Four problems for the $c-\tau$, b and super- $c-\tau$, b factories

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Abstract

The problems suggested by authors for studying at the c-τ, b and super- c-τ, b factories are stated.

I. Comparison of the mechanism of the light scalar meson production in $D_s^+ \to s\bar{s}\, e^+\nu \to [\sigma(600) + f_0(980)]\, e^+\nu \to \pi^+\pi^-\, e^+\nu$ with the mechanism of the light pseudoscalar meson production in $D_s^+ \to s\bar{s}\, e^+\nu \to (\eta/\eta')\, e^+\nu$ shows that $s\bar{s} \to \sigma(600)$ is negligibly small in comparison with $s\bar{s} \to f_0(980)$. As for $f_0(980)$, $s\bar{s} \to f_0(980)$ is not more 30% of $s\bar{s} \to \eta_s$ ($\eta_s = s\bar{s}$). The study of the light scalar mesons in semileptonic decays of the $D^+(D^-)$, $D^0(\bar{D}^0)$, $B^+(B^-)$, $B^0(\bar{B}^0)$ mesons is suggested.

II. Interference phenomenon observed in the $\psi(3770)$ resonance region the $e^+e^- \to D\bar{D}$ reactions is described with models satisfying the elastic unitarity requirement. As a candidate, a model with the mixing $\psi(3770)$ and $\psi(2S)$ resonances is proposed. The selection of theoretical models in the non-$D\bar{D}$ channels $e+e- \to \psi(3770) \to \gamma\chi_{c0}$, $J/\psi\eta$, $\phi\eta$, etc is suggested.

III. The branching ratios (BR) of decays $\psi(3770)$ and $\Upsilon(10580)$ into light (non-$D\bar{D}$ and non-$B\bar{B}$) hadrons caused by the intermediate real $D\bar{D}$ and $B\bar{B}$ states are calculated. We got a band of predictions for the branching ratios : $1\% \lesssim \mathrm{BR} \lesssim 15\%$. The lower bound is 10 times as large as the branching ratio of annihilation into three gluons.

IV. Enfant terrible of charmonium spectroscopy, the resonance $X(3872)$, generated a stream of interpretations and ushered in a new exotic XYZ spectroscopy. In the meantime, many (if not all) characteristics of $X(3872)$ are rather ambiguous. We construct spectra of decays of $X(3872)$ with good analytical and unitary properties which allows to define the branching ratio of the $X(3872) \to D^{*0}\bar{D}^0 + c.c.$ decay studying only one more decay, for example, the $X(3872) \to \pi^+\pi^- J/\psi(1S)$ decay. We next define the range of values of the coupling constant of $X(7872)$ with the $D^{*0}\bar{D}^0$ system. Finally, we show that our spectra are effective means of selection of models for $X(3872)$.

1 Light scalars in semi-leptonic decays of heavy quarkonia

Based on Ref. [1]. It is time to explore the light scalar mesons in the decays of heavy quarkonia. The semi-leptonic decays are of prime interest because they have the clear mechanisms.

1.1 The $D_s^+ \to (\sigma/f_0)\, e^+\nu$ and $D_s^+ \to (\eta/\eta')\, e^+\nu$ decays

Below we study the mechanism of production of the light scalar mesons in the $D_s^+ \to \pi^+\pi^-\, e^+\nu$ decays: $D_s^+ \to s\bar{s}\, e^+\nu \to [\sigma(600) + f_0(980)]\, e^+\nu \to \pi^+\pi^-\, e^+\nu$, and compare it with the mechanism of production of the light pseudoscalar mesons in the $D_s^+ \to (\eta/\eta')\, e^+\nu$ decays: $D_s^+ \to s\bar{s}\, e^+\nu \to (\eta/\eta')\, e^+\nu$, in a model of the Nambu-Jona-Lasinio (NJL) type (see Fig. 1).

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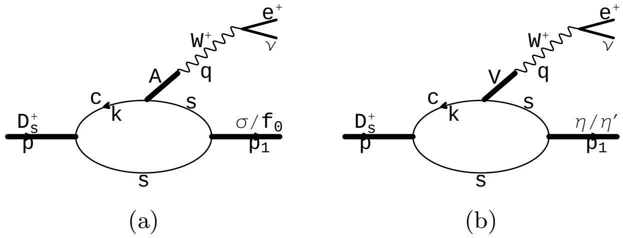


Figure 1: The model of the $D_s^+ \to \sigma/f_0\, e^+\nu$ (a) and $D_s^+ \to (\eta/\eta')\, e^+\nu$ (b) decays.

$$M[D_s^+(p) \to P(p_1)W^+(q) \to P(p_1)\, e^+\nu] = \tfrac{G_F}{\sqrt{2}} V_{cs} V_\alpha L^\alpha\,,$$
$$M[D_s^+(p) \to S(p_1)W^+(q) \to S(p_1)\, e^+\nu] = \tfrac{G_F}{\sqrt{2}} V_{cs} A_\alpha L^\alpha\,,$$
$$V_\alpha = f_+^P(q^2)(p+p_1)_\alpha + f_-^P(q^2)(p-p_1)_\alpha\,, \quad A_\alpha = f_+^S(q^2)(p+p_1)_\alpha + f_-^S(q^2)(p-p_1)_\alpha\,,$$
$$L_\alpha = \bar\nu\gamma_\alpha(1+\gamma_5)e\,, \quad q=(p-p_1)\,.$$

The influence of $f_-^P(q^2)$ and $f_-^S(q^2)$ are negligible for m_{e^+}. The decay rates in the stable P and S states are given by

$$\frac{d\Gamma(D_s^+ \to P\, e^+\nu)}{dq^2} = \frac{G_F^2|V_{cs}|^2}{24\pi^3} p_1^3(q^2)|f_+^P(q^2)|^2,$$
$$\frac{d\Gamma(D_s^+ \to S\, e^+\nu)}{dq^2} = \frac{G_F^2|V_{cs}|^2}{24\pi^3} p_1^3(q^2)|f_+^S(q^2)|^2.$$

For the $f_+^P(q^2)$ and $f_+^S(q^2)$ form factors we use the vector dominance model

$$f_+^P(q^2) = f_+^P(0)\tfrac{m_V^2}{m_V^2-q^2} = f_+^P(0) f_V(q^2)\,, \qquad f_+^S(q^2) = f_+^S(0)\tfrac{m_A^2}{m_A^2-q^2} = f_+^S(0) f_A(q^2)\,,$$

where $V = D_s^*(2112)^\pm$, $A = D_{s1}(2460)^\pm$. Following the NJL type model we write $f_+^P(0)$ and $f_+^S(0)$ in the form

$$f_+^P(0) = g_{D_s^+ c\bar s} F_P g_{s\bar s P}\,, \qquad f_+^S(0) = g_{D_s^+ c\bar s} F_S g_{s\bar s S}\,.$$

We know the structure of η and η': $\eta = \eta_q \cos\phi - \eta_s \sin\phi$, $\eta' = \eta_q \sin\phi + \eta_s\cos\phi$, where $\eta_q = (u\bar u + d\bar d)/\sqrt{2}$ and $\eta_s = s\bar s$. The angle $\phi = \theta_i + \theta_P$, where θ_i is the ideal mixing angle with $\cos\theta_i = \sqrt{1/3}$ and $\sin\theta_i = \sqrt{2/3}$, i.e., $\theta_i = 54.7^\circ$, and θ_P is the angle between the flavor-singlet state η_1 and the flavor-octet state η_8. The Particle Data Group (PDG) gives the θ_P band $-20^\circ \lesssim \theta_P \lesssim -10^\circ$ that gives us the opportunity to extract information about the $s\bar s \to \eta_s$ coupling constant, $g_{s\bar s\eta_s}$, from experiment and to compare with the $s\bar s \to f_0$ coupling constant, $g_{s\bar s f_0}$, extracted from experiment also. We consider the next set of θ_P.

$$\theta_P = -11^\circ: \quad \eta = 0.72\eta_0 - 0.69\eta_s\,, \quad \eta' = 0.69\eta_0 + 0.72\eta_s$$
$$\theta_P = -14^\circ: \quad \eta = 0.76\eta_0 - 0.65\eta_s\,, \quad \eta' = 0.65\eta_0 + 0.76\eta_s$$
$$\theta_P = -18^\circ: \quad \eta = 0.8\eta_0 - 0.6\eta_s\,, \quad \eta' = 0.6\eta_0 + 0.8\eta_s\,.$$

$$BR(D_s^+ \to s\bar s\, e^+\nu \to \eta\, e^+\nu) = (2.67 \pm 0.29)\,, \quad BR(D_s^+ \to s\bar s\, e^+\nu \to \eta'\, e^+\nu) = (9.9 \pm 2.3)\times 10^{-3}.$$

The $D_s^+ \to s\bar s\, e^+\nu \to \pi^+\pi^-\, e^+\nu$ decay amplitude is given by

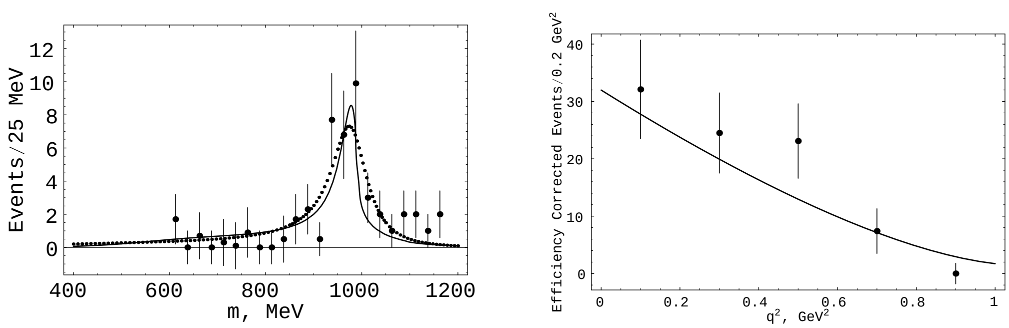
$$M(D_s^+ \to s\bar s\, e^+\nu \to [\sigma(600){+}f_0(980)]\, e^+\nu \to \pi^+\pi^-\, e^+\nu) = \frac{G_F}{\sqrt{2}} V_{cs}\, L^\alpha\, (p{+}p_1)_\alpha\, g_{D_s^+ c\bar s}\, f_A(q^2) \frac{e^{i\delta_B^{\pi\pi}}}{\Delta(m)}$$
$$\times\Big(F_\sigma g_{s\bar s\sigma} D_{f_0}(m) g_{\sigma\pi^+\pi^-} {+} F_\sigma g_{s\bar s\sigma}\Pi_{\sigma f_0}(m) g_{f_0\pi^+\pi^-} {+} F_{f_0} g_{s\bar s f_0}\Pi_{f_0\sigma}(m) g_{\sigma\pi^+\pi^-} {+} F_{f_0} g_{s\bar s f_0} D_\sigma(m) g_{f_0\pi^+\pi^-}\Big),$$

where m is the invariant mass of the $\pi\pi$ system, $\Delta(m) = D_{f_0}(m)D_\sigma(m) - \Pi_{f_0\sigma}(m)\Pi_{\sigma f_0}(m)$, $D_\sigma(m)$ and $D_{f_0}(m)$ are the inverted propagators of the σ and f_0 mesons, $\Pi_{\sigma f_0}(m) = \Pi_{f_0\sigma}(m)$ is the off-diagonal element of the polarization operator, which mixes the σ and f_0 mesons.

Thus, for the $D_s^+ \to \pi^+\pi^- e^+\nu$ decay rate we have

$$\frac{d^2\Gamma(D_s^+ \to \pi^+\pi^- e^+\nu)}{dq^2 dm} = \frac{G_F^2 \, |V_{cs}|^2}{24\,\pi^3} \, g^2_{D_s^+ c\bar{s}} \, |f_A(q^2)|^2 \, p_1^3(q^2,m) \frac{1}{8\pi^2} \, m \, \rho_{\pi\pi}(m) \left| \frac{1}{\Delta(m)} \right|^2$$
$$\times \Big| F_\sigma g_{s\bar{s}\sigma} D_{f_0}(m) g_{\sigma\pi^+\pi^-} + F_\sigma g_{s\bar{s}\sigma} \Pi_{\sigma f_0}(m) g_{f_0\pi^+\pi^-}$$
$$+ F_{f_0} g_{s\bar{s}f_0} \Pi_{f_0\sigma}(m) g_{\sigma\pi^+\pi^-} + F_{f_0} g_{s\bar{s}f_0} D_\sigma(m) g_{f_0\pi^+\pi^-} \Big|^2 ,$$

where $\rho_{\pi\pi}(m) = \sqrt{1 - 4m_\pi^2/m^2}$. The comparisons with the CLEO data [3] on the $\pi^+\pi^-$ mass (m) spectrum and q^2 distribution in $D_s^+ \to \pi^+\pi^- e^+\nu$ are shown in Figs. 2(a) and 2(b), respectively. The results of the analysis of the CLEO data are summarized in the table given below.



(a) (b)

Figure 2: (a) CLEO dotted line: $BR(D_s^+ \to f_0(980)\, e^+\nu \to \pi + \pi^- e^+\nu) = 0.20\%$. Our solid line: 0.17%. (b) The q^2 distribution for $BR(D_s^+ \to f_0(980)\, e^+\nu)$. The axial-vector dominance model (the theoretical curve) describes the data quite satisfactorily.

$Br(D_s^+ \to f_0 e^+\nu \to \pi^+\pi^- e^+\nu) = 0.17\%$			
$\frac{F_\sigma g_{s\bar{s}\sigma}}{F_{f_0} g_{s\bar{s}f_0}}$	$\frac{F^2_{f_0} g^2_{s\bar{s}f_0}}{F^2_\eta g^2_{s\bar{s}\eta}}$	$\frac{F^2_{f_0} g^2_{s\bar{s}f_0}}{F^2_{\eta'} g^2_{s\bar{s}\eta'}}$	$\frac{F^2_\eta g^2_{s\bar{s}\eta}}{F^2_{\eta'} g^2_{s\bar{s}\eta'}}$
0.039	0.67	0.49	0.73
The $\eta - \eta'$ mixing			
θ_P	$-11°$	$-14°$	$-18°$
$\frac{F^2_{f_0} g^2_{s\bar{s}f_0}}{F^2_\eta g^2_{s\bar{s}\eta_s}}$	0.32	0.29	0.24
$\frac{F^2_{f_0} g^2_{s\bar{s}f_0}}{F^2_{\eta'} g^2_{s\bar{s}\eta_s}}$	0.27	0.28	0.31

1.2 Discussion and conclusion

When fitting the CLEO data, we use the parameters of the resonances obtained by us in Ref. [2] in the analysis of the $\pi\pi$ scattering and the $\phi \to \gamma(\sigma + f_0) \to \gamma\pi^0\pi^0$ decay. In addition, we take into acount the Adler self consistency condition (the Adler zero at m^2 near $m_\pi^2/2$). Fitting the shape we fix only one parameter $f_+^\sigma(0)/f_+^{f_0}(0) = (F_\sigma g_{s\bar{s}\sigma})/(F_{f_0} g_{s\bar{s}f_0})$=0.039, 0.014, 0.055, 0.058, 0.032, 0.055 for six fits from Ref. [2]. The 44 events in Fig. 2(a) determine only one parameter $f_+^\sigma(0)/f_+^{f_0}(0)$. The branching ratio fixes $f_+^{f_0}(0)$. So the intensity of the $\sigma(600)$ production is much less than the intensity of the $f_0(980)$ production ($(f_+^\sigma(0)/f_+^{f_0}(0))^2 < 0.003$).

That is we find the direct evidence of decoupling of $\sigma(600)$ with the $s\bar{s}$ pair, which agrees well with the decoupling of $\sigma(600)$ with the $K\bar{K}$ states, obtained in Ref. [2] $g^2_{\sigma K^+K^-}/g^2_{\sigma\pi^+\pi^-}$ =0.04, 0.001, 0.01, 0.01, 0.003, 0.025 for six fits. The decoupling of $\sigma(600)$ with the $K\bar{K}$ states means also the decoupling of $\sigma(600)$ with $\sigma_q = (u\bar{u}+d\bar{d})/\sqrt{2}$ because σ_q results in $g^2_{\sigma K^+K^-}/g^2_{\sigma\pi^+\pi^-} = 1/4$. Fit 1 describes the $\pi^+\pi^-$ spectrum on better than others, $(f^{\sigma}_{+}(0)/f^{f_0}_{+}(0))^2 = (0.039)^2$, $g^2_{\sigma K^+K^-}/g^2_{\sigma\pi^+\pi^-} = 0.04$. So, the CLEO experiment gives new support in favour of the four-quark, $ud\bar{u}\bar{d}$, structure of the $\sigma(600)$ meson.

In the chirally symmetric model of the NJL type the coupling constants of the pseudoscalar and scalar partners with quarks are equal to each other, i.e., $g_{s\bar{s}\eta_s} = g_{s\bar{s}f_{0s}}$, where $f_{0s} = s\bar{s}$. If to neglect the strange quark mass as compared with the charmed quark mass ($m_s/m_c \ll 1$) in the numerators of the integrands for the decay diagrams, then $F_{f_0} = F_{\eta'}$ and we find that $g^2_{s\bar{s}f_0}/g^2_{s\bar{s}\eta_s} \approx 0.3$. So, the $f_{0s} = s\bar{s}$ part in the $f_0(980)$ wave function is near thirty percent. Taking into account the suppression of the $f_0(980)$ meson coupling with the $\pi\pi$ system, $g^2_{f_0\pi^+\pi^-}/g^2_{f_0K^+K^-} = 0.154$, one can conclude that the $f_{0q} = (u\bar{u}+d\bar{d})/\sqrt{2}$ part in the $f_0(980)$ wave function is suppressed also.

So, the CLEO experiment gives new support in favour of the four-quark, $(sd\bar{s}\bar{d} + sd\bar{s}\bar{d})/\sqrt{2}$, structure of the $f_0(980)$ meson, too.

1.3 Outlook

Certainly, there is an extreme need in experiment on the $D^+_s \to s\bar{s}\, e^+\nu \to \pi^+\pi^-\, e^+\nu$ decay with high statistics. Of great interest is the experimental search for the decays $D^0 \to d\bar{u}\, e^+\nu \to a^-_0(980)\, e^+\nu \to \pi^-\eta\, e^+\nu$ and $D^+ \to d\bar{d}\, e^+\nu \to a^0_0(980)\, e^+\nu \to \pi^0\eta\, e^+\nu$ (or the charge conjugate ones), which will give the information about the $a^-_q = d\bar{u}$ (or $a^+_q = u\bar{d}$) component in the $a^-_0(980)$ (or $a^+_0(980)$) wave function and $a^0_q = (u\bar{u} - d\bar{d})/\sqrt{2}$ component in the a^0_0 wave function. Now it is known that $BR(D^0 \to d\bar{u}\, e^+\nu \to \pi^-\, e^+\nu) = (2.89 \pm 0.08) \times 10^{-3}$ and $BR(D^+ \to d\bar{d}\, e^+\nu \to \pi^0\, e^+\nu) = (4.05 \pm 0.18) \times 10^{-3}$.

No less interesting is also search for the decays $D^+ \to d\bar{d}\, e^+\nu \to [\sigma(600) + f_0(980)]\, e^+\nu \to \pi^+\pi^-\, e^+\nu$ (or the charge conjugate ones), which will give the information about the $\sigma_q = (u\bar{u}+d\bar{d})/\sqrt{2}$ and $f_{0q} = (u\bar{u}+d\bar{d})/\sqrt{2}$ components in the $\sigma(600)$ and $f_0(980)$ wave functions respectively. Now it is known that $BR(D^+ \to d\bar{d}\, e^+\nu \to \eta\, e^+\nu) = (1.14 \pm 0.10) \times 10^{-3}$ and $BR(D^+ \to d\bar{d}\, e^+\nu \to \eta'\, e^+\nu) = (2.2 \pm 0.5) \times 10^{-4}$.

Comparative research of light scalar and pseudoscalar mesons in semileptonic decays of B quarkonia at super B-factories is very tempting. Now it is known that $BR(B^0 \to d\bar{u}\, e^+\nu \to \pi^-\, e^+\nu) = (1.44 \pm 0.05) \times 10^{-4}$, $BR(B^+ \to u\bar{u}\, e^+\nu \to \pi^0\, e^+\nu) = (7.79 \pm 0.26) \times 10^{-5}$, $BR(B^+ \to u\bar{u}\, e^+\nu \to \eta\, e^+\nu) = (3.8 \pm 0.6) \times 10^{-5}$ and $BR(B^+ \to u\bar{u}\, e^+\nu \to \eta'\, e^+\nu) = (2.3 \pm 0.8) \times 10^{-5}$.

2 Interference phenomena in the $\psi(3770)$ resonance region

Based on Refs. [4, 5]. In the process $e^+e^- \to D\bar{D}$, one investigates above all the D meson isoscalar electromagnetic form factor F^0_D. A similar representation of the $e^+e^- \to D\bar{D}$ reaction amplitude used for the data description guarantees the unitarity requirement on the model level. The sum of the $e^+e^- \to D^0\bar{D}^0$ and $e^+e^- \to D^+D^-$ reaction cross sections (see the data in Fig. 3) is expressed in terms of F^0_D in the following way

$$\sigma(e^+e^- \to D\bar{D}) = \frac{8\pi\alpha^2}{3s^2}\left|F^0_D(s)\right|^2 \nu(s),$$

where $\nu(s) = [p^3_0(s) + p^3_+(s)]/\sqrt{s}$, $p_{0,+}(s) = \sqrt{s/4 - m^2_{D^{0,+}}}$. Here we discuss the model for F^0_D with the mixing $\psi(3770)$ and $\psi(2S)$ resonances. Below, $\psi(3770)$ is denoted as ψ''.

It is clear that the main sources of the background in the ψ'' region are the tails from the J/ψ, $\psi(2S)$, $\psi(4040)$, $\psi(4160)$ and other resonances. It is easy to incorporate the right

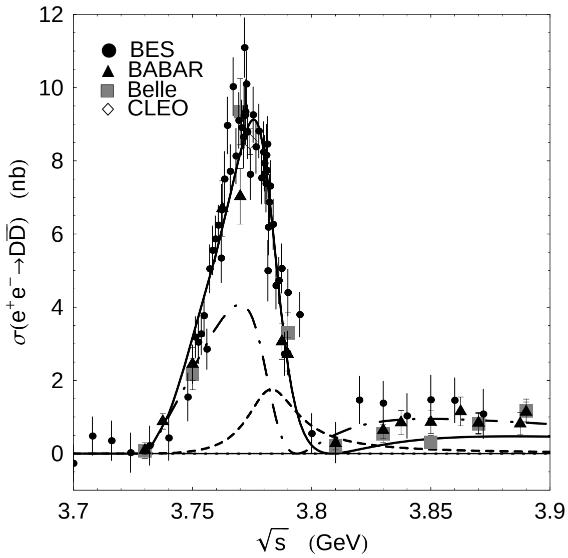


Figure 3: The simplest variant of the model of the mixed ψ'' and $\psi(2S)$ resonances. The solid curve is the fit to the data. The dashed and dot-dashed curves show the ψ'' and $\psi(2S)$ contributions, respectively. Bare parameters: $m_{\psi''} = 3.794$ GeV, $\Gamma_{\psi'' D\bar{D}} = 56.8$ MeV, $\Gamma_{\psi'' e^+e^-} = 0.062$ keV, and $g^2_{\psi(2S)D\bar{D}}/4\pi = 32.2$.

number of resonances in our scheme. Here we present the simplest variant of the model taking into account the background contribution from the nearest neighbor resonance $\psi(2S)$ and also discuss how it can be checked.

In the considered model the ψ'' and $\psi(2S)$ resonances mix via transitions $\psi'' \to D\bar{D} \to \psi(2S)$. Then the corresponding form factor F^0_D is given by

$$F^0_D(s) = \frac{\mathcal{R}_{D\bar{D}}(s)}{D_{\psi''}(s)D_{\psi(2S)}(s) - \Pi^2_{\psi''\psi(2S)}(s)}\,,$$

where $D_{\psi''}(s) = m^2_{\psi''} - s - i\sqrt{s}\Gamma_{\psi'' D\bar{D}}(s)$ and $D_{\psi(2S)}(s) = m^2_{\psi(2S)} - s - i\sqrt{s}\Gamma_{\psi(2S)D\bar{D}}(s)$ are the inverse propagators of ψ'' and $\psi(2S)$, and $\Gamma_{\psi'' D\bar{D}}(s) = g^2_{\psi'' D\bar{D}}\,\nu(s)/(6\pi\sqrt{s})$ and $\Gamma_{\psi(2S)D\bar{D}}(s) = g^2_{\psi(2S)D\bar{D}}\,\nu(s)/(6\pi\sqrt{s})$ their decay widths, respectively. The amplitude $\Pi_{\psi''\psi(2S)}(s) = i g_{\psi'' D\bar{D}} g_{\psi(2S)D\bar{D}}\,\nu(s)/(6\pi)$ describes the $\psi'' - \psi(2S)$ mixing, caused by the $\psi'' \to D\bar{D} \to \psi(2S)$ transitions via the real $D\bar{D}$ intermediate states, and $\mathcal{R}_{D\bar{D}}(s)$ in the above equation for F^0_D has the form: $\mathcal{R}_{D\bar{D}}(s) = (m^2_{\psi''} - s) g_{\psi(2S)\gamma} g_{\psi(2S)D\bar{D}} + (m^2_{\psi(2S)} - s) g_{\psi''\gamma} g_{\psi'' D\bar{D}}$. $m_{\psi''}$, $g_{\psi'' D\bar{D}}$, $g_{\psi''\gamma}$, and $g_{\psi(2S)D\bar{D}}$ are determined by fitting; $m_{\psi(2S)}$ and $g_{\psi''\gamma}$ are fixed by the PDG data.

Note that F^0_D in the considered model is proportional to the first-degree polynomial in s with real coefficients (see $\mathcal{R}_{D\bar{D}}(s)$ above). Hence the dip observed in $\sigma(e^+e^- \to D\bar{D})$ near 3.81 GeV, in all current experiments, can be explained by the $F^0_D(s)$ zero, caused by compensation between the ψ'' and $\psi(2S)$ contributions (see Fig. 3).

From the fitting of the $e^+e^- \to D\bar{D}$ data we all know, at the model level, about the $I=0$ P wave $D\bar{D}$ elastic scattering amplitude T^0_1:

$$T^0_1(s) = e^{i\delta^0_1(s)}\sin\delta^0_1(s) = \frac{\nu(s)}{6\pi}\left[\frac{(m^2_{\psi''} - s) g^2_{\psi(2S)D\bar{D}} + (m^2_{\psi(2S)} - s) g^2_{\psi'' D\bar{D}}}{D_{\psi''}(s)D_{\psi(2S)}(s) - \Pi^2_{\psi''\psi(2S)}(s)}\right].$$

The cross section and phase for $D\bar{D}$ elastic scattering in the P wave are shown in Fig. 4. Unfortunately, these predictions are not possible to verify. However, there are many other reactions which can be measured experimentally. Figure 5 illustrates the ψ'' shapes in non-$D\bar{D}$ decay channels.

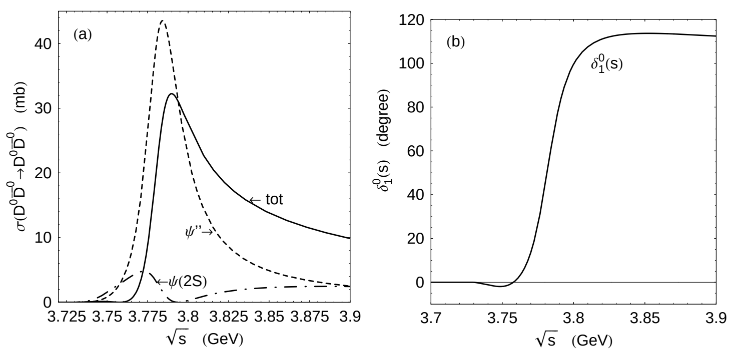


Figure 4: (a) The cross section $\sigma(D^0\bar{D}^0 \to D^0\bar{D}^0) = 3\pi|\sin\delta_1^0(s)|^2/p_0^2(s)$ and (b) the phase $\delta_1^0(s)$ for the simplest variant of the $\psi'' - \psi(2S)$ mixing model.

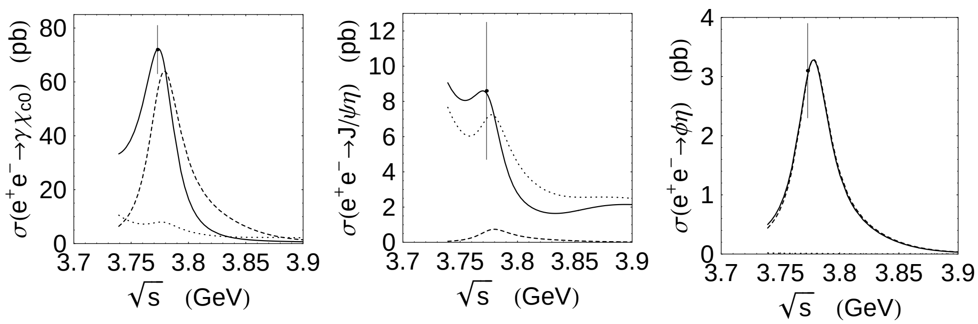


Figure 5: The solid curves show predictions of the model with the mixing ψ'' and $\psi(2S)$ resonances for the ψ'' peak shapes in the $e^+e^- \to \gamma\chi_{c0}$, $e^+e^- \to J/\psi\eta$, and $e^+e^- \to \phi\eta$ cross sections; the dashed and dotted curves show the contributions from ψ'' and $\psi(2S)$, respectively. The points with errors are the CLEO data.

Thus we conclude.

1. The ψ'' resonance shape keep important information about the production mechanism and interference with background. Its description requires taking into account the unitarity.

2. The simplest model mixing the ψ'' and $\psi(2S)$ resonances satisfies the unitarity requirement and describes the current data on the $e^+e^- \to D\bar{D}$ reaction cross section very well. Considering different variants, we extracted from experiment $g^2_{\psi(2S)D\bar{D}}/4\pi \approx 13 - 30$.

3. New high-statistics data on the reactions $e^+e^- \to D\bar{D}$ should help reveal the complex mechanism of the ψ'' production.

4. The measurements of mass spectra in the ψ'' region in the non-$D\bar{D}$ channels, such as $e^+e^- \to \gamma\chi_{c0}$, $J/\psi\eta$, $\phi\eta$, etc., will promote comprehensive study of the ψ'' resonance physics and effective selection of theoretical models.

3 Branching ratios of decays $\psi(3770)$, $\psi(4040)$, and $\Upsilon(10580)$ into light hadrons

Based on Refs [6, 7]. Exclusive decays of the ground-state $c\bar{c}$ and $b\bar{b}$ quarkonia $J/\psi(1S)$ and $\Upsilon(1S)$ into light hadrons are qualitatively similar in that their branching ratios are very small, $\sim 10^{-3} - 10^{-4}$. Since in the framework of the quark-gluon picture such decays are originated from the 3-gluon annihilation (see Fig. 6), a rough estimate gives $\Gamma((Q\bar{Q} \to (q\bar{q}) + (q\bar{q})) \sim$

$\alpha_s^3\Gamma((Q\bar{Q} \to 3gluons)$, where Q (q) denotes heavy (light) quark, means that each of above listed decays has the branching ratio which is much lower than the branching ratio of the decay into 3 gluons.

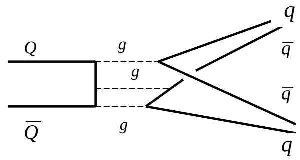


Figure 6: The 3-gluon annihilation.

Let us try to understand this suppression in the language of intermediate hadronic states, i.e. in the framework of dispersion approach. In this approach a amplitude of a decay under discussion can be represented as the sum over the contributions to the dispersion integral coming from the $D\bar{D}$, $D^*\bar{D} + c.c.$, $D^*\bar{D}^*$ etc., intermediate states in the case of the $J/\psi(1S)$ or $B\bar{B}$, $B^*\bar{B} + c.c.$, $B^*\bar{B}^*$ etc., intermediate states in the case of the $\Upsilon(1S)$. We do not see a reason for large suppression of each specific contribution. The most probable explanation of the suppression of the decays under consideration is the strong cancelation between the contributions from intermediate states listed above. However, such a cancelation could be broken when a new channel is opening. If so, the energy window may open where imaginary part of the amplitude is appreciable.

We believe that such a situation is realized for the states lying slightly above the production thresholds of open charm and beauty. Hence, the states $\psi(3770)$ and $\Upsilon(10580)$ are most promising from the point of view of the idea under consideration (see Figs. 7, 8, and 9). All told about the $\psi(3770)$ is transferred to the case of $\Upsilon(10580) \equiv \Upsilon(4S)$ by means of the replacements $\psi(3770) \to \Upsilon(10580), c \to b, D \to B, D_s^* \to B_s^*$, etc.

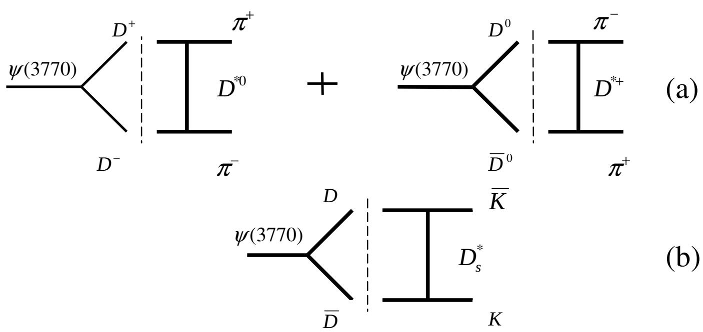


Figure 7: The $\psi(3770) \to PP$ decay in dispersion approach.

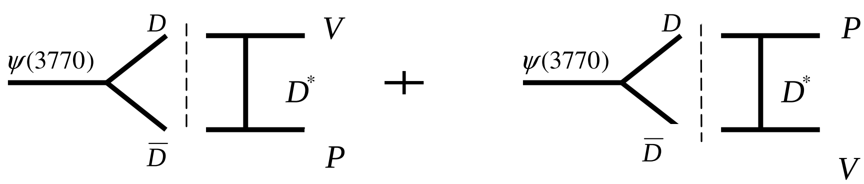


Figure 8: The $\psi(3770) \to VP$ decay in dispersion approach.

Calculating the amplitudes, we took into account two suppressing factors: 1) the absorption in final state, the suppressing factor is 1/2; 2) the exponential vertexes of the t-channel exchanges in Figs. 7, 8, and 9, $\exp\{\lambda t\}$ with $\lambda \approx (1/m_0)^2$, where m_0 is the mass of the lightest threshold in the t-channel, the effective suppressing factor is 7.

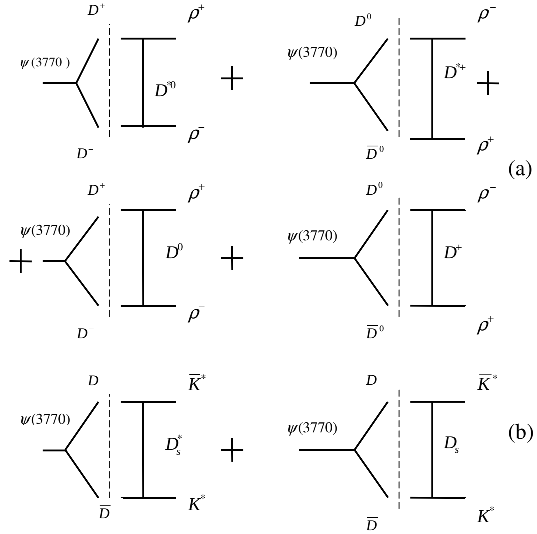


Figure 9: The $\psi(3770) \to VV$ decay in dispersion approach.

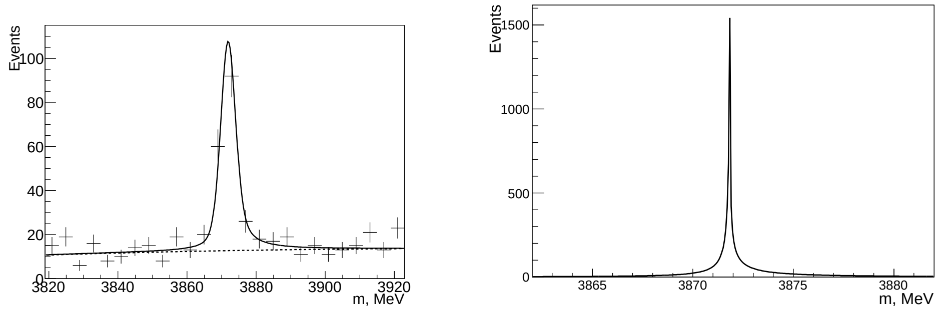
So, if carefully to formulate our results we got a band of predictions for the branching ratios (BR) of decays $\psi(3770)$ and $\Upsilon(10580)$ into light (non-$D\bar{D}$ and non-$B\bar{B}$) hadrons caused by the intermediate real $D\bar{D}$ and $B\bar{B}$: $1\% \lesssim \text{BR} \lesssim 15\%$. The lower bound is 10 times as large as the branching ratio of annihilation into three gluons.

Table 1: The results of our calculation are adduced in the table from Ref. [6]. See also tables in Ref. [7].

Mode	$\psi(3770)$	$\Upsilon(10580)$
$\pi^+\pi^-$	$2\cdot10^{-6}(7\cdot10^{-5})$	$8\cdot10^{-8}(6\cdot10^{-6})$
$K\bar{K}$	$2\cdot10^{-5}$	$2\cdot10^{-6}$
$\omega\pi^0$	$2\cdot10^{-5}(7\cdot10^{-4})$	$5\cdot10^{-6}(4\cdot10^{-4})$
$\omega\eta$	$3\cdot10^{-4}(1\cdot10^{-5})$	$3\cdot10^{-4}(4\cdot10^{-6})$
$\omega\eta'$	$1\cdot10^{-4}(7\cdot10^{-6})$	$2\cdot10^{-4}(2\cdot10^{-6})$
$\rho\pi$	$2\cdot10^{-3}(7\cdot10^{-5})$	$1\cdot10^{-3}(2\cdot10^{-5})$
$\rho\eta$	$1\cdot10^{-5}(3\cdot10^{-4})$	$4\cdot10^{-6}(3\cdot10^{-4})$
$\rho\eta'$	$7\cdot10^{-6}(1\cdot10^{-4})$	$2\cdot10^{-6}(2\cdot10^{-4})$
$\rho^+\rho^-$	$3\cdot10^{-5}(1\cdot10^{-3})$	$1\cdot10^{-4}(8\cdot10^{-3})$
$K^*\bar{K}+c.c$	$3\cdot10^{-4}$	$4\cdot10^{-4}$
$K^*\bar{K}^*$	$7\cdot10^{-4}$	$3\cdot10^{-3}$
$J/\psi(1S)+\pi^0$	$8\cdot10^{-6}(1\cdot10^{-4})$	-
$J/\psi(1S)+\eta$	$4\cdot10^{-5}(1\cdot10^{-6})$	-
$\Upsilon(1S)+\pi^0$	-	$7\cdot10^{-9}(5\cdot10^{-7})$
$\Upsilon(1S)+\eta$	-	$2\cdot10^{-7}(3\cdot10^{-9})$
3 gluons	$2\cdot10^{-4}$	$4\cdot10^{-4}$
total	$4\cdot10^{-3}(3\cdot10^{-3})$	$5\cdot10^{-3}(13\cdot10^{-3})$

4 How learn the branching ratio $X(3872) \to D^{*0}\bar{D}^0 + c.c.$

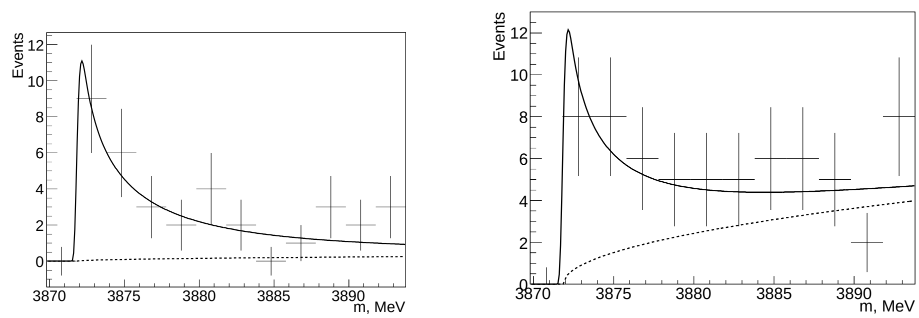
Based on Ref. [8]. The mass spectrum $\pi^+\pi^- J/\psi(1S)$ in the $X(3872) \to \pi^+\pi^- J/\psi(1S)$ decay looks as the ideal Breit-Wigner one, see Fig. 10.



(a) (b)

Figure 10: (a) The Belle data on the invariant $\pi^+\pi^- J/\psi(1S)$ mass (m) distribution. The solid line is our theoretical one with taking into account the Belle energy resolution. (b) Our undressed theoretical line.

The mass spectrum $D^{*0}\bar{D}^0 + c.c.$ in the $X(3872) \to D^{*0}\bar{D}^0 + c.c.$ decay looks as the typical resonance threshold enhancement, see Fig. 11.



(a) (b)

Figure 11: The Belle data on the invariant $D^{*0}\bar{D}^0 + c.c.$ mass (m) distribution. The solid line is our theoretical one with taking into account the Belle energy resolution. (a) $D^{*0} \to D^0\pi^0$. (b) $D^{*0} \to D^0\gamma$.

If structures in the above channels are manifestation of the same resonance, it is possible to define the branching ratio $BR(X(3872) \to D^{*0}\bar{D}^0 + c.c.)$ treating data only these (two) decay channels. We believe that the $X(3872)$ is the axial vector, 1^{++}. In this case the S wave dominates in the $X(3872) \to D^{*0}\bar{D}^0 + c.c.$ decay and hence is described by the Lagrangian $L(x) = g_A X^\mu \Big(D_\mu(x)\bar{D}(x) + \bar{D}_\mu(x)D(x)\Big)$. The width of the $X \to D^{*0}\bar{D}^0 + c.c.$ decay

$$\Gamma(X \to D^{*0}\bar{D}^0 + c.c.\,,\, m) \approx \frac{g_A^2}{8\pi}\frac{\rho(m)}{m}\,, \text{ where } \rho(m) = \frac{2|\mathbf{k}|}{m} = \frac{\sqrt{(m^2 - m_+^2)(m^2 - m_-^2)}}{m^2}\,,$$

$m_\pm = m_{D^{*0}} \pm m_{D^0}$;

$$\frac{dBR(X \to D^{*0}\bar{D}^0 + c.c.\,,\, m)}{dm} = \frac{4}{\pi}\frac{m^2\Gamma(X \to D^{*0}\bar{D}^0,\, m)}{|D_X(m)|^2}\,.$$

The branching ratio of $X(3872) \to D^{*0}\bar{D}^0 + c.c.$

$$BR(X \to D^{*0}\bar{D}^0 + c.c.) = \frac{4}{\pi}\int_{m_+}^{\infty}\frac{m^2\Gamma(X \to D^{*0}\bar{D}^0,\, m)}{|D_X(m)|^2}dm\,.$$

In others $\{i\}$ (non-$D^{*0}\bar{D}^0$) channels the $X(3872)$ state is seen as a narrow resonance that is why we write the mass spectrum in the i channel and the branching ratio of $X(3872) \to i$ as

$$\frac{dBR(X \to i\,,\, m)}{dm} = \frac{2}{\pi}\frac{m_X^2\,\Gamma_i}{|D_X(m)|^2}\,, \qquad BR(X \to i) = \frac{2}{\pi}\int_{m_0}^{\infty}\frac{m_X^2\Gamma_i}{|D_X(m)|^2}dm\,,$$

where Γ_i is the width of the $X(3872) \to i$ decay and m_0 is the threshold of the i state. The inverse propagator $D_X(m)$ is $D_X(m) = m_X^2 - m^2 + Re[\Pi_X(m_X)] - \Pi_X(m) - \imath m_X\Gamma$, where $\Gamma = \Sigma\Gamma_i$ is the total width of the $X(3872)$ decay into all non-$D^{*0}\bar{D}^0$ channels. When $m_+ \leq m$,

$$\Pi_X(m) = \frac{g_A^2}{8\pi^2}\left\{\frac{(m^2-m_+^2)}{m^2}\frac{m_-}{m_+}\ln\frac{m_{D^{*0}}}{m_{D^0}} + \rho(m)\left[\imath\pi + \ln\frac{\sqrt{m^2-m_-^2}-\sqrt{m^2-m_+^2}}{\sqrt{m^2-m_-^2}+\sqrt{m^2-m_+^2}}\right]\right\}.$$

In other areas of m ($m_- \leq m \leq m_+$, $m \leq m_-$ and $m^2 \leq 0$), the function $\Pi_X(m)$ is defined by analytic continuation. In fitting, our branching ratios satisfy unitarity: Sum$= BR(X \to D^{*0}\bar{D}^0 + c.c.) + \sum_i BR(X \to i) = 1$. Fitting the Belle data, we take into account the Belle results that $m_X = 3871.84\,\text{MeV} = m_{D^{*0}} + m_{D^0} = m_+$ and $\Gamma_{X(3872)} < 1.2$ MeV 90%CL that corresponds to $\Gamma < 1.2$ MeV, which controls the width of the $X(2872)$ signal in the $\pi^+\pi^- J/\psi(1S)$ channel and in every non-$D^{*0}\bar{D}^0$ channel, see Fig. 10 (b).

The results of our fit of the Belle data. $B_{seen} = BR(X \to D^{*0}\bar{D}^0 + c.c.\,; m \leq 3891.84\,\text{MeV})$, $B = BR(X \to D^{*0}\bar{D}^0 + c.c.)$, $B(r)_{seen} = \sum_i BR(X \to i\,; 3851.84 \leq m \leq 3891.84\,\text{MeV})$.

Γ	$g_A^2/8\pi$	χ^2/Ndf	$\mathcal{B}_{seen}$	$\mathcal{B}$	$\mathcal{B}(r)_{seen}$	Sum
$1.2_{-0.5}$	$0.9^{+4}_{-0.5}$	44/42	$0.5^{+0.1}_{-0.3}$	$0.8^{+0.2}_{-0.2}$	$0.2^{+0.2}_{-0.2}$	1

The current statistics is not sufficient for serious conclusions. Nevertheless, one can state that our results are consist with experiment. Really, in view of

$$BR(B \to X(3872)K) \times BR(X(3872) \to D^{*0}\bar{D}^0) = (0.80 \pm 0.20 \pm 0.1) \times 10^{-4},$$

$$BR(B^+ \to X(3872)K^+) \times BR(X(3872) \to \pi^+\pi^- J/\psi(1S)) = (8.61 \pm 0.82 \pm 0.52) \times 10^{-6},$$

$$BR(B^+ \to X(3872)K^+) \times BR(X(3872) \to \pi^+\pi^-\pi^0 J/\psi(1S)) = (0.6 \pm 0.2 \pm 0.1) \times 10^{-5},$$

$$BR(B^+ \to X(3872)K^+) \times BR(X(3872) \to \gamma J/\psi(1S)) = (1.78^{+0.48}_{-0.44} \pm 0.12) \times 10^{-6}$$

it follows that $BR(X \to D^{*0}\bar{D}^0 + c.c.\,; m \leq 3892\,\text{MeV})$ is a few times as large as the sum of all non-$D^{*0}\bar{D}^0$ known branching ratios.

So, when fitting the $X(3872) \to D^{*0}\bar{D}^0$ data and data for any $X(3872)$ decay into non-$D^{*0}\bar{D}^0$ state, $X(3872) \to i$, we find Γ and $g_A^2/8\pi$, which define $BR(X(3872) \to D^{*0}\bar{D}^0 + c.c.)$. Generally speaking, we don't need to know $BR(X(3872) \to i)$.

Our approach can serve as the guide in selection of theoretical models for the $X(3872)$ resonance. Indeed, let 3871.68 MeV $< M_X <$ 3871.95 MeV, $\Gamma < 1.2$ MeV and $g_A^2/8\pi < 0.1$ GeV^2 that does not contradict current experiment. But then $BR(X \to D^{*0}\bar{D}^0 + c.c.) < 0.3$, that is, unknown decays of $X(3872)$ into non-$D^{*0}\bar{D}^0$ states are considerable or dominant.

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