].

[29] M. LoVerde, E. Nelson and S. Shandera, JCAP **1306** (2013) 024 [arXiv:1303.3549 [astro-ph.CO]].

[30] P. D. Meerburg and E. Pajer, JCAP **1302**, 017 (2013) [arXiv:1203.6076 [astro-ph.CO]].

[31] Y. Wang, arXiv:1303.1523 [hep-th].

[32] E. Bugaev and P. Klimai, Phys. Rev. D **85**, 103504 (2012) [arXiv:1112.5601 [astro-ph.CO]].

[33] E. Bugaev and P. Klimai, Phys. Rev. D **79**, 103511 (2009) [arXiv:0812.4247 [astro-ph]].

[34] W. H. Press and P. Schechter, Astrophys. J. **187** (1974) 425.

[35] K. Kohri and J. Yokoyama, Phys. Rev. D **61**, 023501 (2000) [astro-ph/9908160].

[36] B. J. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, Phys. Rev. D **81**, 104019 (2010) [arXiv:0912.5297 [astro-ph.CO]].

[37] S. G. Crowder, R. Namba, V. Mandic, S. Mukohyama and M. Peloso, Phys. Lett. B **726**, 66 (2013) [arXiv:1212.4165 [astro-ph.CO]].