Cosmic Antigravity

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Abstract

It is shown that in astronomical systems with rising energy density $F(R)$-modified gravitational theories lead to curvature oscillations with high frequency and large amplitude. Detailed study of spherically symmetric solutions of the modified gravity equations is performed for finite size astronomical objects. It is shown that for the solutions in which curvature scalar oscillates with large amplitude the analysis of Jeans instability is essentialy modified. It is discovered that for some astronomical objects the modified theory could lead to gravitational repulsion, so such objects would form relatively thin shells instead of quasi uniform bodies.

Discovery of cosmic acceleration based on the accumulated astronomical data [1] is the most impressive fact of the last quarter of century, but the driving force behind this phenomenon is still unknown. Among possible explanations of the accelerated expansion of the universe one of the most popular is an assumption of a new form of cosmological energy density with large negative pressure, $P < -\varrho/3$, the so-called dark energy.

A competing mechanism to create and describe the accelerated expansion is presented by modifications of gravity in which the usual action of General Relativity (GR) acquires an additional term, $F(R)$:

$$S = \frac{m_{Pl}^2}{16\pi} \int d^4x \sqrt{-g}\, [R + F(R)] + S_m, \tag{1}$$

where $m_{Pl} = 1.22 \cdot 10^{19}$ GeV is the Planck mass, R is the scalar curvature, and S_m is the matter action. The non-linear function $F(R)$ is chosen in such a way that the modified GR equations have solution $R = const$ in absence of any matter source.

The initial suggestion [2] of gravity modification with $F(R) \sim \mu^4/R$ suffered from strong instabilities in celestial bodies [3]. Because of that, further modifications have been suggested [4, 5] which are free of these instabilities. The suggested modifications, however, may lead to infinite-R singularities in the past cosmological history [6] and in the future in astronomical systems with rising energy/matter density [7, 8]. These singularities can be successfully cured by an addition of R^2-term into the action. Such a contribution naturally appears as a result of quantum corrections due to matter loops in curved space-time [9, 10]. Another mechanism which may in principle eliminate these singularities is particle production by the oscillating curvature [11]. If the production rate is sufficiently high, the oscillations of R are efficiently damped and the singularity could be avoided.

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The R^2 term may also have dominated in the early universe where it could lead to strong particle production. The process was studied long ago in [10, 12]. Renewed interest to this problem arose recently [11], stimulated by the interest in possible effects of additional ultraviolet terms, $\sim R^2$, in infrared-modified $F(R)$ gravity models.

In what follows we consider the version of the modified gravity suggested by Starobinsky [5]:

$$F(R) = -\lambda R_0 \left[1 - \left(1 + \frac{R^2}{R_0^2}\right)^{-n}\right] - \frac{R^2}{6m^2}, \tag{2}$$

where n is an integer, $\lambda > 0$, and $|R_0| \sim 1/t_u^2$, where $t_u \approx 13$ Gyr is the universe age. The R^2-term, absent in the original formulation, has been included to prevent curvature singularities in the presence of contracting bodies [8], and is relevant only at very large curvatures, because we need $m \gtrsim 10^5$ GeV in order to preserve the successful predictions of the standard BBN [11].

The evolution of R is determined from the trace of the modified Einstein equations:

$$3D^2 F_{,R} - R + RF_{,R} - 2F = \widetilde{T}\,, \tag{3}$$

where $D^2 \equiv D_\mu D^\mu$ is the covariant D'Alambertian operator, $F_{,R} \equiv dF/dR$, $\widetilde{T} = \widetilde{T}^\mu_\mu$, $\widetilde{T}_{\mu\nu} = 8\pi T_{\mu\nu}/m_{Pl}^2$, and $T_{\mu\nu}$ is the energy-momentum tensor of matter.

We are particularly interested in the regime $|R_c| \ll |R| \ll m^2$, in which F can be approximated by

$$F(R) \simeq -R_c \left[1 - \left(\frac{R_c}{R}\right)^{2n}\right] - \frac{R^2}{6m^2}\,. \tag{4}$$

We consider a nearly-homogeneous distribution of pressureless matter, with energy/mass density rising with time but still relatively low (e.g. a gas cloud in the process of galaxy or star formation). In such a case the spatial derivatives can be neglected and, if the object is far from forming a black hole, the space-time would be almost Minkowski. Then eq. (3) takes the form

$$3\partial_t^2 F_{,R} - R - \widetilde{T} = 0\,. \tag{5}$$

Let us introduce the dimensionless quantities[1]

$$\begin{aligned} z &\equiv \frac{T(t)}{T(t_{in})} \equiv \frac{\widetilde{T}}{T_0} = \frac{\varrho_m(t)}{\varrho_{m0}}\,, \qquad y \equiv -\frac{R}{T_0}\,, \\ g &\equiv \frac{T_0^{2n+2}}{6n(-R_c)^{2n+1}m^2} = \frac{1}{6n(mt_U)^2}\left(\frac{\varrho_{m0}}{\varrho_c}\right)^{2n+2}, \qquad \tau \equiv m\sqrt{g}\,t\,, \end{aligned} \tag{6}$$

where $\varrho_c \approx 10^{-29}$ g/cm^3 is the cosmological energy density at the present time, ϱ_{m0} is the initial value of the mass/energy density of the object under scrutiny, and $T_0 = 8\pi\varrho_{m0}/m_{Pl}^2$. Next let us introduce the new scalar field:

$$\xi \equiv \frac{1}{2n}\left(\frac{T_0}{R_c}\right)^{2n+1} F_{,R} = \frac{1}{y^{2n+1}} - gy\,, \tag{7}$$

in terms of which eq. (5) can be rewritten in the simple oscillator form:

$$\xi'' + z - y = 0\,, \tag{8}$$

where a prime denotes derivative with respect to τ. The potential of the oscillator is defined by:

$$\frac{\partial U}{\partial \xi} = z - y(\xi). \tag{9}$$

[1]The parameter g should not be confused with $\det g_{\mu\nu}$.

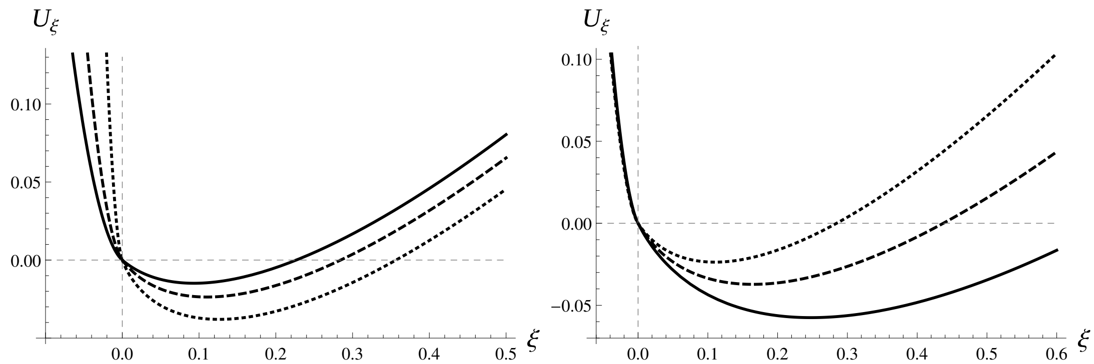

Figure 1: Examples of the variation of potential (12) for different values of parameters. *Left panel* ($n = 2$, $z = 1.5$) solid line: $g = 0.02$, dashed line: $g = 0.01$, dotted line: $g = 0.002$. *Right panel* ($n = 2$, $g = 0.01$) solid line: $z = 1.3$, dashed line: $z = 1.4$, dotted line: $z = 1.5$. The bottom of the potential moves to higher values of U and lower values of ξ, as g and z increase.

The minimum of potential $U(\xi)$ is located at $y(\xi) = z(\tau)$, so it moves with time according to

$$\xi_{min}(\tau) = z(\tau)^{-(2n+1)} - gz(\tau). \tag{10}$$

It is intuitively clear that even if initially ξ takes its GR value $\xi = \xi_{min}$ it would not catch the motion of the minimum and as a result ξ starts to oscillate around it. Dimensionless frequency of small oscillations, Ω, is determined by:

$$\Omega^2 = \left.\frac{\partial^2 U}{\partial \xi^2}\right|_{y=z} = \left(\frac{2n+1}{z^{2n+2}} + g\right)^{-1}. \tag{11}$$

Note that physical frequency is $\omega = \Omega\, m\sqrt{g}$.

One cannot analytically invert eq. (7) to find the exact expression for $U(\xi)$. However, we can find an approximate expression for $gy^{2n+2} \ll 1$ ($\xi > 0$) and $gy^{2n+2} \gg 1$ ($\xi < 0$). The value $\xi = 0$ separates two very distinct regimes, in each of which Ω has a very simple expression and ξ is dominated by either one of the two terms in the r.h.s. of eq. (7). Hence, in those limits the relation $\xi = \xi(y)$ can be inverted giving an explicit expression for $y = y(\xi)$, and therefore the following form for the potential:

$$U(\xi) = U_+(\xi)\Theta(\xi) + U_-(\xi)\Theta(-\xi)\,, \tag{12a}$$

where

$$\begin{aligned} U_+(\xi) &= z\xi - \frac{2n+1}{2n}\left[\left(\xi + g^{(2n+1)/(2n+2)}\right)^{2n/(2n+1)} - g^{2n/(2n+2)}\right], \\ U_-(\xi) &= \left(z - g^{-1/(2n+2)}\right)\xi + \frac{\xi^2}{2g}\,. \end{aligned} \tag{12b}$$

By construction U and $\partial U/\partial\xi$ are continuous at $\xi = 0$. The shape of this potential is shown in Fig. 1.

Our primary goal is to determine the amplitude and shape of the oscillations of y. In contrast to ξ the oscillations of y are strongly unharmonic and for negative and even very small $|\xi|$ the amplitude of y may be very large because $y \approx -\xi/g$, according to eq. (7).

A detailed study of the solutions of the modified gravity equations in the present day universe was performed in ref. [13,14] for finite-size astronomical objects. It was found that if the energy density rises with time, fast oscillations of the scalar curvature are induced, with an amplitude possibly much larger than the usual GR value $R = -\widetilde{T}$. The solution has the form:

$$R = R_{GR}(r)y(t), \tag{13}$$

where $R_{GR} = -\widetilde{T}(r)$ is the would-be solution in the limit of GR, while the quickly oscillating function $y(t)$ may be much larger than unity. According to ref. [14] the maximum value of y in the so-called spike region is:

$$y(t) \sim 6n(2n+1)mt_u \left(\frac{t_u}{t_{contr}}\right) \left[\frac{\varrho_m(t)}{\varrho_{m0}}\right]^{(n+1)/2} \left(\frac{\varrho_c}{\varrho_{m0}}\right)^{2n+2}, \tag{14}$$

where t_u is the universe age, t_{contr} is the characteristic contraction time, so the energy density of the contracting cloud behaves as $\varrho_m(t) = \varrho_{m0}(1 + t/t_{contr})$, with ϱ_{m0} being the initial energy density of the cloud, and $\varrho_c = 10^{-29}$ g/cm^3 being the present day cosmological energy density. Since the mass parameter m should be larger than about 10^5 GeV to avoid a conflict with BBN, so the factor mt_u is huge: $mt_u \geq 10^{47}$ and y can reach a very high value, if not suppressed by a small ratio $(\varrho_c/\varrho_{m0})^{2n+2}$, when n is large.

As shown in ref. [14], such spikes of high amplitude are formed if

$$6n^2(2n+1)^2 \left(\frac{t_u}{t_{contr}}\right)^2 \left[\frac{\varrho_m(t)}{\varrho_{m0}}\right]^{3n+1} \left(\frac{\varrho_c}{\varrho_{m0}}\right)^{2n+2} > 1. \tag{15}$$

The values of the densities ϱ_{m0} and $\varrho_m(t)$ depend upon the objects under scrutiny. If we speak about formation of galaxies or clusters thereof, the following ratios can be expected: $\varrho_{m0}/\varrho_c = 1 - 10^3$ and $\varrho_m(t)/\varrho_{m0}$ varying in the range $1 - 10^5$. Indeed the oscillations of curvature in such systems are excited if their mass density rises with time. For large scale structures this process began when they decoupled from the overall Hubble flow, which mostly took place at redshifts in the interval $z = 10 - 0$, and could result in creation of galaxies with energy density 5 orders of magnitude higher than the present day cosmological one. If we consider formation of stellar or planetary objects from the intergalactic gas with initial density 10^{-24} g/cm^3, then $\varrho_{m0}/\varrho_c = 10^5$ and $\varrho_m(t)/\varrho_{m0}$ can vary in the range $1 - 10^{24}$ or even larger.

If condition (15) is not fulfilled and the spiky solution with high amplitude is not excited, still as calculations of refs. [13,14] show, both numerically and analytically, the amplitude of $y(t)$ would be larger than unity, which is essential for the result presented below about gravitational repulsion inside systems with rising energy density. This feature is well demonstrated by the results of numerical calculations shown in Fig. 2.

The analysis of ref. [13,14] has been done under the assumption that the background space-time is nearly flat and so the background metric is almost Minkowsky. However, the large deviation of curvature from its GR value, found in these works, may invalidate the assumption of an approximately flat background and should be verified. In what follows we consider a spherically symmetric bubble of matter, e.g. a gas cloud or some other astronomical object, which occupies a finite region of space of radius r_m, and study spherically symmetric solution of corresponding equations of motion, assuming that the metric has the Schwarzschild form:

$$ds^2 = A(r,t)dt^2 - B(r,t)dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2). \tag{16}$$

We assume that the metric is close to the flat one, i.e.

$$A_1 = A - 1 \ll 1 \text{ and } B_1 = B - 1 \ll 1 \tag{17}$$

and study if and when this assumption remains true for the solutions with very large values of R found in our previous works [13,14]. It is convenient to use modified gravitational equations

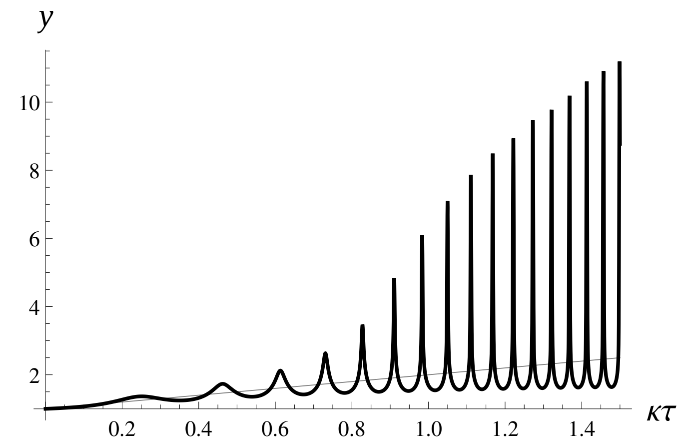

Figure 2: "Spikes" in the solutions. The results presented are for $n = 2$, $g = 0.001$. Note the asymmetry of the oscillations of y around $y = z$ and their anharmonicity.

in the following form (for details see [15]):

$$R_{00} - R/2 = \frac{\widetilde{T}_{00} + \Delta F_{,R} + F/2 - RF_{,R}/2}{1 + F_{,R}}, \tag{18}$$

$$R_{rr} + R/2 = \frac{\widetilde{T}_{rr} + (\partial_t^2 + \partial_r^2 - \Delta)F_{,R} - F/2 + RF_{,R}/2}{1 + F_{,R}}, \tag{19}$$

because their left hand sides contain only first derivatives of the metric coefficients. In the weak field limit, when derivatives of $A(r,t)$ and $B(r,t)$ are sufficiently small so that their square can be neglected, we obtain the following expressions for the R_{00} and R_{rr} components of the Ricci tensor and for the Ricci scalar R:

$$R_{00} \approx \frac{A'' - \ddot{B}}{2} + \frac{A'}{r}, \tag{20}$$

$$R_{rr} \approx \frac{\ddot{B} - A''}{2} + \frac{B'}{r}, \tag{21}$$

$$R \approx A'' - \ddot{B} + \frac{2A'}{r} - \frac{2B'}{r} + \frac{2(1-B)}{r^2}. \tag{22}$$

If the energy density of matter inside the the cloud, i.e. for $r < r_m$, is much larger than the cosmological energy density, the following restrictions are fulfilled:

$$F_{,R} \ll 1 \quad \text{and} \quad F \ll R \tag{23}$$

For static solutions the effects of gravity modifications in this limit are weak and, as we will see in what follows, the solution is quite close to the standard Schwarzschild one in agreement with other works on this subject. We assume that the spatial derivatives of F'_R are small in comparison with the time derivatives. So from eq. (3) it follows that $(\partial_t^2 - \Delta)F_{,R} = (\widetilde{T} + R)/3$ and we find:

$$B_1' + \frac{B_1}{r} = r\widetilde{T}_{00}, \tag{24}$$

$$A_1'' - \frac{A_1'}{r} = -\frac{3B_1}{r^2} + \ddot{B}_1 + \widetilde{T}_{00} - 2\widetilde{T}_{rr} + \frac{\widetilde{T}_{\theta\theta}}{r^2} + \frac{\widetilde{T}_{\varphi\varphi}}{r^2 \sin^2\theta} \equiv S_A. \tag{25}$$

Equation (24) has the solution:

$$B_1(r,t) = \frac{C_B(t)}{r} + \frac{1}{r}\int_0^r dr' r'^2 \widetilde{T}_{00}(r',t). \tag{26}$$

To avoid a singularity at $r = 0$ we have to assume that $C_B(t) \equiv 0$. Then this expression for B_1 formally coincides with the usual Schwarzschild solution, while the equation determining the metric coefficient A_1 allows for an additional freedom:

$$A_1(r,t) = C_{1A}(t)r^2 + C_{2A}(t) + \int_r^{r_m} dr_1\, r_1 \int_{r_1}^{r_m} \frac{dr_2}{r_2}\, S_A(r_2,t)\,. \tag{27}$$

The integration limits are chosen in such a way that the singularity at $r_2 = 0$ is avoided. Using equation (26) with $C_B = 0$ we can rewrite S_A as:

$$S_A = -\frac{3}{r^3}\int_0^r dr' r'^2 \widetilde{T}_{00}(r',t) + \frac{1}{r}\int_0^r dr' r'^2 \ddot{\widetilde{T}}_{00}(r',t) + \widetilde{T}_{00} - 2\widetilde{T}_{rr} + \frac{\widetilde{T}_{\theta\theta}}{r^2} + \frac{\widetilde{T}_{\varphi\varphi}}{r^2\sin^2\theta}\,. \tag{28}$$

Accordingly we obtain the following expression for $A_1(r,t)$:

$$A_1(r,t) = C_{1A}(t)r^2 + C_{2A}(t) + \int_r^{r_m} dr_1\, r_1 \int_{r_1}^{r_m} \frac{dr_2}{r_2}\left(\widetilde{T}_{00}(r_2,t) - 2\widetilde{T}_{rr}(r_2,t) + \frac{\widetilde{T}_{\theta\theta}(r_2,t)}{r^2} + \frac{\widetilde{T}_{\varphi\varphi}(r_2,t)}{r^2\sin^2\theta}\right) - \int_r^{r_m} dr_1\, r_1 \int_{r_1}^{r_m} \frac{dr_2}{r_2}\left(\frac{3}{r_2^3}\int_0^{r_2} dr' r'^2 \widetilde{T}_{00}(r',t) - \frac{1}{r_2}\int_0^{r_2} dr' r'^2 \ddot{\widetilde{T}}_{00}(r',t)\right). \tag{29}$$

It is instructive to check how solutions (26) and (29) reduce to the vacuum Schwarzschild solution in GR. The mass of matter inside a radius r is defined in the usual way:

$$M(r,t) = \int_0^r d^3r\, T_{00}(r,t) = 4\pi \int_0^r dr\, r^2\, T_{00}(r,t)\,. \tag{30}$$

If all matter is confined inside a radius r_m, the total mass is $M \equiv M(r_m)$ and due to mass conservation it does not depend on time. Since $\widetilde{T}_{00} = 8\pi T_{00}/m_{Pl}^2$, we obtain for $r > r_m$, as expected, $B_1 = r_g/r$, where $r_g = 2M/m_{Pl}^2$ is the usual Schwarzschild radius.

Let us turn now to the calculation of A_1 (29). Evidently, for $r > r_m$ the first integral term vanishes because r_2 is also larger than r_m, in fact in this region we have $T_{\mu\nu} = 0$. The integral containing $\ddot{\widetilde{T}}_{00}$ is also zero due to total mass conservation. The remaining integral can be easily taken:

$$\int_r^{r_m} dr_1\, r_1 \int_{r_1}^{r_m} \frac{dr_2}{r_2}\,\frac{3}{r_2^3}\int_0^{r_2} dr' r'^2 \widetilde{T}_{00}(r',t) = \frac{r_g}{r} + \frac{r_g\, r^2}{2r_m^3} - \frac{3r_g}{2r_m}\,. \tag{31}$$

Thus the metric coefficient outside the source is:

$$A_1 = -\frac{r_g}{r} + \left[C_{1A}(t) - \frac{r_g}{2r_m^3}\right] r^2 + \left[C_{2A}(t) + \frac{3r_g}{2r_m}\right]\,. \tag{32}$$

Choosing $C_{1A} = r_g/(2r_m^3)$, to eliminate the r^2-term at infinity, and $C_{2A} = -3r_g/(2r_m)$ we obtain the usual Schwarzschild solution.

In the modified theory the internal solution remains of the same form (26) and (29), where the coefficient C_{1A}, however, may depend non-trivially on time. This coefficient can be found from eq. (22) if the curvature scalar is known. As previously mentioned, we have shown in papers [13, 14] that in systems with rising energy density the curvature scalar may be much larger than its value in GR. Using eqs. (26) and (29) and comparing them to eq. (22) we can conclude that the dominant contribution into such form of the curvature is given by $A'' + 2A'/r$, i.e. $C_{1A}(t) = R(t)/6$, where $R(t)$ is given by eqs. (13), (14).

There is an essential difference between the modified and the standard solutions in vacuum. In the standard case the term proportional to r^2 appears both at $r < r_m$ and $r > r_m$ with the same coefficient and hence it must vanish. On the other hand, for modified gravity such condition is not applicable and the $C_{1A}r^2$-term may be present at $r < r_m$ and absent at $r \gg r_m$. The vacuum solution for R is presumably $R \sim R_c$, where R_c the small cosmological curvature, plus possible oscillating terms.

Thus to summarize, the metric functions inside the cloud are equal to:

$$B(r,t) = 1 + \frac{2M(r,t)}{m_{Pl}^2 r} \equiv 1 + B_1^{(Sch)}, \tag{33}$$

$$A(r,t) = 1 + \frac{R(t)\, r^2}{6} + A_1^{(Sch)}(r,t)\,. \tag{34}$$

In other words we construct the internal solution assuming that it consists of two terms: the Schwarzschild one and the oscillating part generated by the rising density as is shown in our works [13, 14]. The expression for $A_1^{(Sch)}(r,t)$ can be found from (29) with constant $C_{A1} = r_g/2r_m^3$ and $C_{A2} = -3r_g/r_m$, as determined from eq. (32). As for the integrals in eq. (29), we calculated them assuming that matter is nonrelativistic, so the space components of $T_{\mu\nu}$ are negligible in comparison to T_{00}, and that the matter/energy density, $T_{00} \equiv \varrho_m(t)$, is spatially constant but may depend on time. The first two integrals in eq. (29) cancel out and only the integral containing the second time derivative of the mass density survives. So for the Schwarzschild part of the solution we find:

$$A_1^{(Sch)}(r,t) = \frac{r_g r^2}{2r_m^3} - \frac{3r_g}{2r_m} + \frac{\pi \ddot{\varrho}_m}{3m_{Pl}^2}\,(r_m^2 - r^2)^2\,. \tag{35}$$

As we noted, $R(t)$ is typically larger that the GR value: $|R_{GR}| = 8\pi\varrho_m/m_{Pl}^2$, so the second term in eq. (34), $R(t)r^2/6$, gives the dominant contribution into A_1 at sufficiently large r. Indeed, $r^2 R(t) \sim r^2 y R_{GR}$ with $y > 1$, while the canonical Schwarzschild terms are of the order of $r_g/r_m \sim \varrho_m r_m^2/m_{Pl}^2 \sim r_m^2 R_{GR}$.

As is already mentioned, the solution with large oscillating $R(t)$ was obtained [13, 14] under the assumption that the background metric weakly deviates from the flat Minkowsky one. Though this is certainly true for the Schwarzschild part of the solution (35), this may be questioned for the $r^2R(t)/6$ - term. Evidently the flat background metric is not noticeably distorted if $r^2 < 6/R(t)$. If the initial energy density of the cloud is of the order of the cosmological energy density, i.e. $R_{GR} \sim 1/t_u^2$, then the metric would deviate from the Minkowsky one for clouds having radius $r_m > t_u/\sqrt{y}$, where the maximum value of y is given by eq. (14). For systems where very large values of y are reached, the flat space approximation may be broken already for non-interestingly small r. However, at the stage of rising $R(t)$ when $y > 1$ but not huge, the flat space approximation would be valid over all the volume of the collapsing cloud. For large objects or large y, such that $Rr^2/6 \sim 1$, the approximation of flat background metric becomes inapplicable and one has to solve the exact non-linear equations; this situation will be studied elsewhere. If A_1 becomes comparable with unity, the evolution of $R(t)$ may significantly differ from that found in [13, 14], but it seems evident that once a large $y > 1$ is reached, it would remain larger than unity despite a possible back-reaction of the non-flat metric.

In the lowest order in the gravitational interaction the motion (the geodesic equation in metric (16)) of a non-relativistic test particle is governed by the equation:

$$\ddot{r} = -\frac{A'}{2} = -\frac{1}{2}\left[\frac{R(t)r}{3} + \frac{r_g r}{r_m^3}\right], \tag{36}$$

where A is given by eq. (34). Since $R(t)$ is always negative and large, the modifications of GR considered here lead to anti-gravity inside a cloud with energy density exceeding the cosmological one. Gravitational repulsion dominates over the usual attraction if

$$\frac{|R| r_m^3}{3r_g} = \frac{|R| r_m^3 m_{Pl}^2}{6M} = \frac{|R| r_m^3 m_{Pl}^2}{8\pi\varrho\, r_m^3} = \frac{|R|}{\widetilde{T}_{00}} \equiv y > 1\,, \tag{37}$$

so basically whenever oscillations of R start rising, regardless of the initial value of ϱ and to some extent of the specific $F(R)$ considered. Therefore, this is most likely a more fundamental statement, applicable to essentially all $F(R)$ models producing oscillations of R with the amplitude larger than the GR value.

So, in modified gravity and in systems with rising energy density, the curvature scalar would typically exceed the GR value R_{GR}, i.e. $y > 1$, and thus the gravitational repulsion would dominate over the usual Schwarzcshild attraction. The back-reaction of this repulsion would slow down the contraction but evidently do not stop it. Moreover, the repulsion could overtake the contraction at sufficiently large radius. As a result shell type structures could be formed. Hence the gravitational repulsion found here might be responsible for the formation of cosmic voids but the lengthy analysis of realistic scenarios is outside the framework of the presented letter.

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