On Consistent Lagrangian Quantization of Yang–Mills Theories without Gribov Copies

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Abstract

We review the results of our research [A.A. Reshetnyak, IJMPA 29 (2014) 1450184; P.Yu. Moshin, A.A. Reshetnyak, Nucl. Phys. B 888 (2014) 92; P.Yu. Moshin, A.A. Reshetnyak, Phys. Lett. B 739 (2014) 110; P.Yu. Moshin, A.A. Reshetnyak, arXiv:1406.5086[hep-th]], devoted to a consistent Lagrangian quantization for gauge theories with soft BRST symmetry breaking, in particular, for various descriptions of the Yang–Mills theory without Gribov copies. The cited works rely on finite BRST and BRST-antiBRST transformations, respectively, with a singlet λ of nilpotent and a doublet λ_a, $a = 1, 2$, of anticommuting Grassmann parameters, both global and field-dependent. It turns out that global finite BRST and BRST-antiBRST transformations form a 1-parametric and a 2-parametric Abelian supergroup, respectively. Explicit superdeterminants corresponding to these changes of variables in the partition function allow one to calculate precise changes of the respective gauge-fixing functional. These facts provide the basis for a proof of gauge independence of the corresponding path integral under respective BRST and BRST-antiBRST transformations and lead to the appearance of modified Ward identities. It is shown that the gauge independence becomes restored for path integrals with soft BRST and BRST-antiBRST symmetry breaking terms. In this case, the form of transformation parameters is found to induce a precise change of the gauge in the path integral, thus connecting two arbitrary R_ξ-like gauges in the average effective action. Finite field-dependent BRST-antiBRST transformations are used to solve (perturbatively) the Gribov problem in the Gribov–Zwanziger approach. A modification of the path integral for theories with a gauge group, being consistent with gauge invariance and providing a restriction of the integration measure to the first Gribov region with a non-vanishing Faddeev–Popov determinant, is suggested.

Keywords: Faddeev–Popov rules, BRST-antiBRST Lagrangian quantization, Yang–Mills theory, Gribov–Zwanziger model, field-dependent BRST and BRST-antiBRST transformations

1 Introduction

It is well known that the electroweak and strong interactions are described by the Standard Model, with the Quantum Chromodynamics (QCD) as its constituent, and there are no experimental facts in conflict with QCD. While the Standard Model has been justified by the discovery of the Higgs boson, the problem of consistency in QCD is far from its solution, especially in view of the confinement phenomenon. The Lagrangian of QCD (and generally that of the Standard Model) belongs to the class of non-Abelian gauge theories [1, 2, 3, 4] of Yang–Mills (YM) type. Descendants of gauge invariance that emerge as one applies the Faddeev–Popov trick [5]

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to the partition function of Yang–Mills theories [6] are special supersymmetries, known as the BRST symmetry [7, 8] and the BRST-antiBRST symmetry [9, 10, 11]. They provide a basis for contemporary quantization methods applied to gauge theories [2, 12]. These symmetries are characterized by the presence of a Grassmann-odd parameter μ and two Grassmann-odd scalar parameters $(\mu, \bar{\mu})$, respectively. The latter parameters in the extended schemes of generalized Hamiltonian [13] and Lagrangian [14, 15] quantization (see [16] as well) form an $\mathrm{Sp}\,(2)$-doublet $(\mu, \bar{\mu}) \equiv (\mu_1, \mu_2) = \mu_a$, whereas the former parameter is used in the generalized canonical [17], [18] and field-antifield [19] quantization methods. When considered either as constants or as field-dependent functionals, these infinitesimal odd-valued parameters can be used, respectively, to obtain the Ward identities and to establish the gauge-independence of the partition function in the path integral approach.

On the other hand, due to the well-known Gribov problem [21], the covariant quantization of YM and gravity theories on a basis of the FP procedure cannot be realized correctly for the entire spectrum of the momenta distribution in the deep infra-red region for gauge fields once the gauge condition has been imposed using differentiation [22], since there remains an infinitely large number of discrete gauge copies after gauge-fixing, contrary to the case of Quantum Electrodynamics (with an Abelian gauge symmetry). This implies that a non-Abelian addition to the quadratic part of a Lagrangian turns a free well-defined partition function (of Gauss-type) to one for an interacting theory, which does not meet the requirement of positive definiteness for density of the distribution function, due to an infinite number of zero eigenvalues for the FP matrix. Gribov has studied YM theories in the Coulomb gauge and suggested a restriction of the domain of functional integration for gauge fields to the so-called first Gribov region, which has been effectively incorporated into the functional measure as the Heaviside Θ-function $(\mathcal{V}(1 - \sigma(0, A))$ for vanishing momenta k in the notations [21]), thus realizing the "no-pole" condition for the ghost propagator and a correct interpretation of the partition function.

There are other means of solving the Gribov problem: first, the Gribov–Zwanziger (GZ) procedure [23], where the mentioned Θ-function, due to certain non-perturbative arguments, such as a replacement of the Θ-function by the δ-function and the hypothesis [23] "the equivalence of the microcanonical and canonical ensembles in classical statistical mechanics is valid here, so that it is correct to replace the δ-function by the Boltzmann factor", may be applied to the Landau and Coulomb [24] gauges with a hermitian FP operator and a special addition to the standard FP action, known as the Gribov–Zwanziger horizon functional [23, 25]. However, this addition is not gauge-invariant and hence non-invariant under the initial BRST transformations. Second, there is a procedure of imposing an algebraic (rather than differential) gauge on auxiliary scalar fields in a theory with the ghost and antighost fields considered as classical gauge fields, which is non-perturbatively equivalent to a Yang–Mills theory with the same gauge group, but with the FP operator considered as part of the classical Lagrangian [26, 27, 28].

In [29], BRST transformations with a finite field-dependent parameter (FFDBRST) for YM theories with the FP quantum action have been first introduced by using a functional equation for the corresponding infinitesimal parameter, so as to provide the path integral with such a change of variables that would allow one to relate the quantum action given in a certain gauge to the one given in a different gauge, however, without solving this equation in a general setting, which has led to the appearance of numerous similar results (see [30, 31] and references therein). The problem of establishing a relation of the FP action in a certain gauge with the action in a different gauge by using a change of variables corresponding to a FFDBRST transformation has been generally solved in [32], thus providing an exact relation between a finite parameter and a finite change of the gauge-fixing condition. In particular, this result leads to the preservation of the number of physical degrees of freedom in a given YM theory with respect to FFDBRST transformations, which means the impossibility of relating the YM theory to another theory, e.g., to GZ action [25] in the same configuration space. Notice that the study of Gribov copies in YM theories has not been restricted to a certain gauge; see the use of the covariant and maximal Abelian gauges, as well as the Landau and Coulomb gauges in [33, 34, 35, 36, 37, 38].

Notice that the solution of a similar problem for arbitrary constrained dynamical systems in the generalized Hamiltonian formalism [17, 18] has been recently proposed in [39], whereas for general gauge theories (featuring reducible gauge symmetries and/or an open gauge algebra) an exact Jacobian generated by FFDBRST transformations in the path integral given by the BV procedure [19, 20] has been obtained in [40] (see [41] as well) and gives a positive solution of the consistency of *soft BRST symmetry breaking* [42].

In [44], we consider an extension of BRST-antiBRST transformations to the case of finite (both global and field-dependent) parameters in YM theories. In [45, 46, 47], we have done the same for general gauge theories, by using the Lagrangian and generalized Hamiltonian BRST-antiBRST quantization methods; see also [48]. In the present work, the origins of finite BRST and BRST-antiBRST transformations are reviewed in Sections 2 and 4 respectively, and then in Section 3 we use their properties to study their influence on the quantum structure of YM theories and general gauge theories, both with and without BRST(antiBRST) symmetry breaking terms, and using the respective BRST and BRST-antiBRST settings, including the cases of refined and standard GZ theories in Section 5. A modification of the path integral in YM theories, which is consistent with gauge invariance and which provides a restriction of the integration measure to the first Gribov region with a non-vanishing FP determinant, is suggested in Section 6.

We use the condensed notation of DeWitt and the conventions of [40, 44]. Unless otherwise specified by an arrow, derivatives with respect to the fields are taken from the right, and those with respect to the corresponding antifields are taken from the left. The raising and lowering of Sp (2) indices, $s^a = \varepsilon^{ab} s_b$, $s_a = \varepsilon_{ab} s^b$, is carried out using a constant antisymmetric metric tensor ε^{ab}, $\varepsilon^{ac}\varepsilon_{cb} = \delta^a_b$, subject to the normalization $\varepsilon^{12} = 1$. The Grassmann parity of a homogeneous quantity B is denoted as $\varepsilon(B)$.

2 Finite Field-dependent BRST Transformation and its Jacobian

An extended generating functional of Green's functions (GFGF) for a gauge theory defined in a total configuration space $\mathcal{M}$ parameterized by fields ϕ^A, $\varepsilon(\phi^A) = \varepsilon_A$, which, in the formalism of BRST quantization [19], contain the initial classical fields A^i, $\varepsilon(A^i) = \varepsilon_i$, $i = 1, \ldots, n$, the Nakanishi–Lautrup fields B^α, $\varepsilon(B^\alpha) = \varepsilon_\alpha$, $\alpha = 1, \ldots, m < n$, and the pairs of ghost and antighost fields[1] C^α, $\bar{C}^\alpha$, $\varepsilon(C^\alpha) = \varepsilon(\bar{C}^\alpha) = \varepsilon_\alpha + 1$, is given by the rule

$$Z_\Psi(J, \phi^*) = \int d\phi \ \exp\left\{\tfrac{i}{\hbar}\left[S_\Psi(\phi, \phi^*) + J_A\phi^A\right]\right\} \equiv \int \mathcal{I}^\Psi_{\phi,\phi^*} \exp\left\{\tfrac{i}{\hbar}J_A\phi^A\right\} , \tag{1}$$

where $\hbar, J_A, \phi^*_A$ and $\Psi(\phi)$ are, respectively, the Planck constant, external sources to ϕ^A, antifields, $\varepsilon(J_A) = \varepsilon(\phi^*_A) + 1 = \varepsilon_A$, and an admissible Fermionic gauge-fixing functional $\Psi(\phi)$. The usual GFGF is $Z_\Psi(J) = Z_\Psi(J, 0)$ and the quantum action $S_\Psi(\phi, \phi^*)$ is given by

$$S_\Psi(\phi, \phi^*) = S\left(\phi, \phi^* + \tfrac{\delta\Psi}{\delta\phi}\right), \ \text{where } S(\phi, 0) = \mathcal{S}_0(A), \tag{2}$$

with the classical action $\mathcal{S}_0(A)$ invariant under infinitesimal gauge transformations $\delta A^i = R^i_\alpha(A)\xi^\alpha$, $\varepsilon(R^i_\alpha) = \varepsilon_i + \varepsilon_\alpha$, whose generators $R^i_\alpha(A)$ form an algebra of gauge transformations,

$$\begin{aligned} &R^i_{\alpha,j}(A)R^j_\beta(A) - (-1)^{\varepsilon_\alpha\varepsilon_\beta} R^i_{\beta,j}(A)R^j_\alpha(A) = -R^i_\gamma(A)F^\gamma_{\alpha\beta}(A) - S_{0,j}(A)M^{ij}_{\alpha\beta}(A) \ , \\ &\texttt{for } F^\gamma_{\alpha\beta} = -(-1)^{\varepsilon_\alpha\varepsilon_\beta} F^\gamma_{\beta\alpha} \ , \quad M^{ij}_{\alpha\beta} = -(-1)^{\varepsilon_i\varepsilon_j} M^{ji}_{\alpha\beta} = -(-1)^{\varepsilon_\alpha\varepsilon_\beta} M^{ij}_{\beta\alpha} \,. \end{aligned} \tag{3}$$

[1] As well as the towers of additional ghost, antighost and Nakanishi–Lautrup fields, introduced according to the stage of reducibility of a given theory [20].

The bosonic functional $S = S(\phi, \phi^*)$, as well as the quantum action S_Ψ, satisfies the master equation (in two equivalent forms)

$$\Delta \exp\left\{\tfrac{\imath}{\hbar}S\right\} = 0 \Longleftrightarrow \tfrac{1}{2}(S,S) \;=\; \imath\hbar\,\Delta S\,, \tag{4}$$

expressed in terms of an odd Poisson bracket $(\cdot,\cdot)$, also called antibracket, and in terms of an odd Laplacian $\Delta = (-1)^{\varepsilon_A}\frac{\delta_l}{\delta\phi^A}\frac{\delta}{\delta\phi^*_A}$, defined in the field-antifield space [19]. Finite (group) BRST transformations introduced in [40] are invariance transformations for the integrand $\mathcal{I}^{\Psi}_{\phi,\phi^*}$ with account taken of (4) for S_Ψ,

$$\left(\phi^A, \phi^*_A\right) \to \left(\phi'^A, \phi'^*_A\right) = \left(\phi^A \exp\{\overleftarrow{s}_e\Lambda\}, \phi^*_A\right) \Longrightarrow \mathcal{I}^{\Psi}_{\phi\exp\{\overleftarrow{s}_e\Lambda\},\phi^*} = \mathcal{I}^{\Psi}_{\phi,\phi^*}, \tag{5}$$

where the set $\{g(\Lambda)\} = \{\exp\{\overleftarrow{s}_e\Lambda\}\}$ forms a one-parametric Lie supergroup with an odd parameter Λ, despite the fact that the generator $\overleftarrow{s}_e = \frac{\overleftarrow{\delta}}{\delta\phi^A}\frac{\delta S_\Psi}{\delta\phi^*_A}$ of BRST transformations fails to be nilpotent, $\overleftarrow{s}^2_e \neq 0$, due to the presence of $M^{ij}_{\alpha\beta} \neq 0$ in the gauge algebra relations (3) and also due to the presence of the operator Δ. When the parameter Λ is chosen as a field-dependent functional $\Lambda(\phi,\phi^*)$ depending parametrically on antifields, the set of $\{g(\Lambda)\}$ transforms into an non-Abelian supergroup. The superdeterminant of a change of variables corresponding to FFDBRST transformations (5) has been calculated in [40] and reads

$$\operatorname{Sdet}\left\|\frac{\delta\phi^A\exp\{\overleftarrow{s}_e\Lambda(\phi,\phi^*)\}}{\delta\phi^B}\right\| \;=\; \left(1+\Lambda\overleftarrow{s}_e\right)^{-1}\exp\{\overleftarrow{s}_e\Lambda(\phi,\phi^*)\}\Big\{1+\left(\Delta S_\Psi\right)\Lambda\Big\}, \tag{6}$$

with the notation $\frac{\delta S_\Psi}{\delta\phi^*_A} \equiv S^A_\Psi$. For constant Λ, the Jacobian reduces to $\operatorname{Sdet}\left\|\frac{\delta\phi^A\exp\{\overleftarrow{s}_e\Lambda(\phi,\phi^*)\}}{\delta\phi^B}\right\| = \Big\{1+\left(\Delta S_\Psi\right)\Lambda\Big\}$. The requirement of gauge independence for a finite change of the gauge,[2] $\Psi \to \Psi + \Delta_f\Psi$, leads to the compensation equation [40],

$$\mathcal{I}^{\Psi}_{\phi\exp\{\overleftarrow{s}_e\Lambda(\phi,\phi^*)\},\phi^*} = \mathcal{I}^{\Psi+\Delta_f\Psi}_{\phi,\phi^*}, \tag{7}$$

being a functional equation for an unknown $\Lambda(\phi,\phi^*)$,

$$-i\hbar\ln\left[\left(1+\Lambda\overleftarrow{s}_e\right)^{-1}\exp\{\overleftarrow{s}_e\Lambda(\phi,\phi^*)\}\right] = \Big(\exp\Big\{-[\Delta,\,\Delta_f\Psi]\Big\}-1\Big)S_\Psi \tag{8}$$

which has been proven to have a solution [40],

$$\Lambda\left(\phi,\phi^*|\Delta_f\Psi\right) = \Lambda\left(\Delta_f\Psi\right) \quad\text{so that}\quad \Lambda\left(\phi,\phi^*|\Delta_f\Psi\right) = -(\imath/\hbar)\Delta_f\Psi + o(\Delta_f\Psi)\,. \tag{9}$$

This allows one to state the gauge independence of the vacuum functional under finite changes of the gauge Fermion. Next, in theories having a closed algebra of rank 1, i.e., $M^{ij}_{\alpha\beta} = 0$ in (3) and being such that $\Delta S_\Psi = 0$, provided that $\overleftarrow{s}_e{}^2 = 0$, the Jacobian (6), the compensation equation (8), and its solution (9) are reduced to those corresponding to $\hat{\Lambda} = \Lambda(\phi)$, namely,

$$\operatorname{Sdet}\left\|\frac{\delta\phi^A\exp\{\overleftarrow{s}_e\hat{\Lambda}\}}{\delta\phi^B}\right\| \;=\; \left(1+\hat{\Lambda}\overleftarrow{s}_e\right)^{-1}; \qquad \imath\hbar\Big\{\ln\left(1+\hat{\Lambda}\overleftarrow{s}_e\right)\Big\} \;=\; \left(\Delta_f\Psi(\phi)\right)\overleftarrow{s}_e, \tag{10}$$

$$\hat{\Lambda} \;=\; \Delta_f\Psi(\phi)\big\{\left(\Delta_f\Psi(\phi)\right)\overleftarrow{s}_e\big\}^{-1}\Big[\exp\big\{-\tfrac{\imath}{\hbar}\left(\Delta_f\Psi(\phi)\right)\overleftarrow{s}_e\big\}-1\Big]. \tag{11}$$

Relations (10), (11) are identical to those in YM theories [32], when restricted to the case of irreducible gauge theories, provided that $F^{\gamma}_{\alpha\beta,i} = 0$.

[2]This change is inspired by infinitesimal FDBRST transformations [19, 20] with $\Lambda(\phi) = -(\imath/\hbar)\delta\Psi$, for which the vacuum functional is gauge-independent under a variation of the gauge condition, $\Psi \to \Psi+\delta\Psi$: $Z_{\Psi+\delta\Psi}(0,0) = Z_\Psi(0,0)$.

3 Gauge Dependence Problem and Ward Identities for Gauge Theories with Soft BRST Symmetry Breaking

A *soft BRST symmetry breaking* term is introduced into the gauge theory as an bosonic additions, $M = M(\phi, \phi^*)$, to the quantum action, S_Ψ [42] thus determining the GFGF,

$$Z_{M_\Psi,\Psi}(J,\phi^*) \;=\; \int d\phi\; \exp\left\{\tfrac{i}{\hbar}\left[S_\Psi\left(\phi,\phi^*\right)+M_\Psi(\phi,\phi^*)+J\phi\right]\right\} \equiv \int \mathcal{I}^{M_\Psi,\Psi}_{\phi,\phi^*}\exp\left\{\tfrac{i}{\hbar}J\phi\right\}. \quad (12)$$

The BRST breaking term M does not invariant with respect to the same BRST transformations (5), and may or not satisfies to the so-called *soft BRST symmetry breaking equation* respectively for dimensional-like regularization when $\Delta M = 0$, for local M [42], and for more general regularization [43]

$$M\exp\{\overleftarrow{s}_e\Lambda\} = M + M_A(\phi^A\overleftarrow{s}_e)\Lambda \neq M \;\;\text{and}\;\; (M,M)=0 \;\;\text{or}\;\; \Delta\exp\left\{-\tfrac{\imath}{\hbar}M\right\}=0, \quad (13)$$

however, providing an existence of the vacuum functional $Z_{M_\Psi,\Psi}(0,0)$. As the consequence of the BRST breaking the integrand $\mathcal{I}^{M_\Psi,\Psi}_{\phi,\phi^*}$ fails to be invariant for $J_A = 0$,

$$\mathcal{I}^{M_\Psi,\Psi}_{\phi\exp\{\overleftarrow{s}_e\Lambda\},\phi^*} = \mathcal{I}^{0,\Psi}_{\phi\exp\{\overleftarrow{s}_e\Lambda\},\phi^*}\exp\left\{\tfrac{i}{\hbar}M\exp\{\overleftarrow{s}_e\Lambda\}\right\} \overset{(13)}{\neq} \mathcal{I}^{M_\Psi,\Psi}_{\phi,\phi^*}. \quad (14)$$

In spite of this fact, there is a *modified Ward identity* for $Z_{M_\Psi,\Psi}(J)$ which is easily obtained by making in (12) a field-dependent BRST transformation (5) and using the relations (9) and the expression (6) for the Jacobian:

$$\left\langle\left\{1+\tfrac{i}{\hbar}\left[J_A\phi^A+M_\Psi\right]\overleftarrow{s}_e\Lambda\left(\Delta_f\Psi\right)\right\}\left(1+\Lambda\left(\Delta_f\Psi\right)\overleftarrow{s}_e\right)^{-1}\exp\{\overleftarrow{s}_e\Lambda\left(\Delta_f\Psi\right)\}\right\rangle_{M,\Psi,J} = 1\;, \quad (15)$$

where the symbol "$\langle\mathcal{A}\rangle_{M,\Psi,J}$" for a quantity $\mathcal{A}$ stands for a source-antifield- dependent average expectation value with respect to $Z_{M_\Psi,\Psi}(J,\phi^*)$, corresponding to the gauge-fixing Ψ. The modified Ward identity (derived then for Green's functions as well) depends on the field-dependent parameter $\Lambda\left(\Delta_f\Psi\right)$ as the weight functional, and therefore on the change of the gauge condition, $\Delta_f\Psi$. Note, first, that (15) for $M = 0$ permits to obtain new form of modified Ward identity for the general gauge theories within BV quantization, second, the Ward identity takes the form for a constant Λ,

$$\left\langle\left[J_A\phi^A+M_\Psi\right]\overleftarrow{s}_e\right\rangle_{M,\Psi,J}=0 \Longleftrightarrow \Big(J_A+M_A\big(\tfrac{\hbar}{\imath}\tfrac{\delta}{\delta J},\phi^*\big)\Big)\Big(\tfrac{\hbar}{\imath}\tfrac{\delta}{\delta\phi^*_A}-M^{A*}\big(\tfrac{\hbar}{\imath}\tfrac{\delta}{\delta J},\phi^*\big)\Big)Z_{M_\Psi,\Psi}=0, \quad (16)$$

$$\text{for } M_A\big(\tfrac{\hbar}{\imath}\tfrac{\delta}{\delta J},\phi^*\big) \equiv \frac{\delta M(\phi,\phi^*)}{\delta\phi^A}\Big|_{\phi\to\frac{\hbar}{\imath}\frac{\delta}{\delta J}} \;\;\text{and}\;\; M^{A*}\big(\tfrac{\hbar}{\imath}\tfrac{\delta}{\delta J},\phi^*\big) \equiv \frac{\delta M(\phi,\phi^*)}{\delta\phi^*_A}\Big|_{\phi\to\frac{\hbar}{\imath}\frac{\delta}{\delta J}}, \quad (17)$$

which is identical with the Ward identity for $Z_{M_\Psi,\Psi}(J,\phi^*)$ in [40, 43], whereas for $\Delta M = 0$ one should to extract from the left-hand side of the latter identity, the term, $M_A\,M^{A*} = 0$, deriving the identity from [42].

The identity (15) together with equivalence theorem arguments [49] implies an equation which describes the gauge dependence of $Z_{M_\Psi,\Psi}(J)$ for a finite change of the gauge $\Psi \to \Psi' = \Psi + \Delta_f\Psi$, namely,

$$\begin{aligned} Z_{M_{\Psi'},\Psi'}(J,\phi^*) - Z_{M_\Psi,\Psi}(J,\phi^*) &= Z_{M_\Psi,\Psi}(J,\phi^*)\left\langle\tfrac{i}{\hbar}J_A\phi^A\overleftarrow{s}_e\Lambda\left(\phi,\phi^*|-\Delta_f\Psi\right)\right\rangle_{M,\Psi,J} \\ &= (-1)^{\varepsilon_A}J_A\Lambda\left(\tfrac{\hbar}{\imath}\tfrac{\delta}{\delta J},\phi^*|-\Delta_f\Psi\right)\left(\frac{\delta}{\delta\phi^*_A}-\frac{\imath}{\hbar}M^{A*}_\Psi\right)Z_{M_\Psi,\Psi}(J,\phi^*), \end{aligned} \quad (18)$$

if the following representation for the soft BRST symmetry breaking term M in the reference frame described by the gauge, $\Psi + \Delta_f\Psi$, holds

$$M_{\Psi+\Delta_f\Psi}(\phi,\phi^*) \;=\; M_\Psi(\phi,\phi^*)\exp\{\overleftarrow{s}_e\Lambda\left(\Delta_f\Psi\right)\} \;=\; M_\Psi(\phi,\phi^*)\left[1+\overleftarrow{s}_e\Lambda\left(\Delta_f\Psi\right)\right]. \quad (19)$$

We obtain the main result of .[40] that on the extremals $J = 0$ finite change of the GFGF for the gauge theory with (soft) BRST symmetry breaking vanishes therefore preserving the gauge independence property in case $M \neq 0$ providing that the form of BRST symmetry breaking term M transforms under change of the gauge: $\Psi \to \Psi + \Delta_f\Psi$ by the rule (19). The equation (19) determines the rule of transformation of any quantity under change of the gauge.

For the effective action (generating functional of vertex Green's functions), $\Gamma_{M_\Psi,\Psi} = \Gamma_{M_\Psi,\Psi}(\phi,\phi^*)$, obtained via Legendre transformation of $\ln Z_{M_\Psi,\Psi}$ with respect to J_A,

$$\Gamma_{M_\Psi,\Psi}(J,\phi^*) = \frac{\hbar}{\imath}\ln Z_{M_\Psi,\Psi}(J,\phi^*) - J\phi, \quad \text{with} \quad \phi^A = \frac{\hbar}{\imath}\frac{\delta \ln Z_{M_\Psi,\Psi}}{\delta J_A}, \quad \frac{\delta\Gamma_{M_\Psi,\Psi}}{\delta\phi^A} = -J_A. \quad (20)$$

the Ward identity (16) takes the form [42, 40] in terms of antibracket and operatorial fields $\widehat{\phi}^A$:

$$\tfrac{1}{2}(\Gamma_M,\Gamma_M) = \frac{\delta\Gamma_M}{\delta\Phi^A}\widehat{M}^{A*} + \widehat{M}_A\frac{\delta\Gamma_M}{\delta\Phi^*_A} - \widehat{M}_A\widehat{M}^{A*}\ , \quad (21)$$

$$\text{for } \widehat{M}_A \equiv \frac{\delta M(\phi,\phi^*)}{\delta\phi^A}\Big|_{\phi\to\widehat{\phi}}, \qquad \widehat{M}^{A*} \equiv \frac{\delta M(\phi,\phi^*)}{\delta\phi^*_A}\Big|_{\phi\to\widehat{\phi}}, \quad (22)$$

$$\text{and } \widehat{\phi}^A = \phi^A + \imath\hbar\,(\Gamma''^{-1}_{M,\Psi})^{AB}\frac{\delta_l}{\delta\phi^B},\ (\Gamma''_{M,\Psi})_{AB} = \frac{\delta_l}{\delta\phi^A}\Big(\frac{\delta\Gamma_{M,\Psi}}{\delta\phi^B}\Big):\ (\Gamma''^{-1}_{M,\Psi})^{AC}(\Gamma''_{M,\Psi})_{CB} = \delta^A_B. \quad (23)$$

In turn, the finite change of the effective action $\Gamma_{M,\Psi}$ (as well as of $Z_{M_\Psi,\Psi}$) under change of the gauge Fermion, $\Delta_f\Psi$, without using FDBRST transformations concept and as well the transformation rules for BRST breaking term M (19) was firstly derived in [40] (see Eq. (3.31)) from which the linear in $\Delta_f\Psi$ and $\Delta_f M$ approximation looks as

$$\Delta_f\Gamma_{M_\Psi,\Psi} = \frac{\delta\Gamma_{M_\Psi,\Psi}}{\delta\phi^A}\widehat{F}^A\,\Delta_f\Psi(\widehat{\phi}) - \widehat{M}_A\widehat{F}^A\Delta_f\Psi(\widehat{\phi}) + \Delta_f M_\Psi(\widehat{\phi},\phi^*), \quad (24)$$

$$\text{where } \widehat{F}^A = -\frac{\delta}{\delta\phi^*_A} + (-1)^{\varepsilon_B(\varepsilon_A+1)}(\Gamma''^{-1}_{M,\Psi})^{BC}\Big(\frac{\delta_l}{\delta\phi^C}\frac{\delta\Gamma_{M,\Psi}}{\delta\phi^*_A}\Big)\frac{\delta_l}{\delta\phi^B} \quad (25)$$

and it was argued in [42, 43] to be non-vanishing on the extremals $\Gamma_{M_\Psi,\Psi;A} = 0$. However, a sufficient condition to vanish of $\Delta_f\Gamma_{M_\Psi,\Psi}|_{\Gamma_{M_\Psi,\Psi;A}=0}$ shown in [40],

$$\Delta_f M_\Psi(\widehat{\phi},\phi^*) = \widehat{M}_A\widehat{F}^A\Delta_f\Psi(\widehat{\phi})\ , \quad (26)$$

is always fulfilled and appears nothing else that average expectation value of the linear in $\Delta_f\Psi$ relation (19) with account for (9), presented as, $\Delta_f M_\Psi(\phi,\phi^*) = -(\imath/\hbar)M_\Psi\overleftarrow{s}_e\Delta_f\Psi$.

For YM theories the form of modified Ward identities and result of gauge dependence study remain valid [46] and simplify because of the nilpotency of Slavnov generator $\overleftarrow{s} = \overleftarrow{s}_e$, the jacobian of FDBRST transformations and solution of the compensation equation for change of the gauge take the form (10), (11).

4 Finite Field-Dependent BRST-antiBRST Transformation and its Jacobian

The GFGF for irreducible gauge theories with closed algebra within BRST-antiBRST Lagrangian quantization [14, 15] is given by,

$$Z_F(J) = \int d\phi\ \exp\left\{\tfrac{i}{\hbar}\left[S_F(\phi) + J_A\phi^A\right]\right\}\ . \quad (27)$$

with BRST-antiBRST-invariant quantum action

$$S_F(\phi) = S_0(A) - 1/2F_\xi\overleftarrow{s}_a\overleftarrow{s}^a = S_0(A) + S_{\rm gf}(A,B) + S_{\rm gh}(A,C) + S_{\rm add}(C)\,, \quad (28)$$

determined on the total configuration space parameterized as above respectively by the classical, $Sp(2)$-duplet of ghost-antighost, Nakanishi-Lautrup fields $\phi^A = (A^i, C^{\alpha a}, B^\alpha)$ and being the same as in FP method under identification $(C^{\alpha 1}, C^{\alpha 2}) = (C^\alpha, \overline{C}^\alpha)$. The quantities S_0, F appear by classical gauge-invariant action and admissible gauge-fixing Bosonic functional chosen here in quadratic approximation, in case of YM theory (with $A^i = A^{\mu n}(x)$ given on D-dimensional Minkowski space for $\eta_{\mu\nu} = diag(-,+,...,+)$ and taking its values in the algebra Lie of $SU(N)$ gauge group

$$S_0 = -1/4 \int d^D x F^n_{\mu\nu} F^{\mu\nu n}, \text{ for } F^{\mu\nu n} = \partial^{[\mu} A^{\nu]n} + f^{nop} A^{\mu 0} A^{\nu p},\ n = 1, ..., N^2-1, \quad (29)$$

$$F_\xi(A,C) = -\frac{1}{2} \int d^D x \ \left(A^m_\mu A^{m\mu} - \xi/2 \varepsilon_{ab} C^{ma} C^{mb}\right) \quad (30)$$

corresponding to R_ξ-family of the gauges (with $\chi_\xi(A,B) = \partial_\mu A^{\mu a} + \frac{\xi}{2} B^a = 0$) within FP rules for YM theories. The gauge-fixing term $S_{\rm gf}$, the ghost term $S_{\rm gh}$, and the interaction term $S_{\rm add}$, quartic in C^{ma} in (28) (vanishing for Landau gauge $\xi = 0$ and therefore for $S_F|_{\xi=0}$ coinciding with FP BRST-invariant action $S_{FP}(\phi)$) are determined by,

$$\left(S_{\rm gf},\ S_{\rm gh}\right) = \int d^D x \Big(\left[\left(\partial^\mu A^m_\mu\right) + \xi/2 B^m \right] B^m,\ \tfrac{1}{2} \left(\partial^\mu C^{ma}\right) D^{mn}_\mu C^{nb} \varepsilon_{ab} \Big), \quad (31)$$

$$S_{\rm add} = -\tfrac{\xi}{48} \int d^D x f^{mnl} f^{lrs} C^{sa} C^{rc} C^{nb} C^{md} \varepsilon_{ab} \varepsilon_{cd}. \quad (32)$$

The action (29) is invariant with respect to the infinitesimal gauge transformations $\delta A^m_\mu = D^{mn}_\mu \zeta^n$ with arbitrary functions $\xi^\alpha \equiv \zeta^n$ $(\varepsilon_\alpha = 0)$ on $R^{1,D-1}$, whereas the infinitesimal BRST-antiBRST transformations, $\delta\phi^A = \phi^A \overleftarrow{s}^a \mu_a$, for YM theories in terms of anticommutiong generators $\overleftarrow{s}^a : \overleftarrow{s}^a \overleftarrow{s}^b + \overleftarrow{s}^b \overleftarrow{s}^a = 0$,

$$\left(A^m_\mu, B^m\right) \overleftarrow{s}^a = \left(D^{mn}_\mu C^{na}, 1/2 f^{nml} \left[B^l C^{na} + (1/6) f^{lrs} C^{sb} C^{ra} C^{nc} \varepsilon_{cb}\right]\right),$$

$$C^{ma} \overleftarrow{s}^b = \left(\varepsilon^{ab} B^m - (1/2) f^{mnl} C^{la} C^{nb}\right), \quad (33)$$

leave the action S_F and integrand $\mathcal{I}^F_\phi$ in $Z_F(0) = \int \mathcal{I}^F_\phi$ by invariant only in the 1-st order in μ_a.

To restore the total BRST-antiBRST invariance of S_F and $\mathcal{I}^F_\phi$ in the whole orders in μ_a we introduced in [44] finite transformations of ϕ^A with a doublet λ_a of anticommuting parameters, $\lambda_a \lambda_b + \lambda_b \lambda_a = 0$,

$$\phi^A \to \phi'^A = \phi'^A(\phi|\lambda) : \phi'(\phi|0) = \phi,\ \left[\phi'^A \frac{\overleftarrow{\partial}}{\partial\lambda_a}\right]_{\lambda=0} = \phi^A \overleftarrow{s}^a \text{ and } \left[\phi'^A \frac{\overleftarrow{\partial}}{\partial\lambda_a} \frac{\overleftarrow{\partial}}{\partial\lambda_b}\right] = \tfrac{1}{2} \varepsilon^{ab} \phi^A \overleftarrow{s}^2 \quad (34)$$

as the solution of the functional equation

$$G\left(\phi'\right) = G(\phi) \quad \texttt{if} \quad s^a G(\phi) = 0 \quad (35)$$

for any regular functional $G(\phi)$. The general solution of (35) permits to restore *finite BRST-antiBRST transformations* in a unique way $\phi^A \to \phi'^A$,

$$\phi'^A = \phi^A \left(1 + \overleftarrow{s}^a \lambda_a + \tfrac{1}{4} \overleftarrow{s}^2 \lambda^2\right) \equiv \phi^A \exp\{\overleftarrow{s}^a \lambda_a\}, \quad (36)$$

where a set of elements $\{g(\lambda_a)\} = \{\exp\{\overleftarrow{s}^a \lambda_a\}\}$ forms Abelian two-parametric supergroup with odd generating elements λ_a. The BRST-antiBRST invariance of $\mathcal{I}^F_\phi$ means the validity of

$$\mathcal{I}^F_{\phi g(\lambda_a)} = \mathcal{I}^F_\phi. \quad (37)$$

where we have used the fact established in [44] that under global finite transformations, corresponding to $\lambda_a = \text{const}$, the integration measure remains invariant:

$$\text{Sdet}\left(\frac{\delta\phi \exp\{\overleftarrow{s}^a \lambda_a\}}{\delta\phi}\right) = 1 \ \text{ and } \ d\phi' = d\phi. \quad (38)$$

At the same time for finite field-dependent transformations, we show in [44] that for the particular case of functionally dependent parameters $\lambda_a = \Sigma\overleftarrow{s}_a$, $(\lambda_1\overleftarrow{s}^1 + \lambda_2\overleftarrow{s}^2 = -\Sigma\overleftarrow{s}^2)$ with a certain even-valued potential, $\Sigma = \Sigma(\phi)$, which is inspired by infinitesimal field-dependent BRST-antiBRST transformations with the parameters

$$\mu_a = \frac{i}{2\hbar}\varepsilon_{ab}\left(\Delta_f F\right)_{,A} X^{Ab} = \frac{i}{2\hbar}\left(\Delta_f F\right)\overleftarrow{s}_a\,, \tag{39}$$

for which with accuracy up to linear in $\Delta_f F$ terms the gauge independence of the integrand (therefore of the vacuum functional $Z_F(0)$) follows $\mathcal{I}^F_{\phi g(\mu(\Delta_f F))} = \mathcal{I}^{F+\Delta_f F}_{\phi} + o(\Delta_f F)$. In case of finite field-dependent transformations with group element $g(\Sigma\overleftarrow{s}_a)$ a set of which forms now non-Abelian 2-parametric supergroup, the superdeterminant of the change of variables takes the form

$$\mathrm{Sdet}\left(\frac{\delta(\phi g(\Sigma\overleftarrow{s}_a)}{\delta\phi}\right) = \left[1 - \frac{1}{2}\Sigma\overleftarrow{s}^2\right]^{-2},\ d\phi' = d\phi\ \exp\left\{\tfrac{i}{\hbar}\left[i\hbar\ln\left(1-\tfrac{1}{2}\Sigma\overleftarrow{s}^2\right)^2\right]\right\}\ . \tag{40}$$

Again, the functionally dependent FDBRST-antiBRST transformations may be used due to s_a-exact form of the jacobian (40) for the establishing of the gauge independence of the vacuum functional $Z_F(0)$ from the requirement of the BRST-antiBRST version of the compensation equation under change of the gauge Boson, $F \to F + \Delta_f F$, validity:

$$\mathcal{I}^F_{\phi g(\Sigma\overleftarrow{s}_a)} = \mathcal{I}^{F+\Delta_f F}_{\phi} \Longleftrightarrow i\hbar\ \ln\left(1-\Sigma\overleftarrow{s}^2/2\right)^2\ =\ \left(\Delta_f F\overleftarrow{s}^2/2\right), \tag{41}$$

whose solution for unknown Bosonic FD parameter $\Sigma(\phi)$, and therefore for $Sp(2)$-doublet of $\lambda_a(\phi) = \Sigma\overleftarrow{s}_a$ with accuracy up to for s_a-exact terms looks as [44]:

$$\Sigma\left(\phi|\Delta_f F\right) = -2\Delta_f F\left((\Delta_f F)\overleftarrow{s}^2\right)^{-1}\left[\exp\left(-\tfrac{1}{4i\hbar}(\Delta_f F)\overleftarrow{s}^2\right) - 1\right]. \tag{42}$$

And visa-verse having considered the equation (41) for unknown $\Delta_f F$ with given Σ we obtain

$$\Delta_f F\left(\phi\right) = -2i\hbar\ \Sigma\left(\Sigma\overleftarrow{s}^2\right)^{-1}\ln\left(1-\Sigma\overleftarrow{s}^2/2\right)^2\ . \tag{43}$$

Thus, the field-dependent transformations with the parameters $\lambda_a = \Sigma\overleftarrow{s}_a$ amount to a precise change of the gauge-fixing functional. E.g. to relate $Z_{F_\xi}(J)$ with $Z_{F_{\xi+\Delta\xi}}(J)$ in R_ξ- family of the gauges we should to fulfill FFDBRST-antiBRST transformations with parameters, $\lambda_a = \lambda_a(\xi)$

$$\lambda_a = \tfrac{\Delta\xi}{4i\hbar}\varepsilon_{ab}\int d^Dx\ B^nC^{nb}\sum_{n=0}^{\infty}\tfrac{1}{(n+1)!}\left[\tfrac{\Delta\xi}{4i\hbar}\int d^Dy\left(B^uB^u - \tfrac{1}{24}f^{uwt}f^{trs}C^{sc}C^{rp}C^{wd}C^{uq}\varepsilon_{cd}\varepsilon_{pq}\right)\right]^n. \tag{44}$$

Being base on (42) the modified Ward identity for $Z_F(J)$ depending on FD parameters $\lambda_a(\phi|\Delta_f F)$, following from it usual Ward identities for constant λ_a and gauge dependence problem under finite change of the gauge [44, 46] in terms of the notations similar to one in (15) :

$$\Big\langle\Big\{1 + \tfrac{i}{\hbar}J_A\left[\phi^A\overleftarrow{s}^a\lambda_a(\Sigma) + \tfrac{1}{4}\phi^A\overleftarrow{s}^2\lambda^2(\Sigma)\right] - \tfrac{1}{4}\left(\tfrac{i}{\hbar}\right)^2\varepsilon_{ab}J_A\phi^A\overleftarrow{s}^aJ_B\phi^B\overleftarrow{s}^b\lambda^2(\Sigma)\Big\} \tag{45}$$

$$\times\left(1-\tfrac{1}{2}\Sigma\overleftarrow{s}^2\right)^{-2}\Big\rangle_{F,J} = 1, \qquad J_A\left\langle\phi^A\overleftarrow{s}^a\right\rangle_{F,J} = 0, \tag{46}$$

$$\Delta Z_F(J) = \tfrac{i}{\hbar}Z_F\left\langle J_A\left[\phi^A\overleftarrow{s}^a\hat{\lambda}_a + \tfrac{1}{4}\phi^A\overleftarrow{s}^2\hat{\lambda}^2\right] - (-1)^{\varepsilon_B}\left(\tfrac{i}{4\hbar}\right)J_BJ_A\left(\phi^A\overleftarrow{s}^a\phi^B\overleftarrow{s}^b\right)\varepsilon_{ab}\hat{\lambda}^2\right\rangle_{F,J} \tag{47}$$

for the notations $\hat{\lambda}_a \equiv \lambda_a\left(\phi|-\Delta_f F\right)$.

5 (Refined) Gribov–Zwanziger Theory in BRST, BRST-anti-BRST Formulations in Many Parametric Family of Gauges

GZ theory is determined by the GZ action $S_{GZ}(\phi)$ on the same configuration space as for YM theory, given in the Landau gauge $\chi^m(A) = \partial_\mu A^{\mu m} = 0$ (with using of the Minkowsky space-time notations rather formally because of GZ theory is determined only in Euclidian space $\mathbb{R}^D$)

$$S_{GZ}(\phi) = S_{FP}(\phi) + M_0(A),\ M_0(A) = \gamma^2\big(f^{mrl}A^r_\mu K^{mn-1}f^{nsl}A^{\mu s} + D(N^2-1)\big) \tag{48}$$

with the additive non-local BRST-non-invariant with respect to BRST transformations $\big(A^{\mu m}$, C^m, $\overline{C}^m$, $B^m\big)\overleftarrow{s} = \big(D^{\mu mn}C^n, \frac{1}{2}f^{mno}C^nC^o, B^m,\ 0\big)$ term:

$$M_0\overleftarrow{s} = \gamma^2 f^{mrl}f^{lse}\big[2D^{rq}_\mu C^q(K^{-1})^{ms} - f^{gpn}A^r_\mu(K^{-1})^{mg}K^{pq}C^q(K^{-1})^{ns}\big]A^{e\mu} \neq 0, \tag{49}$$

known as the GZ horizon functional, implying an inclusion of the Gribov horizon [21] in terms of the FP operator $(K)^{mn} = \partial_\mu D^{\mu mn}$ for $(K^{-1})^{mo}(K)^{on} = \delta^{mn}$ and the so-called thermodynamic (Gribov mass) parameter γ, introduced in a self-consistent way by the gap equation [23]. The idea to improve the GZ theory is due to the facts that, first, it fails to eliminate all Gribov's copies, and, second, a non-zero value for the Gribov parameter γ is a manifestation of nontrivial properties of the vacuum [37] of the theory. The latter means that there exist additional reasons for non-perturbative effects, which can be encoded in a set of dimension-2 condensate, $\langle A^{\mu a}A^a_\mu\rangle$, in the case of a non-local GZ action with the YM gauge fields $A^{\mu a}$ only[3]

$$S_{GZ}(\phi) \to S_{RGZ}(\phi) = S_{GZ} + \tfrac{m^2}{2}A^m_\mu A^{\mu m}. \tag{50}$$

To determine GZ and RGZ models in any gauges in a gauge independence manner compatible with (18) let us consider a family of linear gauges given by the equation

$$\chi^m(A,B) \ = \ \Lambda_\mu(\partial,\alpha,\beta,n)A^{\mu m} + \frac{\xi}{2}B^m = 0 \text{ with } \Lambda_\mu(\partial,\alpha,\beta,n) = \alpha\partial_\mu + \beta\frac{\kappa_{\mu\nu}}{n^2}n^\nu. \tag{51}$$

Here, we have 3 real, α,β,ξ, and 1 vector, n^μ, gauge parameters.

Particular cases of R_ξ-gauges and *generalized Coulomb gauges* gauges can be obtained from the general many-parameter family under the choices

$$(\alpha;\beta) = (1;0) \to \ R_{\xi}\text{–gauges};\ \ (\beta,\xi) = (-\alpha,0), \kappa_{\mu\nu} = n^\rho\partial_\rho\eta_{\mu\nu}, n^2 < 0, \tag{52}$$

The Landau and Feynman gauges are obtained from the first family for the respective choices $\xi = 0;1$, whereas the Coulomb, $\chi^m_C(A,B) = \partial_i A^{im} = 0$ for $\mu = (0,i)$ from $n^\mu = (1,0,...,0)$.

The FP action, GZ horizon functional $M_g(\phi)$ and, therefore GZ and RGZ model in any gauges, including ones from the set of (51) starting from ones in the Landau (or Coulomb [24], where horizon functional has the same form (48) but for $(K)^{ab}_C = \partial_i D^{iab}$ and $D-1$ instead of D) gauge is determined by (19) with help of FDBRST transformations with odd parameter $\hat{\Lambda}$ from (11) with $\overleftarrow{s}$ defined before (49):

$$S_{FP}(\phi,\alpha,\beta,n^\mu,\xi) = S_0 + \Psi_g(\phi)\overleftarrow{s},\ \ \text{for}\ \ K^{mn}_g = \Lambda^\mu(\partial,\alpha,\beta,n)D^{mn}_\mu, \tag{53}$$

$$M_g(\phi) = M_0(A)\exp\{\overleftarrow{s}\hat{\Lambda}\} = M_0(A)\Big(1 + \overleftarrow{s}\Delta_f\Psi\big\{(\Delta_f\Psi)\overleftarrow{s}\big\}^{-1}\Big[\exp\{-\tfrac{\imath}{\hbar}(\Delta_f\Psi)\overleftarrow{s}\} - 1\Big]\Big), \tag{54}$$

$$\tfrac{m^2}{2}A^m_\mu A^m_\mu\exp\{\overleftarrow{s}\hat{\Lambda}\} = \tfrac{m^2}{2}A^m_\mu\Big(A^m_\mu + \partial^\mu C^m\Delta_f\Psi\big\{(\Delta_f\Psi)\overleftarrow{s}\big\}^{-1}\big[\exp\{-\tfrac{\imath}{\hbar}(\Delta_f\Psi)\overleftarrow{s}\} - 1\big]\Big), \tag{55}$$

[3] As well as in a similar set of dimension-2 condensates, $\langle A^{\mu m}A^m_\mu\rangle$, $\langle\bar{\varphi}^{\mu mn}\varphi^{mn}_\mu\rangle - \langle\bar{\omega}^{\mu mn}\omega^{mn}_\mu\rangle$, for a local GZ action [37], $S_{GZ}(\phi,\hat{\phi})$ with an equivalent local representation for the horizon functional in terms of the functional S_γ, defined in an extended configuration space with auxiliary variables $\phi^{\bar{A}}$ described in [25],[40].

for $\Psi_g(\phi) = \bar{C}^m\chi^m(A,B)$ and where

$$\Delta_f\Psi = \Psi_g - \Psi_0 = \bar{C}^m\big(\{(\alpha-1)\partial_\mu + \beta\tfrac{\kappa_{\mu\nu}}{n^2}n^\nu\}A^{\mu m} + \tfrac{\xi}{2}B^m\big), \quad (56)$$

$$\Delta_f\Psi\overleftarrow{s} = \Big\{B^m\Big(\{(\alpha-1)\partial_\mu + \beta\tfrac{\kappa_{\mu\nu}}{n^2}n^\nu\}A^{\mu m} + \tfrac{\xi}{2}B^m\Big) + \bar{C}^m\big((\alpha-1)\partial_\mu + \beta\tfrac{\kappa_{\mu\nu}}{n^2}n^\nu\big)D^{\mu mn}C^n\Big\}. \quad (57)$$

The GZ: $S_{g;GZ} = S_{FP}(\alpha,\beta,n^\mu,\xi) + M_g(\phi)$, and RGZ: $S_{g;RGZ} = S_{g;GZ} + \frac{m^2}{2}A^m_\mu A^m_\mu \exp\{\overleftarrow{s}\hat{\Lambda}\}$, actions in any from $\chi^m(A,B)$-set of the gauges present one from the main results in this section[4] Considering the generalization of GZ and RGZ theories within BRST-antiBRST quantization, note because of the gauge-fixing functional F_0 (30) corresponds to the Landau gauge, we introduce the GZ horizon functional in the same manner as in [23] for the FP procedure in the Euclidian space coinciding with $M_{F_0} = M_0(A)$ (48) as well as GZ action appears by the same (48. We determine the GZ theory in any F_ξ gauges (R_ξ-gauges) in a way compatible with the gauge-independence of the generating functional of Green's functions in F_0, where Gribov horizon in the gauge F_ξ should be determined as

$$M_{F_\xi} = M_{F_0}\Big(1 + \tfrac{1}{2i\hbar}\left(\overleftarrow{s}^a\right)\left(\Delta F_\xi\overleftarrow{s}_a\right)\textstyle\sum_{n=0}^\infty \frac{1}{(n+1)!}\left(-\frac{1}{4i\hbar}\Delta F_\xi\overleftarrow{s}^2\right)^n - \frac{1}{16\hbar^2}\left(\overleftarrow{s}^2\right)\left(\Delta F_\xi\right)^2$$
$$\times\Big[\textstyle\sum_{n=0}^\infty \frac{1}{(n+1)!}\left(-\frac{1}{4i\hbar}\Delta F_\xi\overleftarrow{s}^2\right)^n\Big]^2\Big), \quad (58)$$

where ΔF_ξ is readily determined with account taken of (33), (44); see for details [44]. The construction of the Gribov horizon functional M_{F_ξ} (h_ξ in [44]) in the gauge F_ξ, starting from M_0 in the gauge F_0, may be considered as a generalization of the result [50] obtained in the BRST setting of the problem. In turn, the RGZ model in any from F_ξ-gauge are readily constructed as, $S_{RGZ,\xi}$ with account for

$$\tfrac{m^2}{2}A^m_\mu A^m_\mu \exp\{\overleftarrow{s}^a\lambda_a(\xi)\} = \tfrac{m^2}{2}\Big[A^m_\mu\big(A^m_\mu + \partial^\mu C^{ma}\lambda_a(\xi)\big) + \tfrac{1}{4}A^m_\mu A^{\mu m}\overleftarrow{s}^2\lambda^2(\xi)\Big], \quad (59)$$

which is differed by the last term proportional to $\lambda^2(\xi)$ from BRST transformed composite field (55).

6 Modified Faddeev-Popov Rules for Gauge Theory with Gauge Group

Starting from the Gribov anzats for YM theory with the functional $\mathcal{V}(\partial_\mu\partial^\mu)$ in the Eq. (31) [21] which restricts the integration in the path integral, $Z_F(0)$ (27) in BRST-antiBRST quatization (or $Z_\Psi(0)$ within FP method with S_{FP} instead of S_F) only to the first Gribov region C_0 we suppose that it may be presented as Θ-function: $\mathcal{V}(\partial_\mu\partial^\mu) = \Theta(\partial_\mu\partial^\mu) = \Theta(1-\sigma(\lambda_0(A)))$, where a quantity $\lambda_0(A)$ appears by the least real part of positive proper eigen-value of the FP operator $K^{mn}(A)$ in a gauge $\chi^n = 0$: $0 \geq \mathrm{Re}\lambda^n_0 \geq \mathrm{Re}\lambda^n_1 \geq \ldots \geq \mathrm{Re}\lambda^n_k \geq \ldots$ in the spectrum problem on,

$$K^{mn}(A)u^n_k = \delta^{mn}\lambda^n_k(A)u^n_k, \text{ for } K^{mn}(A) = (\delta\chi^m)/(\delta A^{\mu o})D^{\mu on},\ k\in\mathbb{Z}. \quad (60)$$

We determine the GFGF $\mathcal{Z}\Psi(J)$ with restricted region of the integration (where $\mathrm{Det}\|K^{mn}(A)\|$ > 0 everywhere) for the gauge Lie algebra $g = su(N)$ without Gribov's copies as,

$$\mathcal{Z}_\Psi(J) \;=\; \int dA\Theta[1-\sigma(\lambda_0(A))]\delta\big(\chi(A)\big)\mathrm{Det}K(A)\exp\left\{\tfrac{\imath}{\hbar}(S_0(A) + jA)\right\} \quad (61)$$

[4]These results call for a verification of the fact that M_g actually selects the first Gribov region for $A^{\mu a}$ in any $\chi^m(A,B)$-gauge, since extracting this region by means of the functional $M_0(A)$ has been determined non-perturbatively [25], whereas a corresponding explicit and rigorous proof, e.g., for an R_ξ-gauge $M(A,\xi)$ to provide a restriction for $A^{\mu a}$ in the first Gribov region, $\Omega(\xi)$: $= \Big\{A^{\mu a}\big|\chi^a(A,B)\big|_{\alpha=1,\beta=0} = 0, K^{ab}(\xi) \geq 0\Big\}$, has not been presented in the literature.

$$= \int dA\delta\big(\chi(A)\big) \det\Big\{\Theta^{\frac{1}{\dim g}}[1-\sigma(\lambda_0(A))]K(A)\Big\} \exp\big\{\frac{\imath}{\hbar}(S_0(A)+jA)\big\}$$
$$= \int d\phi \exp\big\{\tfrac{\imath}{\hbar}(S_0(A)+\bar{C}K'C+\chi(A)B+jA)\big\}, \qquad (62)$$

with $K'^{mn} = \Theta^{\frac{1}{\dim g}}[1-\sigma(\lambda_0(A))]K^{mn}(A)$, being by modified Faddeev-Popov operator in the Lagrangian formalism, for $\dim g = N^2-1$. Of course, it is a problem to solve the spectrum problem (60) and to construct the functional $\sigma(\lambda_0(A))$ but corresponding results for some gauge group exist.

The classical action, $S_0(A)$ is still invariant with respect to by the restricted to the region C_0 infinitesimal *modified gauge transformations* with *modified generators of gauge transformations*:

$$\delta_m A^{\mu n}(x) = \Theta^{\frac{1}{\dim g}}[1-\sigma(\lambda_0(A))]D^{\mu no}(x)\zeta^o(x), \qquad (63)$$
$$\text{with } \mathcal{R}^{\mu m0}(x,y) = \Theta^{\frac{1}{\dim g}}[1-\sigma(\lambda_0(A))]D^{\mu no}(x)\delta(x-y).$$

The integrand $\mathfrak{I}^\Psi_\phi$ in $\mathcal{Z}_\Psi(J) = \int \mathfrak{I}^\Psi_\phi \exp\{\frac{\imath}{\hbar}J\phi\}$ is invariant with respect to *modified BRST transformations*:

$$\delta_B\big(A^{\mu m}, C^m, \overline{C}^m, B^m\big) = \Theta^{\frac{1}{\dim g}}[1-\sigma(\lambda_0(A))]\big(D^{\mu mn}C^n, \frac{1}{2}f^{mno}C^nC^o, B^m,\ 0\big)\Lambda, \qquad (64)$$

where we have taken into account for the calculation of the jacobian of the change of variables in $\mathcal{Z}_\Psi(J)$ that the terms $(\delta\Theta(...))/(\delta A^i)$ should be proportional to $\delta(0)$ and within appropriate choice of the regularization should vanish. We see that the gauge independence property for the vacuum functional should be follow as well as a consistency of the unitarity problem due to non-appearance of non-physical degrees of freedom as for the GZ model and suppose to continue this research in a forthcoming paper.

7 Conclusion

We have reviewed the results of our research devoted to finite FDBRST transformations in the BV formalism and calculated the Jacobian of a change of variables, used afterwards to obtain a new form of the Ward identities for the generating functionals of Green's functions. For these functionals, we study the issue of gauge dependence, and this enables us to solve the consistency problem of an introduction of (soft) BRST symmetry breaking terms in the BV method. We have also proposed the concept of finite BRST-antiBRST and FFDBRST-antiBRST transformations for Yang–Mills theories in the Sp(2)-covariant Lagrangian quantization. The Jacobian of a change of variables corresponding to FFDBRST-antiBRST transformations with functionally-dependent parameters is calculated in a precise manner. It has been established that quantum YM actions in different gauges are related to each other by FFDBRST-antiBRST transformations with functionally dependent parameters obtained as solutions of the compensation equation. A new Ward identity and the gauge dependence problem for finite changes of the gauge for the generating functional of Green's functions have been obtained and studied. The Gribov–Zwanziger theory and a refined Gribov–Zwanziger theory in BRST and in BRST-antiBRST descriptions for a many parametric family of linear gauges (explicitly including the covariant and Coulomb gauges), starting from M_0 in the Landau gauge, are suggested in a way consistent with the gauge independence of the respective S-matrices as a consequence of BRST(antiBRST) symmetry breaking. A modification is proposed for the Faddeev–Popov rules to a definition that involves such a gauge in the path integral and such BRST transformations that are free from the Gribov copies and do not excite the longitudinal degrees of freedom.

Acknowledgement

The author thanks V.A. Rubakov and the Organizing Committee for kind hospitality. He is also grateful to P.Yu. Moshin, D. Bykov, D. Francia, and to the participants of the International Seminar "QUARKS 2014" for useful discussions. The study has been supported by the LRSS grant under Project No. 88.2014.2.

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