Localization on a thick brane in the presence of defect

O. O. Novikov[a*], A. A. Andrianov[ab†], V. A. Andrianov[a‡]

[a] *V.A. Fock Department of Theoretical Physics, Saint-Petersburg State University*
ul. Ulianovskaya, 198504 St. Petersburg, Russia

[b] *Institut de Ciències del Cosmos, Universitat de Barcelona*
Marti i Franques, 1, 08028 Barcelona, Spain

Abstract

The influence of a thin brane defect and gravity on the formation of a domain wall ("thick brane") by matter composed of two scalar fields is studied. The mixing of the scalar and gravitational degrees of freedom leads to the nonperturbative effects in the invariant scalar spectrum . In the case of the negative defect brane tension the corresponding potential takes the form of a potential well with infinitely tall walls containing the discrete spectrum of localized states isolated from the rest of the bulk.

The hypothesis that our universe is a four-dimensional space-time hypersurface (3-brane) embedded in a fundamental multi-dimensional space is a quite popular basis for the models of BSM physics, see, for example, [1] and references therein. The relatively light matter fields can be trapped in the vicinity of such a hypersurface due to the nonlinear interaction of the multidimensional fields resulting in the formation of a domain wall ("thick" brane). The gravity happens to play an important role in a (de) localization of matter fields. [2, 3, 4, 5, 6].

In this work we consider a model of the domain wall ("thick" brane) formation in five-dimensional noncompact space-time by the scalar matter minimally coupled to gravity [7], [8].The scalar matter is assumed to be composed of two fields with $O(2)$ symmetric self interaction and with manifest $O(2)$ symmetry breaking. As was shown in [9], [10], [11] the gravity cause singular repulsion towards the remote AdS horizon so that localized modes on a brane may be absent and a Goldstone mode of translational symmetry breaking disappears. The singular barrier is associated with zeroes of the metric factor derivative ρ' that can be shifted by the introduction of a four-dimensional space-time defect in the form of a rigid thin brane with small tension. Such a defect breaks manifestly the invariance under five-dimensional diffeomorphisms, in particular, translational invariance along the fifth dimension, and is assumed to trigger the domain wall formation in its vicinity.

1 Formulation of the model

Consider the five-dimensional space supplied with a pseudo Riemann metric g_{AB}, which is reduced to η_{AB} in flat space and for the rectangular coordinate system,

$$X^A = (x^\mu, y), \quad x^\mu = (x^0, x^1, x^2, x^3), \tag{1}$$

It is assumed that the size of extra dimension y is large or infinite.

[*]**e-mail**: oonovikov@gmail.com
[†]**e-mail**: andrianov@icc.ub.edu
[‡]**e-mail**: v.andriano@rambler.ru

We define the dynamics of two scalar fields $\Phi(X)$ and $H(X)$ with a minimal interaction to gravity by the following action functional,

$$S[g,\Phi,H] = \int d^5X\sqrt{|g|}\left(-\tfrac{1}{2}M_*^3 R + \mathcal{L}_{mat}(g,\Phi,H)\right) - 3M_*^3\lambda_b \int\limits_{y=0} d^4X\sqrt{\left|{}^{(4)}g\right|} + S_{GH}, \quad (2)$$

$$\mathcal{L}_{mat} = Z\left(\tfrac{1}{2}(\partial_A\Phi\partial^A\Phi + \partial_A H\partial^A H) - V(\Phi,H)\right), \quad (3)$$

$$S_{GH} = \tfrac{1}{2}M_*^3 \int\limits_{y=0} d^4X\sqrt{\left|{}^{(4)}g\right|}\left[\Gamma^\mu_{\mu 5}g^{55} - \Gamma^5_{\mu\nu}g^{\mu\nu}\right]_\pm \quad (4)$$

where R stands for a scalar curvature, $|g|$ and $\left|{}^{(4)}g\right|$ are the determinants of the 5-dim and induced 4-dim metric tensors, S_{GH} is a compensating Gibbons-Hawking-York term [12, 13]. M_* denotes a five-dimensional gravitational Planck scale. The thin brane defect is taken in the form of cosmological constant on a 3-brane parameterized by λ_b. The brane is assumed to be rigid i.e. its fluctuations are suppressed. We will restrict ourselves to the following minimal potential,

$$V = -M^2\Phi^2 - \Delta_H H^2 + \frac{1}{2}(\Phi^2 + H^2)^2 + M^4, \quad M^2 > \Delta_H, \quad Z = \frac{3\kappa M_*^3}{M^2}, \quad (5)$$

where $\kappa \sim M^3/M_*^3 \ll 1$ characterizes the interaction of gravity and matter fields. We will also assume that the tension of the defect brane is small $\lambda_b \sim \kappa M$.

In order to build a thick 3+1-dimensional brane we study the classical vacuum configurations which do not violate spontaneously 4-dimensional Poincare invariance. A background solution for the metric is searched for in the following ansatz,

$$ds^2 = e^{-2\rho(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2. \quad (6)$$

This kind of background metric suits well for interpretation of scalar fluctuation spectrum and corresponding resonance effects (i.e. scattering states) [8].

$$\rho'' = \tfrac{Z}{3M_*^3}(\Phi'^2 + H'^2), \quad \tfrac{2Z}{3M_*^3}V(\Phi,H) = \rho'' - 4(\rho')^2, \quad (7)$$

$$\Phi'' - 4\rho'\Phi' = \tfrac{\partial V}{\partial\Phi}, \quad H'' - 4\rho' H' = \tfrac{\partial V}{\partial H}. \quad (8)$$

One can prove [8], that only three of these equations are independent. These bulk equations should be supplemented with the Israel matching conditions on the thin brane that for our ansatz read,

$$\left[\rho'\right]_\pm = 2\lambda_b, \quad \left[\Phi'\right]_\pm = 0, \quad \left[\Phi''\right]_\pm = 8\lambda_b\Phi'(0), \quad \left[H'\right]_\pm = 0. \quad (9)$$

In the limit of zero gravity the equations on classical backgrounds smoothly reproduce the corresponding equations in the model without gravity.

Depending on the relation between quadratic couplings M^2 and Δ_H there are two types of solutions of eqs. (8) inhomogeneous in y [7]. In the zero gravity limit the first solution dominates when $\Delta_H \leq M^2/2$,

$$\Phi = \pm M\tanh(My) + O(\kappa M), \quad H(y) = 0. \quad (10)$$

To the leading order in κ it generates the following conformal factor ,

$$\rho_1(y) = \frac{2\kappa}{3}\left\{\ln\cosh(My) + \frac{1}{4}\tanh^2(My)\right\} + \lambda_b|y| + O\left(\kappa^2\right), \quad (11)$$

which is chosen to be an even function of y in order to preserve the so-called τ symmetry. The spontaneous breaking of this symmetry is associated with fermion mass generation [7, 14]. In the scalar sector the τ symmetry involves the intrinsic parity reflection of both fields and the reflection of fifth coordinate,

$$\Phi(y) \to -\Phi(-y), \quad H(y) \to -H(-y).$$

Evidently the parity reflection leaves the bosonic action with the potential (5) invariant and holds as a symmetry for the kink (10). In the presence of gravity induced by a background matter the τ symmetry survives for even conformal factors.

The second kink profile arises only when $M^2/2 \le \Delta_H \le M^2$ (in the zero gravity limit), i.e. $2\Delta_H = M^2 + \mu^2,\ \mu^2 < M^2$,

$$\Phi_0(y) = \pm M \tanh\left(\beta M y\right), \quad H_0(y) = \pm \frac{\mu}{\cosh\left(\beta M y\right)}, \quad \beta = \sqrt{1 - \frac{\mu^2}{M^2}}, \tag{12}$$

and it breaks the τ symmetry. Therefrom one can find the conformal factor to the leading order in κ in the following form,

$$\rho_1\left(y\right) = \frac{\kappa}{3}\left\{\left(3-\beta^2\right)\ln\cosh\left(\beta M y\right) + \frac{1}{2}\beta^2\tanh^2(\beta M y)\right\} + \lambda_b|y| + O\left(\kappa^2\right). \tag{13}$$

These two solutions correspond to different phases with critical point at $\Delta_H = M^2/2 + O\Big(\kappa\Big)$. We are interested in the phase with the nonzero v.e.v. of the second field H that can be used for the fermion mass generation. Let us choose further on the positive signs of $\Phi(y), H(y)$ at $y \to +\infty$.

The next approximation to the solutions in the second phase can be obtained with the help of the perturbation theory in the parameters κ expressing the strength of gravity,λ_b/M controlling the tension of the defect brane and μ/M parameterizing the deviation from the critical point. It is useful to introduce the dimensionless coordinate $\tau = M\beta y$.

$$\Phi(\tau) = M\sum_{l,m,n=0}^{\infty}\kappa^l\left(\tfrac{\lambda_b}{M}\right)^m\left(\tfrac{\mu}{M}\right)^{2n}\Phi_{l,m,n}(\tau), \tag{14}$$

$$H(\tau) = M\sum_{l,m,n=0}^{\infty}\kappa^l\left(\tfrac{\lambda_b}{M}\right)^m\left(\tfrac{\mu}{M}\right)^{2n+1}H_{l,m,n}(\tau), \tag{15}$$

$$\rho(\tau) = \kappa\sum_{n,m=0}^{\infty}\kappa^n\left(\tfrac{\mu}{M}\right)^{2m}\rho_{n+1,m}(\tau), \tag{16}$$

$$\Delta_H = \Delta_{H,c}(\kappa) + \tfrac{1}{2}\mu^2, \quad \Delta_{H,c}(\kappa) = \tfrac{1}{2}M^2\sum_{m,n=0}^{\infty}\kappa^m\left(\tfrac{\lambda_b}{M}\right)^n\Delta_H^{m,n}, \tag{17}$$

$$\Phi_{n,0,0} \equiv \Phi_n, \quad H_{n,0,0} \equiv H_n, \quad \rho_{n,0} \equiv \rho_n \tag{18}$$

$$\tfrac{1}{\beta^2} = \sum_{l,m,n=0}^{\infty}\kappa^l\left(\tfrac{\lambda_b}{M}\right)^m\left(\tfrac{\mu}{M}\right)^{2n}\left(\tfrac{1}{\beta^2}\right)_{l,m,n}; \tag{19}$$

The computation gives the following leading order corrections [14, 15],

$$\Delta_{H,c} = \frac{1}{2} - \frac{22}{27}\kappa - \frac{4}{3}(1+2\ln 2)\frac{\lambda_b}{M} + O\Big(\kappa^2, \frac{\lambda_b}{M}\kappa, \frac{\lambda_b^2}{M^2}\Big), \quad \frac{1}{\beta^2}\Big|_{\mu=0} = 1 + \frac{4}{3}\kappa + 2\frac{\lambda_b}{M} + O\Big(\kappa^2, \frac{\lambda_b}{M}\kappa, \frac{\lambda_b^2}{M^2}\Big) \tag{20}$$

2 Fluctuation equations

Let us consider small localized deviations of the fields from the background values and find the action squared in these fluctuations. The fluctuations of the metric $h_{AB}\left(X\right)$ and of the scalar fields $\phi\left(X\right)$ and $\chi\left(X\right)$ against the background solutions of EoM are introduced in the following way,

$$g_{AB}\left(X\right)dx^A dx^B = e^{-2\rho(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 + e^{-2\rho(y)}h_{AB}\left(X\right)dx^A dx^B; \tag{21}$$

$$\Phi(X) = \Phi(y) + \phi(X)\,;\; H(X) = H(y) + \chi(X)\,. \tag{22}$$

Since 4dim Poincare symmetry is not broken, we select the corresponding 4dim part of the metric $h_{\mu\nu}$ and employ the notation for gravi-vectors $h_{5\mu} \equiv v_\mu$ and gravi-scalars $e^{-2\rho}h_{55} \equiv S$. The major simplification can be achieved by separation of different spin components of the field $h_{\mu\nu}$ and v_μ. It can be accomplished by description of ten components of 4-dim metric in terms of the traceless-transverse tensor, vector and scalar components [9, 16],

$$h_{\mu\nu} = b_{\mu\nu} + F_{\mu,\nu} + F_{\nu,\mu} + E_{,\mu\nu} + \eta_{\mu\nu}\psi, \quad v_\mu = v_\mu^\perp + \partial_\mu \eta, \tag{23}$$

$$b_{\mu\nu}^{;\mu} = b = 0 = F_\mu^{;\mu} = v_\mu^{\perp,\mu}. \tag{24}$$

After this separation in the action is performed the scalar sector that we are interested in decouples from the fields with higher spins up to quadratic orders in fluctuations.

The action (2) is invariant under diffeomorphisms therefore the original variables contain redundant degrees of freedom. Infinitesimal diffeomorphisms correspond to the Lie derivative along an arbitrary vector field $\tilde{\zeta}^A(X)$, defining the coordinate transformation $X \to X = X + \tilde{\zeta}(X)$. If a thin brane defect is switched on in our model we treat it as partially breaking the gauge (infinitesimal diffeomorphism) symmetry. Particularly to preserve the position of the brane at $y = 0$ one has to restrict the allowed diffeomorphism to $\tilde{\zeta}_5|_{y=0} = 0$. The further analysis of the scalar spectrum is conveniently performed in the following gauge invariant variables:

$$\check{\eta} = E' - 2\eta + \frac{e^{2\rho}}{\rho'}\psi, \quad \check{\phi} = \phi + \frac{\Phi'}{2\rho'}\psi, \quad \check{\chi} = \chi + \frac{H'}{2\rho'}\psi, \quad \check{S} = S - \frac{1}{\rho'}\psi' + \frac{\rho''}{(\rho')^2}\psi. \tag{25}$$

Note that these variables may have singularities. The states are considered physical if they have finite action. The scalar field $\check{\eta}$ happens to be a Lagrange multiplier for the constraint,

$$\rho'\check{S} = \frac{2Z}{3M_*^3}(\Phi'\check{\phi} + H'\check{\chi}). \tag{26}$$

Thus after resolving this constraint only two independent scalar fields remain. To normalize kinetic terms the fields should be redefined $\check{\psi} = \Omega\hat{\psi}$, $\check{\chi} = \Omega\hat{\chi}$, where $\Omega = Z^{-1/2}e^\rho$. Then the scalar action is reduced to the following form,

$$\sqrt{|g|}\mathcal{L}_{(2),scal} = \frac{1}{2}\Big(\partial_\mu\hat{\phi}\partial^\mu\hat{\phi} + \partial_\mu\hat{\chi}\partial^\mu\hat{\chi} - e^{-2\rho}\begin{pmatrix}\hat{\phi}\\ \hat{\chi}\end{pmatrix}^T\Big(-\partial_y^2 + 2\rho'\partial_y + \hat{\mathcal{M}}\Big)\begin{pmatrix}\hat{\phi}\\ \hat{\chi}\end{pmatrix}\Big), \tag{27}$$

$$\hat{\mathcal{M}} = \partial^2 V + \hat{\mathcal{M}}_{NP} - \rho'' + 3(\rho')^2, \quad \hat{\mathcal{M}}_{NP} = \frac{2Z}{3M_*^3}(-\partial_y + 4\rho')\left[\frac{1}{\rho'}\begin{pmatrix}(\Phi')^2 & \Phi'H'\\ \Phi'H' & (H')^2\end{pmatrix}\right], \tag{28}$$

$\hat{\mathcal{M}}_{NP}$ is a correction to the mass operator that generally speaking can change the spectrum of the scalar fluctuations non-perturbatively.

Let us perform the mass spectrum expansion,

$$\begin{pmatrix}\hat{\phi}(X)\\ \hat{\chi}(X)\end{pmatrix} = e^\rho \sum_m \Psi^{(m)}(x)\begin{pmatrix}\phi^{(m)}(y)\\ \chi^{(m)}(y)\end{pmatrix}, \quad \partial_\mu\partial^\mu\Psi^{(m)} = -m^2\Psi^{(m)}, \tag{29}$$

where the factor $\exp(\rho)$ is introduced to eliminate first derivatives in the equations. We obtain the following equations,

$$\Big(-\partial_y^2 + \hat{\mathcal{M}} - \rho'' + (\rho')^2\Big)\begin{pmatrix}\phi^{(m)}\\ \chi^{(m)}\end{pmatrix} = e^{2\rho}m^2\begin{pmatrix}\phi^{(m)}\\ \chi^{(m)}\end{pmatrix}, \tag{30}$$

Due to the exponent in the mass term any eigenfunction with $m^2 > 0$ is at best a resonance state though it could be quasi-localized in a finite volume around a local minimum of the potential. In [8] the probability for quantum tunneling of quasi-localized light resonances with masses $m \ll M$ was estimated as $\sim \exp\{-\frac{3}{\kappa}\ln\frac{2M}{m}\}$ which for phenomenologically acceptable values of $\kappa \sim 10^{-15}$ and $M/m \gtrsim 30$ means an enormous suppression.

To obtain the spectrum we also have to consider the boundary terms taking into account Gibbons-Hawking supplement,

$$S^{(bound)}_{(2),scal} = -3M_*^3 \int\limits_{bound} d^4x\sqrt{g^{(4)}}\,\left[\rho'\check{S}^2\right]_{\pm} \tag{31}$$

Combining boundary terms arising in variations of $\check{\phi}, \check{\chi}$ and solving the constraint we obtain the following matching conditions,

$$\left[\partial_y\phi^{(m)}\right]_{\pm} = -\frac{2Z}{3M_*^3}\frac{\Phi'|_{y=0}}{\rho'|_{0+}}\left(\Phi'\phi^{(m)} + H'\chi^{(m)}\right)\Big|_{y=0} - 4\rho'|_{0+}\phi^{(m)}, \tag{32}$$

$$\left[\partial_y\chi^{(m)}\right]_{\pm} = -\frac{2Z}{3M_*^3}\frac{H'|_{y=0}}{\rho'|_{0+}}\left(\Phi'\phi^{(m)} + H'\chi^{(m)}\right)\Big|_{y=0} - 4\rho'|_{0+}\chi^{(m)} \tag{33}$$

3 Scalar fluctuations in the ϕ channel

In the phase with $\langle H\rangle = 0$ eqs. of motion (8) entail $\partial^2 V/\partial\Phi\partial H = 0$ and the two scalar sectors decouple. In this case the equation on $\phi^{(m)}$ (30) can be written using eqs. (7,8) in the following factorized form,

$$\left(-\partial_y + \frac{\rho''}{\rho'} - \frac{\Phi''}{\Phi'} + 2\rho'\right)\left(\partial_y + \frac{\rho''}{\rho'} - \frac{\Phi''}{\Phi'} + 2\rho'\right)\phi^{(m)} = e^{2\rho}m^2\phi^{(m)}, \tag{34}$$

Assuming $\lambda_b = \kappa M b$ in the zero gravity limit $\kappa \to 0$ the solutions in the ϕ channel can be obtained exactly. Eq. (34) in this limit can be written as,

$$Q_b Q_b^\dagger \phi^{(m)} = \frac{m^2}{M^2}\phi^{(m)}, \quad Q_b = -\partial_\tau + \frac{\tilde{\rho}_1''}{\tilde{\rho}_1' + b} - \frac{\Phi''}{\Phi'}, \quad Q_b^\dagger = \partial_\tau + \frac{\tilde{\rho}_1''}{\tilde{\rho}_1' + b} - \frac{\Phi''}{\Phi'}, \tag{35}$$

where $\tau = M\beta y$ and $\rho = \kappa\tilde{\rho}_1 + \kappa b\tau + O(\kappa^2)$.

We can use the factorization of the potential to construct the super-partner potential [17, 18] that happens not to depend on b,

$$Q_b^\dagger Q_b = -\partial_\tau^2 + 4 - \frac{2}{\cosh^2\tau}. \tag{36}$$

The solutions with $m^2 \neq 0$ can be constructed from the solutions for the super-partner potential,

$$\phi^{(m)} = Q_b\check{\phi}^{(m)}, \quad Q_b^\dagger Q_b\check{\phi}^{(m)} = \frac{m^2}{M^2}\check{\phi}^{(m)} \tag{37}$$

The super-potential can be factorized in a different way that connects its solution with the solutions of the constant potential,

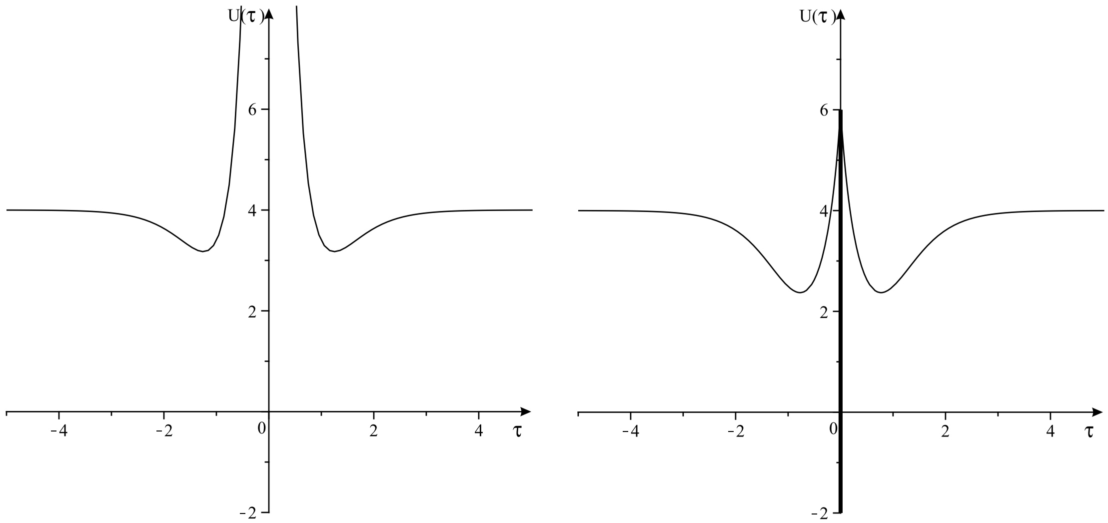
$$\tilde{Q} = -\partial_\tau + \tanh\tau, \quad \tilde{Q} = \partial_\tau + \tanh\tau, \quad Q_b^\dagger Q_b = \tilde{Q}\tilde{Q}^\dagger + 3, \quad \tilde{Q}^\dagger\tilde{Q} = 1 \tag{38}$$

This allows us to obtain the solutions of the continuous spectrum,

$$f_b^{(m)} = Q_b\tilde{Q}\sin k\tau, \quad g_b^{(m)} = Q_b\tilde{Q}\sin k\tau\cos k\tau, \quad m^2 = M^2(4 + k^2) \tag{39}$$

Their linear combination vanishing at $y = 0$ reads,

$$f_b^{(m),0} = b(1 + k^2)f^{(m)} - kg^{(m)}, \quad (f_b^{(m),0})'|_{y=0} = -k(k^2 + 1)(k^2 + 4)b. \tag{40}$$



a b

Figure 1: The potential for scalar fluctuations in the ϕ-channel (a) without defect brane (b) for positive values of the defect brane tension

Both solutions can also be considered for imaginary k corresponding to the mass lower than $2M$. Without matching it is possible to construct the solution decreasing at infinity on one side of the brane however it exponentially increases on another side. With the help of matching it may be possible to construct a normalizable solution with the matching conditions restricting the values of k.

The spectrum of the potential $Q_b^\dagger Q_b$ also contains the states with $m^2 = 3M^2$ that cannot be retrieved from the constant potential. For the branon potential it gives two possible solutions,

$$\phi_b^{(\sqrt{3}M)} = Q_b \frac{1}{\cosh\tau}, \quad \tilde{\phi}_b^{(\sqrt{3}M)} = Q_b\Big(\sinh\tau + \frac{x}{\cosh\tau}\Big). \tag{41}$$

The first one is decreasing at infinity while the second one is increasing exponentially.

Besides the solutions constructed from the super-potential states can also happen to be zero-modes. They can be written in the model-independent way,

$$\omega_b = \frac{\Phi'}{\rho'}, \quad \tilde{\omega}_b = \omega_b \int^\tau d\tau' \frac{1}{\omega_b^2(\tau')} \tag{42}$$

Notice that ω_b may have singularities where ρ' vanishes but decreases at infinity and vice versa $\tilde{\omega}_b$ vanishes simultaneously with ρ' but asymptotically goes to infinity. With appropriate boundaries they still can contribute to the fluctuations.

If there is no defect $b = 0$ the potential has a singular barrier $\sim \frac{2}{\tau^2}$ preventing the appearance of localized solutions (see [11] and Fig. 1.a). Both $\phi_0^{(\sqrt{3}M)}$ and ω_0 happen to be singular at $\tau = 0$ behaving as $\sim 1/\tau$. As mentioned above such singularities are allowed for invariant variables (25) however the corresponding action (27) is not integrable. Likewise $\tilde{\omega}_0$ is not normalizable. Also there is no state constructed from the solutions with imaginary k. Thus there is no localized solution in this case including a (normalizable) Goldstone zero-mode related to spontaneous breaking of translational symmetry.

If the defect has positive tension $b > 0$ the singular potential is shifted to the region which is cut off after matching (see Fig. 1.b). Thus the potential becomes regular except for the δ-well at $\tau = 0$ due to the matching conditions. The zero-mode takes the form,

$$\phi^{(0)} = \begin{cases} \omega_b, & \tau > 0 \\ -\omega_{-b}, & \tau < 0 \end{cases}, \quad \phi^{(0)}|_{y=0} = \frac{\Phi'(0)}{b}, \tag{43}$$

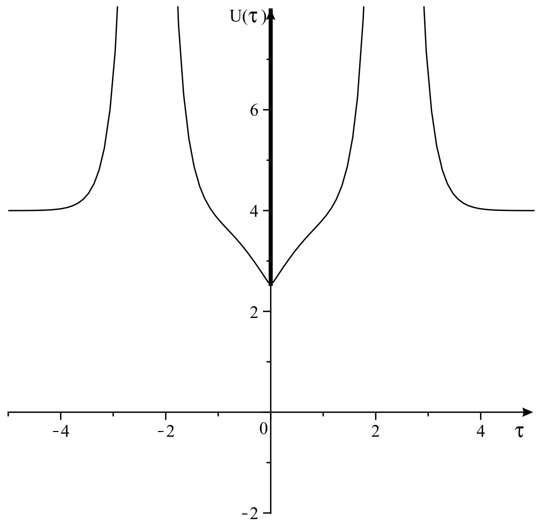


Figure 2: The potential for scalar fluctuations in the ϕ-channel for small negative values of the defect brane tension

and satisfies the matching conditions.

The possible heavy localized state $\phi_b^{(\sqrt{3}M)}$ as a symmetric solution does not satisfy the matching conditions and as an antisymmetric solution has a discontinuity $\left[\phi_b^{(\sqrt{3}M)}\right]_\pm = 2/b$. Thus there is no a localized state with this mass. Yet it is possible to construct the massive localized state from the continuous spectrum solutions with imaginary k. Such a state is given by,

$$\phi^{(m_h)} = \begin{cases} f_b^{(m_h),0}, \\ -f_{-b}^{(m_h),0} \end{cases}, \quad k_h = i\tilde{k}_h = i\frac{-1+\sqrt{1+4a^2}}{2a}, \quad m_h^2 = M^2\frac{-1+\sqrt{1+4a^2}+6a^2}{2a^2} \tag{44}$$

In the limit $b \to 0$ the zero-mode becomes singular and the corresponding action becomes non-integrable. The massive localized state in this limit coincides with the continuous spectrum threshold state $g_0^{(0)}$ with mass $m_h^2 \to 4M^2$.

We can also consider the case of a small negative brane tension $b < 0$. Notice that $b > -\frac{2}{3}$ is chosen in order to keep the Anti-de-Sitter geometry at large τ. In this case we obtain two singular barriers at $\pm\tau_b$, $\tanh^3\tau_b - 3\tanh\tau_b - 3b = 0$ that divide the bulk space-time into three parts essentially separated from each other (see Fig. 2). The spectrum outside the well made by these barriers is continuous. More interestingly inside the well there exists a discrete spectrum of states that vanish at $\tau = \pm\tau_b$ and remain zero outside the potential well. There exist two sets of these solutions that are constructed by matching the solutions $f_b^{(m)}$ and $g_b^{(m)}$. The first set is composed of the solutions $g_b^{(m)}$ with different signs of b on different sides of the brane

$$\phi^{(m)} = \begin{cases} g_b^{(m),0}, & 0 < \tau < \tau_b \\ g_{-b}^{(m),0}, & -\tau_b < \tau < 0\,, \\ 0, & |\tau| > \tau_b \end{cases} \quad m^2 = M^2(4+k^2). \tag{45}$$

The condition on k takes the form,

$$\tan k\tau_b = -\tanh\tau_b \tag{46}$$

and thus the spectrum is,

$$k_n = -\frac{\arctan(\tanh\tau_b)}{\tau_b} + \pi n, \quad n \in \mathbb{Z}, \quad m_n^2 = M^2(4+k_n^2) \tag{47}$$

The second set of solutions from continuous spectrum is constructed by matching solutions (40) vanishing at $\tau = 0$ and with different signs of b on different sides of the brane,

$$\tilde{\phi}^{(m)} = \begin{cases} f_b^{(m),0}, & \tau > 0 \\ f_{-b}^{(m),0}, & \tau < 0 \end{cases}, \quad m^2 = M^2(4 + k^2). \tag{48}$$

The condition on k can be written in the form,

$$\tan k\tau_b = k \frac{(1 + k^2)\tanh^3 \tau_b - 3k^2 \tanh \tau_b}{-3k^2 - (1 + k^2)\big(3\tanh^2 \tau_b - \tanh^4 \tau_b\big)} \simeq \frac{3\tanh \tau_b - \tanh^2 \tau_b}{\tanh^4 \tau_b - 3\tanh^2 \tau_b - 3} \frac{1}{\tau_b} \cdot (k\tau_b), \tag{49}$$

where $k > 0$.

For very small τ_b it can be written simply as $\tan \tau_b k = \tau_b k$. As expected for $b \to 0$, $\tau_b \to 0$ these states become very heavy and decouple. This condition on k is satisfied by the two solutions with imaginary $k = i, 2i$ that are allowed because the function has a compact support by construction. However they happen to be trivial $\phi = 0$.

4 Light scalar state

In the phase with $\langle H \rangle = 0$ the equation for the χ-channel,

$$\Big[-\partial_\tau^2 + \frac{1}{\beta^2 M^2} e^{-2\rho}\Big(-2\Delta_H + 2\Phi^2\Big) + 4(\rho')^2 - 2\rho''\Big]\chi_m = \frac{m^2}{M^2\beta^2} e^{2\rho}\chi^{(m)}, \tag{50}$$

does not gain nonperturbative correction and in the limit $\kappa \to 0$ coincides with the corresponding equation in the model without gravity.

For the minimal model in the phase with $\langle H \rangle = 0$ the χ-channel contains zero-mode that remains massless when next order in κ corrections are taken into account [14]. In the phase with $\langle H \rangle \neq 0$ this state gains mass and nonzero ϕ component that can be computed using the perturbation theory in the parameter μ/M controlling the deviation from the critical point.

$$\chi^{(m)} = \textstyle\sum_{n,k}^\infty \kappa^n \left(\frac{\mu}{M}\right)^k \chi_{n,k}, \quad \phi^{(m)} = \textstyle\sum_{n,k}^\infty \kappa^n \left(\frac{\mu}{M}\right)^{k+1} \phi_{n,k}, \tag{51}$$

$$m^2 = M^2 \textstyle\sum_{n,k}^\infty \kappa^n \left(\frac{\mu}{M}\right)^k (m^2)_{n,k}. \tag{52}$$

assuming $\lambda_b = \kappa M b$.

The computation that we do not present here because of its complexity gives the same leading order of mass as in the model without gravity in the case of the defect brane with positive tension and without the defect brane,

$$(m^2)_{0,1} = 2, \tag{53}$$

whereas for $b < 0$ the result happens to nontrivially depend on b. For $b \longrightarrow 0$ there is a smooth limit $(m^2)_{0,1} \longrightarrow 2$ while for $b \longrightarrow -\frac{2}{3}$,

$$(m^2)_{0,1} \longrightarrow \frac{14}{5} + \frac{16}{5}(\ln 2)^2 + \frac{16}{5}\ln 2 \approx 6.5555 \tag{54}$$

In the absence of a defect brane the next leading order of mass happen to be

$$(m^2)_{0,2} = -128\sqrt{3}\text{arctanh}\frac{\sqrt{3}}{3} + 146 + \frac{4}{3}\ln 2 \cdot (1 + \ln 2) - \frac{\pi^2}{9} \approx +0.4817, \tag{55}$$

whereas similar computation for the model without gravity gives,

$$(m^2)_{0,2}^{NG} = -\frac{130442}{121275} \approx -1.0756. \tag{56}$$

Thus we reveal another manifestation of the discontinuity in the scalar fluctuation mass spectrum between a theory without gravity since the very beginning and a theory in the zero gravity limit.

5 Conclusion

In this work we investigated the nonperturbative effects in the scalar spectrum of a thick brane due to the gravity and the defect in the form of a rigid thin brane. We have shown the essential role that the presence of the defect plays in the localization of light scalar states. The case of the negative tension happens to be the most curious one. Then the singular barriers in the potential form a potential well with infinitely tall walls containing the discrete spectrum of localized states completely isolated from the bulk. Though that would be an ideal mechanism for matter localization on a brane it is not clear how it can be generalized on other fields.

Acknowledgments

We acknowledge the financial support by Grants RFBR, project 13-02-00127 and project 13-01-00136 as well as by the Saint Petersburg State University grant 11.38.660.2013. One of us (A.A.) was partially supported by projects FPA2010-20807.

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