Spontaneous radiatively induced breaking of conformal invariance in the Standard Model

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Abstract

A mechanism of radiatively induced breaking of the conformal symmetry in the Standard Model is suggested. The system of one scalar (Higgs) and one fermion (top-quark) fields with Yukawa and ϕ^4 interactions is considered. The infrared instability of the Coleman-Weinberg effective potential for this system leads to the appearance of a finite renormalization scale and thus to breaking of the conformal symmetry. Finite condensates of both scalar and spinor fields appear. It is shown that the top quark condensate can supersede the tachyon mass of the Higgs field. The Higgs boson is treated as an elementary scalar and the standard mechanism of electroweak symmetry breaking remains unchanged. The difference from the Standard Model appears in the value of the Higgs boson self-coupling constant.

1 Introduction

Recently the major LHC experiments reported upon the discovery of a boson with the mass of about 125 GeV [1, 2]. Further experimental studies [3, 4] showed that this particle behaves very much like the Higgs boson of the Standard Model (SM) [5]. Nevertheless, the question about the fundamental mechanism(s) of mass generation is far beyond the final resolution. Moreover, there is a number of indirect evidences that the conformal symmetry (CS) might be the proper feature of the true fundamental theory, while the SM is just an effective theory with a softly broken CS, see *e.g.* [6] and references therein.

According to the general wisdom, all SM particles (may be except neutrinos) own masses due to "interaction" with the Higgs boson vacuum expectation value. The latter emerges after the spontaneous breaking of the $O(4)$ symmetry in the scalar sector [7, 8]. In the SM, one deals with the potential

$$V_{\rm Higgs}(\Phi) = \lambda(\Phi^\dagger\Phi)^2 + \mu^2\Phi^\dagger\Phi, \tag{1}$$

where one component of the complex scalar doublet field $\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix}$ acquires a non-zero vacuum expectation value $\langle\Phi^0\rangle = v/\sqrt{2}$ if $\mu^2 < 0$ (the vacuum stability condition $\lambda > 0$ is assumed). Note that the tachyon-like mass term in the potential is crucial for this construction. In contrast to the $O(4)$ spontaneous symmetry breaking (SSB), it breaks the conformal symmetry explicitly being the only one *fundamental* dimensionful parameter in the SM. We recall that the explicit conformal symmetry breaking in the Higgs sector gives rise to the naturalness (or fine tuning) problem in the renormalization of the Higgs boson mass. That is certainly the most unpleasant feature of the SM.

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2 The naturalness problem

Let us look at some details of the naturalness problem. In the one-loop approximation the Higgs boson mass gets huge corrections due to quadratically divergent amplitudes:

$$M_H^2 = (M_H^0)^2 + \frac{3\Lambda^2}{8\pi^2 v^2}\left[M_H^2 + 2M_W^2 + M_Z^2 - 4m_t^2\right], \tag{2}$$

where Λ is an ultraviolet cut-off parameter. Certainly it is unnatural to have a huge hierarchy between M_H and M_H^0[1]. There are two general ways to solve the problem:
— either to exploit some (super)symmetry to cancel out the huge terms,
— or to introduce some new physics at a scale not very far from the electroweak (EW) one, *i.e.* making Λ being not large.

One can find in the literature quite a lot of models for both options. Actually, since the (super)symmetry is not observed experimentally at the EW scale, the first way besides the introduction of the symmetry requires application of a mechanism of its breaking at some energy scale close to the EW one. On the other hand, the experimental data coming from modern accelerators and rare decay studies disfavors most of scenarios of new physics with scales up to about 1 TeV and even higher. Moreover, it was shown that the measured value of the Higgs boson mass makes the SM being self-consistent up to very high energies of the order 10^{11} GeV [9] or even up to the Planck mass scale [10, 11]. Direct and indirect experimental searches push up and up possible energy scale of new physical phenomena. So the naturalness problem becomes nowadays more and more prominent. And the question, why the top quark mass, the Higgs boson mass, the vacuum expectation value v, and the electroweak scale itself are of the same order becomes more and more intriguing.

The correction (2) comes from Feynman diagrams with boson single-propagator loops (tadpoles) and from the two-point function with two top-quark propagators. The latter actually is reduced to a top quark tadpole:

$$\begin{aligned} &-N_c\int_{\Lambda_t}\frac{d^4k}{i\pi^2}\,\frac{\mathrm{Tr}(\hat{k}+m_t)((\hat{p}-\hat{k})+m_t)}{(k^2-m_t^2)((p-k)^2-m_t^2} \to -4N_c\int_{\Lambda_t}\frac{d^4k}{i\pi^2}\,\frac{1}{k^2-m_t^2} + \mathcal{O}(m_t^2) \\ &= -4N_c A_0(m_t^2,\Lambda_t^2) + \mathcal{O}(m_t^2), \end{aligned} \tag{3}$$

where A_0 is the standard Passarino-Veltman function.

One the other hand, we have the following standard formal definition of the quark condensate:

$$\langle \bar{q}\, q\rangle \equiv -N_C\int_{\Lambda_q}\frac{d^4k}{i\pi^2}\,\frac{\mathrm{Tr}(\hat{k}+m_q)}{k^2-m_q^2+i\varepsilon} \sim -4N_C m_q A_0(m_q^2,\Lambda_q^2). \tag{4}$$

So, the top quark contribution to Eq. (2) is *formally* provided by its condensate value. Our conjecture is that the formal correspondence has a deep physical meaning and reveals itself not only here.

We claim that the very existence of the quark condensate comes out from the QFT rules. But its value of course depends on the details of the model. In particular, the value of the light quark condensate is rather well known from low-energy strong interactions, $\sqrt[3]{\langle \bar{q}\, q\rangle} \simeq -250$ MeV. The possibility to extract this number from observables if provided by the presence of [non-trivial] non-perturbative interactions at the corresponding energy scale. On the other hand, nothing at all is known from the phenomenology concerning the value of the top quark condensate. That is just because the energy scale of the top quark mass allows only perturbative QCD interactions,

[1] We stress that M_H^0 here should be directly related to the tachyon-like mass parameter in the initial Lagrangian, where it appears as a *fundamental* scale.

which are not sensitive to the condensate[2]. Note also, that it is clear that the scale of the light quark condensate is provided by the $\Lambda_{\rm QCD}$ scale: $\langle\bar{q}\,q\rangle \sim -M_q\Lambda_{\rm QCD}^2$, where M_q is the constituent quark mass which in its turn also has the same scale $M_q \sim \Lambda_{\rm QCD}$.

3 Coleman-Weinberg effective potential in the SM

Let us now consider a simple model with one scalar field ϕ and one fermion field f. We demand the conformal symmetry for the model. The symmetry allows[3] the existence of two types of interactions in this models: the ϕ^4 self-interaction of the scalar and the Yukawa term. So we start with the classical potential

$$V_{\rm cl} = \lambda\phi_c^4/4! + y\phi_c\bar{f}_c f_c, \tag{5}$$

where we used the notation of Ref. [12], in particular the subscript "c" underlines that ϕ_c and f_c are classical fields and they obey the conformal symmetry. The standard one-loop calculation of the effective potential gives two contributions: the one from scalar loops, and the one from fermion loops:

$$\Delta V_{\rm sc} = \frac{1}{2}\int\frac{d^4k}{(2\pi^4)}\ln\left(1+\frac{\lambda\phi_c^2}{2k^2}\right) \to \frac{\lambda\Lambda^2}{256\pi^2}\phi_c^2 + \frac{\lambda^2\phi_c^4}{256\pi^2}\left(\ln\frac{\lambda\phi_c^2}{2\Lambda^2}-\frac{1}{2}\right), \tag{6}$$

$$\Delta V_f = -4N_C{\rm Tr}\int\frac{d^4k}{(2\pi^4)}\ln\left(1+\frac{y\phi_c(\hat{k}+m_f)}{k^2-m_f^2}\right) \to -4N_C\frac{ym_f\Lambda^2}{16\pi^2}\phi_c$$
$$-4N_C\frac{y^2m_f^2\phi_c^2}{32\pi^2}\left(\ln\frac{ym_f\phi_c}{\Lambda^2}-\frac{1}{2}\right)+\dots \tag{7}$$

Due to the condition of the classical conformal symmetry, we have to remormalize the first term on the right hand side of Eq. (6) to zero. On the other hand, it is clear that the effective potential possesses infrared divergence at $\phi = 0$. So, some energy scale M should be introduced to renormalize the logarithmic term. As demonstrated by S. Coleman and E. Weinberg, that induces a spontaneous breaking of the conformal symmetry and leads to the appearance of a non-zero mass and a non-zero vacuum expectation value of ϕ. Obviously, if we have $\langle\phi\rangle \neq 0$ in a model with Yukawa interactions, we automatically get a mass for the fermion. Then the second contribution (7) to the effective potential emerges. Note that the conformal symmetry doesn't require to drop (*i.e.* renormalize to 0) the first term on the right hand side there, since it is proportional to m_f which is zero in the unbroken phase. This term, is again nothing else but the tadpole contribution, *i.e.* the fermion condensate. In this way we have two phases: the classical one with

$$m_\phi = m_f \equiv 0, \qquad \langle\phi\rangle \equiv 0, \qquad \langle\bar{f}\,f\rangle \equiv 0$$

and the one with spontaneously broken CS:

$$m_\phi \sim m_f \sim M, \qquad \langle\phi\rangle \sim M, \qquad \langle\bar{f}\,f\rangle \sim -M^3,$$

where M is the renormalization scale, and we assumed that the coupling constants are not extremely small $\lambda \sim y \sim 1$.

So, we clearly see that system (5) is unstable in the infrared region, which leads to the effect of dimensional transmutation. According to the logic of the original paper [12], the scale comes to the model somewhere from outside the theory via the renormalization procedure. Another crucial point is the question about stability and perturbativity in the phase with the broken conformal symmetry.

[2]Due to the Furry theorem the coefficient before the fermion tadpole with vector vertex is zero. While the tadpole itself (with a scalar vertex) can be non-zero.

[3]For terms in the Lagrangian of a renormalizable QFT model *allowed* is practically equivalent to *must have*.

4 Dimensional Transmutation in the SM

We suggested [13] the following minimal modification of the Standard model: let us drop the tachyon mass term from the Lagrangian. We take the most intensive interaction of the Higgs filed h

$$L_{\rm int} = -\frac{\lambda}{4}h^4 - \frac{y_t}{\sqrt{2}}h\,\bar{t}t, \tag{8}$$

where h is related to the initial field Φ in the standard way. In this case we have a model with a classical conformal invariance. We claim that the apparent breaking of this symmetry can happen spontaneously because of the infrared instability at the quantum level. Obviously, in such a case, see *e.g.* Ref. [14], the softly broken classical symmetry will protect the Higgs boson mass from rapid running in the UV region. So let us look at a stable solution in the broken phase. The leading contribution to the Coleman-Weinberg effective potential comes from the top quark tadpole:

$$V_{\rm eff}(h) \approx \frac{\lambda}{4}h^4 + \frac{y_t}{\sqrt{2}}\langle\bar{t}\,t\rangle h. \tag{9}$$

Naturally we choose the electroweak energy scale as the (re)normalization point. Than all dimensionful parameters in the effective potential are defined by this scale. The extremum condition for the potential $dV_{\rm cond}/dh|_{h=v} = 0$ yields the relation

$$\lambda v^3 = -\frac{y_t}{\sqrt{2}}\langle\bar{t}\,t\rangle. \tag{10}$$

It follows from the fact that the Higgs field has a zero harmonic v in the standard decomposition of the field h over harmonics $h = v + H$, where H represents excitations (non-zero harmonics) with the condition $\int d^3xH = 0$. The Yukawa coupling of the top quark $y_t = 0.995(5)$ is known from the experimental value of top quark mass $m_t = vy_t/\sqrt{2} \simeq 173.2(9)$ GeV [15], and $v = (\sqrt{2}G_{\rm Fermi})^{-1/2} \approx 246.22$ GeV is related to the Fermi coupling constant derived from the muon life time measurements. So, the spontaneous symmetry breaking yields the potential minimum which results in the non-zero vacuum expectation value v and the Higgs boson mass. In fact, the substitution $h = v + H$ gives

$$V_{\rm eff}(h) = V_{\rm eff}(v) + \frac{m_H^2}{2}H^2 + \lambda^2 vH^3 + \frac{\lambda}{4}H^4, \tag{11}$$

which defines the scalar particle mass as

$$m_H^2 = 3\lambda v^2. \tag{12}$$

We stress that this relation is different from the one ($m_H^2 = 2\lambda v^2$) which emerges in the SM with the standard Higgs potential (1).

With the aid of Eqs. (10) and (12), the squared scalar particle mass can be expressed in terms of the top quark condensate:

$$m_H^2 = -\frac{3y_t\langle\bar{t}\,t\rangle}{\sqrt{2}v}. \tag{13}$$

To get $m_H = 125.7$ GeV we need

$$\langle\bar{t}\,t\rangle \approx -(122\ {\rm GeV})^3. \tag{14}$$

As discussed above, such a large value of the top quark condensate does not affect the low energy QCD phenomenology. Since heavy quark condensates do not contribute to observed

QCD quantites (*e.g.* via sum rules), we do not have any experimental or theoretical limits on the top condensate value, see [16] and references therein.

Note that the energy scale of the top quark condensate appears to be the same as the general electroweak one. We believe that the scale of light quark condensate is related to the scale of the conformal anomaly in QCD. At the same time those anomalous properties of the QCD vacuum lead to the constituent mass of a light quark to be of the order 300 MeV. As concerning the top quark, some anomalous properties of the *relevant* vacuum give rise to the mass of this quark[4] and to the condensate being of the same energy scale.

One can note that even we have dropped the scalar field mass term from the classical Lagrangian, it will re-appear after quantization and subsequent renormalization. In fact, such a counter-term in the Higgs sector is necessary. But as described in Ref. [14], the conformal symmetry of the classical Lagrangian will lead to just the proper quantity in the mass term being consistent with all other quantum effects. A similar situation takes place in QCD: the chiral symmetry at the quark level re-appear at the hadronic level even so that the breaking is obvious [17].

In conclusion, we suggest to apply the Coleman-Weinberg mechanism of dimensional transmutation to induce spontaneous conformal symmetry breaking in the Standard Model. This enables us to avoid the problem of the regularization of the divergent tadpole loop integrals by relating them to condensate values hopefully extracted from experimental observations. The top quark condensate can supersede the tachyon-like mass term in the Higgs potential. The suggested mechanism allows to establish relations between condensates and masses including the Higgs boson one. In a sense, we suggest a simple bootstrap between the Higgs and top fields (and their condensates). We underline that we consider the Higgs boson to be an elementary particle without introduction of any additional interaction beyond the SM contrary to various technicolor models. After the spontaneous symmetry breaking in the tree level Lagrangian, the difference from the SM appears only in the value of the Higgs boson self-coupling λ. The latter hardly can be extracted from the LHC data, but it will be certainly measured at a future linear e^+e^- collider.

Acknowledgments A.Arbuzov is grateful to the Dynasty foundation for financial support.

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