

On evaporation of Randall-Sundrum black holes

P. I. Dyadina^{a*}, B. N. Latosh^{b†}, S. O. Alexeyev^{a‡}

^a Sternberg Astronomical Institute, Lomonosov Moscow State University

Universitetsky Prospekt, 13, Moscow 119991, Russia

^b Physics Department, Institute of Natural Sciences, Ural Federal University

19 Mira street, 620002, Ekaterinburg Russian Federation

Abstract

We investigate the possibilities of experimental search for new physics which is predicted by Randall-Sundrum II gravity model. For this purpose we obtained the law of evaporation for Dadhich-Rezania black hole solution and black hole solution by S. Abdolrahimi et al. within RS II. Thus we look for cosmological tests of these theory via a study of gamma-ray bursts from primordial black holes radiation.

1 Introduction

The theory of general relativity (GR) is a huge step in the understanding of our Universe as it allows to go beyond the Newtonian picture of the world.

But our understanding of the fundamental laws still has several shortcomings. Already in 1930s the problem of galactic rotation curves appeared [1]. Moreover the accelerated expansion of the universe (i.e., dark energy) was found from cosmological observations recently [2] and we still can't merge gravity with quantum mechanics. So it became obvious that GR should be modified.

Brane world models are one of the ways to expand GR. One of the first brane world model was proposed by Shaposhnikov and Rubakov [3]. In brane world scenario matter is localized on three-dimensional surface in multidimensional space. There are several brane world models and the Randall-Sundrum scenario [4], [5] is a rather common one. In the Randall-Sundrum model matter and three fundamental interactions are localized on the three-dimensional surface in five-dimensional spacetime (bulk) and gravity can propagate into the bulk. In the RS model extra dimension is not compact. The RS scenario includes two models. Model RSI contains two branes. It was created to solve the hierarchy problem of particle physics. In RSII model there is only one brane in the bulk. We use RSII because in this model possible difference between the behavior of gravity GR appears only because of the extra dimension. We research Dadhich-Rezania solution [6], [17] and S. Abdolrahimi, C. Cattoën, D. N. Page, S. Yaghoobpour-Tari solution (ACPY) [7] within RSII model.

*e-mail: guldur.anwo@gmail.com

†e-mail: latosh.boris@gmail.com

‡e-mail: salexeyev@gmail.com

2 Models

Dadhich-Rezania solution is one of the first black hole solutions. It is exact asymptotically flat solution describing an uncharged, nonrotating black hole:

$$ds^2 = - \left(1 - \left(\frac{2M}{r} + \left(\frac{q}{\tilde{M}_{\text{Pl}}} \right) \frac{1}{r^2} \right) \right) dt^2 + \frac{dr^2}{1 - \left(\frac{2M}{r} + \left(\frac{q}{\tilde{M}_{\text{Pl}}} \right) \frac{1}{r^2} \right)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (1)$$

where \tilde{M}_{Pl} is the multidimensional Planck mass, M is the black hole mass and q is the tidal charge.

ACPY solution is asymptotically AdS solution. It was obtained by numerical simulation. To search for the experimental consequences of modified gravity theories we used the effects of black hole evaporation.

$$\begin{cases} ds^2 = \left[1 - \frac{1}{(-\Lambda r^2)} \frac{r - 2M}{r - 1.5M} \left(F - r \frac{dF}{dr} \right) \right] \left(1 - \frac{2M}{r} \right)^{-1} dr^2 - \\ \quad - \left(1 - \frac{2M}{r} \right) dt^2 + \left[r^2 + \frac{1}{(-\Lambda)} F \right] d\Omega^2, \\ F \approx 1 - 1.062 \left(\frac{2M}{r} \right) + 0.554 \left(\frac{2M}{r} \right)^2 - 0.120 \left(\frac{2M}{r} \right)^3 + \dots + 2.900 \left(\frac{2M}{r} \right)^{11}. \end{cases} \quad (2)$$

The function $F(r)$ describes the perturbation caused by the bulk.

In this work we consider the effects of black hole evaporation [8] to search the experimental consequences of modified gravity theories.

3 Black hole evaporation

In general relativity the effects of evaporation of the black holes with stellar masses are negligible. In extended gravity models typical lifetimes of black holes can differ from GR and the consequences of the evaporation can be observable. We can look for the effects of black hole evaporation on primordial black holes [9]. Primordial black holes are supposed to appear in the early universe due to density fluctuations with masses much smaller than the solar mass. The effects of evaporation play a significant role for them as the evaporation rate is inversely proportional to the black hole mass. The final stage of evaporation should be accompanied by the emission of high energy particles including gamma particles with energies of the TeV order. Events with similar energies occur at large distances from us ($z \leq 9.4$ [10]) and are called gamma ray bursts (GRB). Many GRB are identified with supernovae. But the nature of the most distant GRB remains a mystery. The last stage of the evaporation of primordial black holes may be the decision of this problem. Extended gravity models predict the existence of different masses black holes [11], [15], [16] which evaporate completely during the lifetime of the universe. Therefore the cosmological data can be used to search for events that can be explosion of primordial black holes in the final stages of evaporation.

Using data from the telescope Fermi LAT T. N. Ukwatta, J. H. MacGibbon and others got the closest distance at which the telescope is able to detect the evaporation of primordial black holes [12].

$$d \simeq 0.04 \left(\frac{\Omega}{\text{sr}} \right)^{-0.5} \left(\frac{E}{\text{GeV}} \right)^{0.7} \left(\frac{T}{\text{TeV}} \right)^{0.8} \text{pc}, \quad (3)$$

where d is the distance, Ω is the angular resolution of the telescope, E is the energy range of the telescope, T is the temperature of the black hole. After spending the same reverse procedure and using a telescope detected gamma-ray bursts, we can conclude that the detectable value of mass of primordial black holes should differ from the value predicted by general relativity (with a mass of 10^{15} g), more than five orders of magnitude.

4 Method

To determine the initial mass of the primordial black hole evaporating in our days, we need to measure its temperature. The original Hawking method contains complex calculations and is not very convenient. We used a modification of the method by S. Shankaranarayanan, T. Padmanabhan, and K. Srinivasan [13]. This method was tested on Schwarzschild and Reissner-Nordström black holes and gave the same result as the Hawking method. For this method we use the quasi-classical approximation of the wave function of a particle near the the black hole and consider particle tunneling through the potential barrier of the event horizon. Thus we get the temperature of the black hole.

In this method we use several approximations. As rotating black holes lose angular moment faster than the mass so we can consider nonrotating black holes. The emitted particles can be considered massless and spinless. In 1976 Page showed that black holes in GR emit massless particles with zero spin [14]. For extended theories of gravity we can consider the emission of such particles because we determine the temperature of a black hole on their distribution. The temperature of a black hole does not depend on the type of particles. So we use the wave function without spin part for the quantum description of a massive particle in the black hole field. We also assume that there is no radiation into the bulk and that the lifetime of virtual pairs is negligible because radiation occurs due to the process of creation and annihilation of virtual particles. This method is not applicable for rotating black holes. And it is not applicable for quantum black holes whose lifetimes are comparable with lifetimes of pairs of virtual particles.

We can neglect the matter surrounding black hole because the evaporation effects are important for low-mass black holes and such black holes are not able to keep any significant amount of matter in orbit. Pairs of virtual particles appear near a black hole horizon permanently and we can consider the tunneling of one particle of the pair under the horizon of the black hole. The lifetime of virtual particles is extremely small. So we can assume instantaneous tunneling.

Now we consider the metric in the following form

$$ds^2 = A(r)dt^2 - \frac{dr^2}{B(r)} - f(r)(d\theta^2 + \sin^2\theta d\varphi^2) , \quad (4)$$

Within this method the temperature of black hole depends only on the geometric parameters of the black hole:

$$T = -\frac{1}{4\pi \text{res} \frac{1}{\sqrt{AB}}} . \quad (5)$$

Using described method, we investigate Dadhich-Rezania solution and ACPY solution.

5 Results

Let's start with the solution which was found Naresh Dadhich, Roy Maartens, Philippos Papadopoulos, and Vahid Rezania [6]. This solution is similar to the Reissner-Nordström metric. But it contains a so called with tidal charge instead of the electric charge. For this solution,

the rate of the mass loss is:

$$-\frac{dM}{dt} = \frac{1}{216} \frac{k_B}{\pi^3 M^2} + \mathcal{O}(M^{-6}). \quad (6)$$

The leading term in (6) cannot produce the needed 5-order difference. The initial mass of the Dadhich-Rezania black hole that evaporates completely during the lifetime of the Universe proves this fact:

$$M_{\text{DR}} = 5.3 \times 10^{14} \text{g}. \quad (7)$$

As the obtained difference is much less than the cutoff threshold the “tidal charge” influence is vanishing and cannot have experimentally verifiable consequences.

S. Abdolrahimi, C. Cattoën, D. N. Page and S. Yaghoobpour-Tari have recently received a large black hole solution [7]. It describes the astrophysical black holes in RSII as a perturbation solution of Einstein’s equations within the approach $\text{AdS}_5\text{-CFT}_4$ in five-dimensional space - time with negative cosmological constant. Evaporation law for the metric Abdolrahimi - Page has the form:

$$-\frac{dM}{dt} \simeq \frac{1}{256} \frac{k_B}{\pi^3 M^2} \quad (8)$$

This evaporation law completely coincides with the evaporation law for Schwarzschild black hole solution. The initial mass of the ACPY black hole also coincides with initial mass of the Schwarzschild black hole and equals

$$M_{\text{DR}} = 5.0 \times 10^{14} \text{g}. \quad (9)$$

6 Conclusions

Our results allow to conclude that the accuracy of the current cosmological data is still insufficient in order to distinguish the general theory of relativity from RSII by analyzing the evaporation of primordial black holes.

References

- [1] F. Zwicky, *Helvetica Physica Acta* **6**, 110 (1933);
J.H. Oort, *Bull. Astron. Inst. Netherlands* **6**, 249 (1932).
- [2] M. Kowalski et al (Supernova Cosmology Project), *Astrophys.J.* **686**, 749 (2008).
- [3] V.A. Rubakov, M.E. Shaposhnikov, *Phys.Lett.* **B125**, 136 (1983).
- [4] L. Randall, R. Sundrum, *Phys. Rev. Lett.* **83**, 3370 (1999).
- [5] L. Randall, R. Sundrum, *Phys. Rev. Lett.* **83**, 4690 (1999).
- [6] N. Dadhich, R. Maartens, P. Papadopoulos, V. Rezania, *Phys. Lett.* **B487**, 1 (2000).
- [7] S. Abdolrahimi, C. Cattoën, D. N. Page, S. Yaghoobpour-Tari, *JCAP* **06**, 039 (2013).
- [8] S. W. Hawking, *Nature* **248**, 30 (1974); *Comm. Math. Phys.* **43**, 199 (1975).
- [9] B.J. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, *Phys. Rev.* **D81**, 104019, (2010).
- [10] A. Cucchiara, A.J. Levan, D.B. Fox et al., *Astrophys. J.* **736**, 7 (2011).

- [11] J. H. MacGibbon and B. J. Carr, *Astrophys. J.* **371**, 447 (1991).
- [12] T. N. Ukwatta, J. H. MacGibbon, W. C. Parke et al., *AIP Conf.Proc.* **1133**, 440 (2009).
- [13] S. Shankaranarayanan, T. Padmanabhan and K. Srinivasan, *Class. Quant. Grav.* **19**, 2671 (2002).
- [14] D.N. Page, *Phys. Rev.* **D13**, 198 (1976);
 D.N. Page, *Phys. Rev.* **D14**, 3260 (1976);
 D.N. Page, *Phys. Rev.* **D16**, 2402 (1977).
- [15] S.O. Alexeyev, M.V. Pomazanov, *Phys. Rev.* **D55**, 2110 (1997);
 S.O. Alexeyev, M.V. Sazhin, *Gen. Relativ. Grav.* **8**, 1187 (1998);
 S.O. Alexeyev, A. Barrau, K.A. Rannu, *Phys. Rev.* **D79**, 067503 (2009);
 S.O. Alexeyev, K.A. Rannu, *JETP* **114**, 406 (2012).
- [16] S. Alexeyev, A. Barrau, G. Boudoul, O. Khovanskaya, M. Sazhin, *Class. Quant. Grav.* **19**, 4431 (2002).
- [17] S.O. Alexeyev, D.A. Starodubtseva, *JETP* **111**, 576 (2010).