On a possibility to calculate mass ratios of fundamental fermions and the fine structure constant in the compensation approach

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Abstract

The problem of a calculation of parameters of the Standard Model is considered in the framework of the compensation approach. A principal possibility to calculate mass ratios of fundamental quarks and leptons is demonstrated, as well of mixing angles of quarks, *e.g.* of the Cabibbo angle. A possibility of a spontaneous generation of an effective interaction of electroweak gauge bosons W^a and B is demonstrated. In case of a realization of a non-trivial solution of a set of compensation equations, parameter $\sin^2\theta_W$ is defined. The non-trivial solution is demonstrated to provide a satisfactory value for the electromagnetic fine structure constant α at scale M_Z: $\alpha(M_Z) = 0.00772$. The results being obtained may be considered as sound arguments on behalf of a possibility of a calculation of parameters of the Standard Model.

1 Introduction

In works [1] - [6], N.N. Bogoliubov compensation principle [7, 8] was applied to studies of a spontaneous generation of effective non-local interactions in renormalizable gauge theories. The method and applications are also described in full in the book [9].

In particular, papers [4] - [5] deal with an application of the approach to the electro-weak interaction and a possibility of spontaneous generation of effective anomalous three-boson interaction of the form

$$-\frac{G}{3!}\,F\,\epsilon_{abc}\,W^a_{\mu\nu}\,W^b_{\nu\rho}\,W^c_{\rho\mu}\,; \qquad (1)$$
$$W^a_{\mu\nu}\,=\,\partial_\mu W^a_\nu-\partial_\nu W^a_\mu\,+g\,\epsilon_{abc}W^b_\mu W^c_\nu\,.$$

with uniquely defined form-factor $F(p_i)$, which guarantees effective interaction (1) acting in a limited region of the momentum space. It was done in the framework of an approximate scheme, which accuracy was estimated to be $\simeq (10-15)\%$ [1]. Would-be existence of effective interaction (1) leads to important non-perturbative effects in the electro-weak interaction. It is usually called anomalous three-boson interaction and it is considered for long time on phenomenological grounds [10]. Our interaction constant G is connected with conventional definitions in the following way

$$G\,=\,-\,\frac{g\,\lambda}{M_W^2};\quad \lambda_\gamma=-\,0.022\pm0.019\,;\quad \lambda_Z=-\,0.09\pm0.06\,; \qquad (2)$$

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where $g \simeq 0.65$ is the electro-weak coupling and we also show the best experimental limitations for parameter λ [11].

Solution of the analogous compensation procedure in QCD correspond to $g(z_0) = 3.8$ [6]. For the electro-weak interaction we have [5]

$$g(z_0) = 0.60366\,; \quad z_0 = 9.6175\,; \quad |\lambda| = 2.88 \cdot 10^{-6}\,. \tag{3}$$

Here z_0 is a dimensionless parameter, which is connected with value of a boundary momentum, that is with effective cut-off Λ according to the following definition [5]

$$\frac{2\,G^2\,\Lambda^4}{1024\,\pi^2} = \frac{g^2\,\lambda^2\,\Lambda^4}{512\,\pi^2\,M_W^4} = z_0\,. \tag{4}$$

As a rule the existence of a non-trivial solution of a compensation equation impose essential restrictions on parameters of a problem. Just the example of these restrictions is the definition of coupling constant $g(z_0)$ in (3). In the present work we consider possibilities of definition of important physical parameters: mixing angles and mass ratios of elementary constituents of the Standard Model.

2 A model for mass relations of quarks and leptons

Following the approach [1] - [6] let us formulate the compensation equations for would-be four-fermion interaction of two types of quarks and two leptons, that is we consider one generation of fundamental fermions. We call them "u", "d", "e" and "ν", which in the standard way are represented by their left ψ_L and right ψ_R components. We admit initial masses for all participating fermions to be zero and we will look for possibility of them to acquire masses $m_i, i = 1, ...4$ due to interaction with scalar Higgs-like composite field.

Then let us consider a possibility of spontaneous generation of the following interaction, which is constructed by analogy with the Nambu – Jona-Lasinio effective interaction [12]

$$\begin{aligned} L^F_{eff} \,=\, & G_1 \bar u_L\, u_R\, \bar u_R\, u_L + G_2 \bar d_L\, d_R\, \bar d_R\, d_L + G_4 \bar e_L\, e_R\, \bar e_R\, e_L + G_7\, \bar\nu_L\, \nu_R\, \bar\nu_R\, \nu_L + \\ & G_3(\bar u_L\, u_R\, \bar d_R\, d_L + \bar d_L\, d_R\, \bar u_R\, u_L) + G_5(\bar u_L\, u_R\, \bar e_R\, e_L + \bar e_L\, e_R\, \bar u_R\, u_L) + \\ & G_6(\bar e_L\, e_R\, \bar u_R\, u_L + \bar e_R\, e_L\, \bar u_L\, u_R) + G_8(\bar\nu_L\, \nu_R\, \bar d_R\, d_L + \bar d_L\, d_R\, \bar\nu_R\, \nu_L) + \\ & G_9(\bar\nu_L\, \nu_R\, \bar u_R\, u_L + \bar u_L\, u_R\, \bar\nu_R\, \nu_L) + G_{10}(\bar\nu_L\, \nu_R\, \bar e_R\, e_L + \bar e_L\, e_R\, \bar\nu_R\, \nu_L). \end{aligned} \tag{5}$$

Here all coupling constants G_i have dimension of the inverse mass squared M^{-2}.

Now we would like to find out, if the four-fermion interaction (8) could be spontaneously generated. In doing this we again proceed with the add-subtract procedure

$$\begin{aligned} & L = L_0 \,+\, L_{int}\,; \quad L_{int} \,=\, L_{0int} \,+\, L^F_{eff}\,; \\ & L_0 = \sum_{u,d} \bar q(x)(\imath\partial_\alpha\gamma_\alpha - m)q(x) + \sum_{e,\nu} \bar l(x)(\imath\partial_\alpha\gamma_\alpha - m)l(x) - L^F_{eff}\,; \end{aligned} \tag{6}$$

here L_{0int} is an initial interaction Lagrangian. Then we have to compensate the undesirable term L_{eff} in the newly defined free Lagrangian. In diagram form the compensation equation for four fermions participating the interaction in one-loop approximation is presented in Fig. 1.

Let us define effective cut-off Λ in integrals of equation (8). We shall see below, that Λ may be defined in the course of solution of compensation equations. With account of this definition we introduce the following dimensionless variables

$$\begin{aligned} & y_1 = \frac{G_1\Lambda^2}{8\,\pi^2};\; y_2 = \frac{G_2\Lambda^2}{8\,\pi^2};\; y_3 = \frac{G_3\Lambda^2}{8\,\pi^2};\, z_1 = \frac{G_4\Lambda^2}{8\,\pi^2};\; z_2 = \frac{G_7\Lambda^2}{8\,\pi^2};\; z_3 = \frac{G_{10}\Lambda^2}{8\,\pi^2}; \\ & x_1 = \frac{G_5\Lambda^2}{8\,\pi^2};\; x_2 = \frac{G_9\Lambda^2}{8\,\pi^2}\,;\; x_3 = \frac{G_6\Lambda^2}{8\,\pi^2};\; x_4 = \frac{G_8\Lambda^2}{8\,\pi^2};\; \xi_1 = \frac{m_2}{m_1};\; \xi_2 = \frac{m_3}{m_1};\; \xi_3 = \frac{m_4}{m_1}. \end{aligned} \tag{7}$$

Then we consider scalar bound state consisting of all possible fermion-antifermion combinations $\bar{u}u$, $\bar{d}d$, $\bar{e}e$ and $\bar{\nu}\nu$. The corresponding set of Bethe-Salpeter equations is shown in Fig. 2. In this way we come to the following set of ten compensation equations presented in Fig. 1 and four Bethe-Salpeter equations shown in Fig. 2. Let us note, that in Fig. 2 we present also wouldbe contributions of gauge bosons exchanges, which in the calculations of the present section are not taken into account. Note also, that terms with factor A arise from vertical diagrams in Fig. 1. Let us remind, that the sign minus before linear terms in compensation equations is connected with opposite signs of terms corresponding to effective interactions in the new free Lagrangian and in the new interaction Lagrangian.

$$
\begin{aligned}
&-y_1 + A y_1^2 + 3(y_1^2 + y_3^2) + x_1^2 + x_2^2 = 0;\; -y_2 + A y_2^2 \xi_1^2 + 3(y_2^2 + y_3^2) + x_3^2 + x_4^2 = 0;\\
&-y_3 + A y_3^2 \xi_1 + 3 y_3 (y_1 + y_2) + x_1 x_3 + x_2 x_4 = 0;\; A z_1^2 \xi_2^2 + 3(x_1^2 + x_3^2) + z_1^2 + z_3^2 = z_1;\\
&A z_2^2 \xi_3^2 + 3(x_2^2 + x_4^2) + z_2^2 + z_3^2 = z_2;\; A z_3^2 \xi_2 \xi_3 + 3(x_1 x_2 + x_3 x_4) + z_1 z_3 + z_2 z_3 = z_3; \qquad (8)\\
&A x_1^2 \xi_2 + 3(x_1 y_1 + x_3 y_3) + x_1 z_1 + x_2 z_3 = x_1;\; A x_2^2 \xi_3 + 3(x_2 y_1 + x_3 y_3) + x_1 z_1 + x_2 z_3 = x_2;\\
&A x_3^2 \xi_1 \xi_2 + 3(x_1 y_3 + x_4 y_3) + x_1 z_3 + x_2 z_2 = x_3;\\
&A x_4^2 \xi_1 \xi_3 + 3(x_2 y_3 + x_4 y_2) + x_3 z_3 + x_4 z_2 = x_4; \quad A = \frac{m_u^2}{4\Lambda^2} \ln \frac{\Lambda^2}{\bar{m}^2};\; B = 1 + \frac{m_0^2}{2\Lambda^2} \ln \frac{\Lambda^2}{\bar{m}^2};\\
&\frac{1}{B} = 3(y_1 + \xi_1 y_3) + \xi_2 x_1 + \xi_3 x_2;\; \frac{\xi_1}{B} = 3(y_3 + \xi_1 y_2) + \xi_2 x_3 + \xi_3 x_4; \qquad (9)\\
&\frac{\xi_2}{B} = 3(x_1 + \xi_1\, x_3) + \xi_2 z_1 + \xi_3 z_3;\; \frac{\xi_3}{B} = 3(x_2 + \xi_1 x_4) + \xi_2 z_3 + \xi_3 z_2;
\end{aligned}
$$

where m_0 is the bound state mass and $\bar{m}$ is an average mass of participating fermions.

Let us comment the appearance of mass parameters ξ_i in terms, corresponding to vertical diagrams in Fig. 1. Due to the orthogonality of matrices $\frac{1+\gamma_5}{2}$, $\frac{1-\gamma_5}{2}$ terms containing $\hat{q}$ cancel and we are left only with mass terms in spinor propagators. Introduction of the average $\bar{m}$, instead of substituting in proper places different masses m_i, means of course an approximation. However due to logarithmic dependence on this parameter, this approximation seems to be reasonable. Factor A has to be very small and factor B has to be close to unity, because $\Lambda \gg m_i$. Ten equations (8) correspond to the set of compensation equations, while four equations (9) represent the Bethe-Salpeter equations. Let us remind, that after performing the compensation procedure, which means exclusion of four-fermion vertices in the newly defined free Lagrangian, we use the resulting coupling constants in the newly defined interaction Lagrangian with the opposite sign.

The appearance of ratios ξ_i in Bethe-Salpeter part (9) of the set presumably needs explanation. We assume, that the scalar composite state, which in our approach serves as a substitute of the elementary Higgs scalar, consists of all existing quark-antiquark and lepton-antilepton pairs $\bar{\psi}_L\, \psi_R$. Then coupling of this scalar with different fermions will give their masses according to well known relation

$$g_a \;=\; \frac{g\, m_a}{\sqrt{2}\, M_W}\,. \qquad (10)$$

On the other hand, Bethe-Salpeter wave functions are proportional to coupling constants g_a, where a is just the constituent particle. Thus we change a ratio of coupling constants by a ratio of corresponding masses ξ_i.

In Section 3 we consider interaction of the Higgs field also with electroweak gauge bosons. Thus we assume, that the Higgs scalar consist of all existing fundamental massive fields. So in future studies it should be necessary to consider a set of Bethe-Salpeter equations including all possible constituents. Presumably it would be advisable to take into account also contributions of gauge interactions, which schematically presented in triangle diagrams of Fig. 2.

Now let us consider solutions of set (8, 9). First of all let us remind, that parameter A is very small, so we look for solutions, which are stable in the limit $A \to 0$. We also will consider only real solutions, because our variables just correspond to physical observable quantities. Namely, we have for $A = 0.0001$ the following real solutions

$$\begin{aligned}
&y_1 = 0.12500,\ y_2 = y_1,\ y_3 = -\,y_1,\ z_1 = y_1,\ z_2 = y_1,\ z_3 = -\,y_1,\ x_1 = y_1,\\
&x_2 = -\,y_1,\ x_3 = -y_1,\ x_4 = y_1,\ xi_1 = -\,1,\ \xi_2 = 1,\ \xi_3 = -\,1,\ B = 1.00001. \qquad (11)\\
&y_1 = 0.12500,\ y_2 = y_1,\ y_3 = -y_1,\ z_1 = y_1,\ z_2 = y_1,\ z_3 = y_1,\ x_1 = y_1,\\
&x_2 = y_1,\ x_3 = -y_1,\ x_4 = -y_1,\ \xi_1 = -1,\ \xi_2 = 1,\ \xi_3 = 1,\ B = 1.00001. \qquad (12)\\
&y_1 = 0.24999,\ y_2 = 0.33333,\ y_3 = 0,\ z_1 = 0.24999,\ z_2 = 0.56468,\ z_3 = -0.38570,\\
&x_1 = -0.24999,\ x_2 = x_3 = x_4 = 0,\ \xi_1 = 0.86603,\ \xi_2 = -\,1,\ \xi_3 = 0,\ B = 1.00003. \qquad (13)\\
&y_1 = 0.24999,\ y_2 = 0.33333,\ y_3 = 0,\ z_1 = 0.24999,\ z_2 = 0.99998,\ z_3 = 0,\\
&x_1 = -0.24999,\ x_2 = x_3 = x_4 = 0,\ \xi_1 = 0,\ \xi_2 = 1,\ \xi_3 = 0.5,\ B = 1.000025. \qquad (14)\\
&y_1 = 0.33332,\ y_2 = 0,\ y_3 = 0,\ z_1 = 0.24999,\ z_2 = 0.99998,\ z_3 = 0,\\
&x_1 = x_2 = x_3 = x_4 = 0,\ \xi_1 = 0,\ \xi_2 = \xi_3 = 0.57735,\ B = 1.000033. \qquad (15)\\
&y_1 = 0.33332,\ y_2 = 0.057288,\ y_3 = 0,\ z_1 = 0.26344,\ z_2 = 0.56470,\ z_3 = -0.38570,\\
&x_1 = x_2 = 0,\ x_3 = 0.12285,\ x_4 = -\,0.17986, \xi_1 = \xi_2 = \xi_3 = 0,\ B = 1.00003. \qquad (16)\\
&y_1 = 0.29077,\ y_2 = 0.29077,\ y_3 = -\,0.04256,\ z_1 = 0.25534,\ z_2 = z_3 = x_2 = x_4 = 0,\\
&x_1 = 0.17801,\ x_3 = 0.17801,\ \xi_1 = 1,\ \xi_2 = 1.4344,\ \xi_3 = 0,\ B = 1.00003. \qquad (17)\\
&y_1 = 0.19313,\ y_2 = 0.18758,\ y_3 = 0.14295,\ z_1 = 0.857858,\ z_2 = z_3 = x_2 = x_4 = 0,\\
&x_1 = -0.14116,\ x_3 = 0.14393,\ \xi_1 = 1.069,\ \xi_2 = 0.26728,\ \xi_3 = 0,\ B = 1.00002. \qquad (18)
\end{aligned}$$

Note, that the first three solutions (11,12, 13) contain mass ratios ξ_i with negative signs, that is quite unnatural for fermions entering to one generation. In solutions (14, 15) there is no place for massless neutrino. For the moment, the most suitable ones are the three last solutions (16, 17, 18). All these solutions have nonnegative parameters ξ_i and at least one lepton being massless, that might be a neutrino. The solution (16) gives one (the first) fundamental fermion (quark) being much heavier, than three others, that reminds situation of the third generation with the very heavy t quark. The solution (17) gives charged lepton mass approximately the same as those of quarks, that may hint the situation in the second generation with approximately equal masses of the muon and of the s-quark. The solution (18) gives two different masses for the quark pair, while the wouldbe charged lepton has the mass approximately four times smaller than that of the first quark. This resembles situation for the first generation. Indeed, let us take for the electron mass its physical value $m_e = 0.51\,MeV$. Then we have from (18)

$$m_e = 0.51\,MeV;\ m_u = \frac{m_e}{\xi_2} = 1.90\,MeV;\ m_d = \frac{m_e\,\xi_1}{\xi_2} = 2.04\,MeV. \qquad (19)$$

The wouldbe u-quark mass fits into error bars of its definition, while the wouldbe d-quark mass is rather lighter than its physical value [11]. Note, that in our estimates we have not taken into account the phenomenon of mixing of down quarks (d, s, b).

We would also draw attention to the important point, that for all solutions parameter B is close to unity, just as we have expected [1]. With decreasing of parameter A, which is proportional to ratio squared of the mass of the first quark and cut-off Λ, parameter B tends to unity exactly. Emphasize, that solutions (17, 18) are stable in respect to $A \to 0$.

Let us estimate also an order of magnitude of mixing angles between generations. For the purpose we introduce in effective interaction (5) additional terms, corresponding to the wouldbe

[1] Solutions with B being not close to unity are rejected here as well as in what follows.

s, d mixing.

$$\Delta L = \frac{8\pi^2}{\Lambda^2}\Big(y_{12}\big((\bar{s}'_R d'_L + \bar{d}'_R s'_L)\bar{d}'_L d'_R + (\bar{s}'_L d'_R + \bar{d}'_L s'_R)\bar{d}'_R d'_L\big) + y_{32}\big((\bar{s}'_R d'_L + \bar{d}'_R s'_L)\bar{s}'_L s'_R + (\bar{s}'_L d'_R + \bar{d}'_L s'_R)\bar{s}'_R s'_L\big) + y_{52}(\bar{s}'_L d'_R \bar{d}'_R s'_L + \bar{s}'_R d'_L \bar{d}'_L s'_R) + t_{32}(\bar{d}'_L d'_R \bar{s}'_L s'_R + \bar{d}'_R d'_L \bar{s}'_R s'_L)\Big); \quad (20)$$

We have also mixing in mass terms of the two spinor fields d', s', where ϕ is the Cabibbo angle

$$-m_u\big(\bar{u}u + \xi_1 \bar{d}'d' + \xi_4 \bar{s}'s' + \xi_6(\bar{s}'d' + \bar{d}'s')\big); \; d' = \cos\phi d + \sin\phi s\,; \; s' = -\,\sin\phi d + \cos\phi .s \quad (21)$$

Now we have in addition to parameters in (20) parameter y_2 from (7), which corresponds to term $\bar{d}d\bar{d}d$ and we also introduce the analogous parameter y_{21}, corresponding to term $\bar{s}s\bar{s}s$. These variables will be fixed by results (16 - 18). We now neglect all other transitions but those between d and s states and thus we have the following set of equations

$$\begin{aligned}
&A y_{12} + 3(y_{12}y_2 + y_{32}t_{32} + 2y_{52}y_{12}) = y_{12};\; A y_{32} + 3(y_{12}t_{32} + y_{32}y_{21} + 2\,y_{52}y_{32}) = y_{32};\\
&-y_{52} + A y_{52} + 3(y_{12}^2 + y_{32}^2 + 2\,y_{52}^2)\,;\; -t_{32} + A t_{32} + 3(y_2 t_{32} + y_{21}t_{32} + y_{12}y_{32}); \qquad (22)\\
&\frac{\xi_1}{B} = 3(y_2\xi_1 + t_{32}\xi_4 + 2\,y_{12}\xi_6);\; \frac{\xi_4}{B} = 3(t_{32}\xi_1 + y_{21}\xi_4 + 2\,y_{32}\xi_6);\\
&\frac{\xi_6}{B} = 3(y_{12}\xi_1 + y_{32}\xi_4 + 2\,y_{52}\xi_6)\,.
\end{aligned}$$

We consider only real solutions and choose such ones, which allow physical interpretation. Fixing values for y_2 and y_{21} from results (17, 18) and value A we obtain seven equations for seven variables: y_{12}, y_{32}, y_{52}, t_{32}, B, ξ_1/ξ_6, ξ_4/ξ_6. Let us check if there will be a reasonable mixing of solutions (17, 18) that is between the first two generations according to our guess. With $y_2 = 0.18758$, $y_{21} = 0.29077$, $A = 0.000005$, $\xi_6 = 1$ we have the following solution

$$\begin{aligned}
&y_{12} = -0.0000003158,\; y_{32} = 0.078656,\; y_{52} = 0.0212768,\\
&t_{32} = -0.000000943,\; \xi_1 = -0.0000282,\; \xi_4 = 3.69675,\; B = 1.00003\,. \qquad (23)
\end{aligned}$$

As well as solutions (17, 18), this solution is also stable in respect to $A \to 0$. It is easy to see, that parameters $\xi_{1,4}$ give values of a mixing angle s and a ratio of masses R according to the following set of equations

$$\begin{aligned}
&s\,=\,\sin\phi\,;\quad R\,=\,\frac{m_s}{m_d};\quad (\xi_1 - \xi_4)s\sqrt{1-s^2} + \xi_6(1-2s^2) = 0, \qquad (24)\\
&R = \frac{y + \sqrt{x^2+1}}{\sqrt{x^2+1} - y};\quad x = \frac{\xi_4 - \xi_1}{2},\; y = \frac{\xi_1 + \xi_4}{2}\,.
\end{aligned}$$

For data (23) we have the following two solutions

$$s_1\,=\,0.2454,\quad R_1\,=\,15.6; \qquad (25)$$
$$s_2\,=\,-\,0.9694,\quad R_2\,=\,15.6. \qquad (26)$$

Let us note, that $s_1^2 + s_2^2 = 1$ and $R_1 = R_2$ exactly.

Solution (25) may be compared with real situation of $(d,\,s)$ mixing, because mass ratio $R = m_s/m_d$ is close to its actual value and the mixing angle is also not far from actual Cabibbo angle value [11]

$$\sin\phi_c\,=\,s\,=\,0.2254 \pm 0.0006\,;\quad \frac{m_s}{m_d}\,=\,R\,=\,19.8^{+2.4}_{-2.8}\,. \qquad (27)$$

More details can be found in recent work [13].

The examples being just considered shows possibility of definition of mass ratios and of some mixing angles in the compensation approach.

3 Weinberg mixing angle and the fine structure constant

Let us demonstrate a simple model, which illustrates how the well-known Weinberg mixing angle could be defined. Let us consider a possibility of a spontaneous generation of the following effective interaction of electroweak gauge bosons

$$L_{eff}^{W} = G_1 W_\mu^a W_\mu^d W_{\rho\sigma}^a W_{\rho\sigma}^d + G_2 W_\mu^a W_\mu^a W_{\rho\sigma}^b W_{\rho\sigma}^b +$$
$$G_3\, W_\mu^a W_\mu^a B_{\rho\sigma} B_{\rho\sigma} + G_4\, Z_\mu Z_\mu W_{\rho\sigma}^b W_{\rho\sigma}^b + G_5\, Z_\mu Z_\mu B_{\rho\sigma} B_{\rho\sigma}. \quad (28)$$

where we maintain the residual gauge invariance for the electromagnetic field. Here indices a, d correspond to charged W-s, that is they take values 1, 2, while index b corresponds to three components of W defined by the initial formulation of the electro-weak interaction. Let us remind the relation, which connect fields W^0, B with physical fields of the Z boson and of the photon

$$W_\mu^0 = \cos\theta_W\, Z_\mu + \sin\theta_W\, A_\mu; \quad B_\mu = -\,\sin\theta_W\, Z_\mu + \cos\theta_W\, A_\mu. \quad (29)$$

Interactions of type (28) were earlier introduced on phenomenological grounds in works [14]. Let us introduce an effective cut-off Λ in the same way as we have done in the previous section and use for definition of Λ relation (4). Here we shall proceed just in the same way as earlier. Then let us consider a possibility of a spontaneous generation of interaction (28). In doing this we again proceed with the add-subtract procedure, which was used throughout works [1] - [5]. Now we start with usual form of the Lagrangian, which describes electro-weak gauge fields W^a and B

$$L = L_0\, +\, L_{int}\,; \quad L_0 = -\,\frac{1}{4}\big(W_{0\mu\nu}^a\, W_{0\mu\nu}^a\big) \,-\, \frac{1}{4}\big(B_{\mu\nu}\, B_{\mu\nu}\big); \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu; \quad (30)$$
$$L_{int} = -\frac{1}{4}\big(W_{\mu\nu}^a W_{\mu\nu}^a - W_{0\mu\nu}^a W_{0\mu\nu}^a\big); \quad W_{0\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a. \quad (31)$$

and $W_{\mu\nu}^a$ is defined in (1). Then we perform the add-subtract procedure of expression (28)

$$L = L_0'\, +\, L_{int}'\,; \quad L_0' = L_0\, -\, L_{eff}^W\,; \quad (32)$$
$$L_{int}'\; =\; L_{int}\, +\, L_{eff}^W\,. \quad (33)$$

Now let us formulate compensation equations. We are to demand, that considering the theory with Lagrangian L_0' (32), all contributions to four-boson connected vertices, corresponding to interaction (28) are summed up to zero. That is the undesirable interaction part in the would-be free Lagrangian (32) is compensated. Then we are rested with interaction (28) only in the proper place (33) We have the following set of compensation equations, which corresponds to diagrams being presented in the first six rows of Fig. 3

$$-\,x_1\, +\, x_1^2\; =\; 0\,;\; -\,x_2\, +\, 2\,x_2^2\, +\, 2\,x_1 x_2\, +\, (1-a^2)\,x_3 x_4\, +\, a^2\,x_2 x_4\; =\; 0\,;$$
$$-\,x_3\, +\, x_1 x_3\, +\, 2\,x_2 x_3\, +\, a^2\,x_2\,x_5\, +\, (1-a^2)\,x_3 x_5\; =\; 0\,; \quad (34)$$
$$x_1 x_4\, +\, 2\,x_2 x_4\, +\, a^2\,x_4 x_5\; = x_4\,;\; 2\,x_3 x_4\, +\, a^2\,x_4\,x_5\, +\, (1-a^2)\,x_5^2\; =\; x_5\,;$$
$$x_i\; =\; \frac{3\,G_i\,\Lambda^2}{64\,\pi^2}\,; \quad a\; =\; \cos\theta_W\,.$$

Factor 2 in several terms of equations here corresponds to sum by weak isotopic index $\delta_a^a = 2$, $a = 1, 2$.

Then, following the reasoning of the approach, we assume, that the Higgs scalar corresponds to a bound state consisting of a complete set of fundamental particles. Here we take into account the electro-weak bosons. There are two Bethe-Salpeter equations for this bound state, because constituents are either $W^a\,W^a$ or $Z\,Z$. These equations are presented in the last two rows of Fig. 3. In approximation of very large cut-off Λ these equations have the following form

$$x_1\,+\,(2\,+\,a)\,x_2\,+\,\frac{1-a^2}{a}\,x_3\,+\,\beta\,=\,1\,;\quad (2\,+\,a)\,x_4\,+\,\frac{1-a^2}{a}\,x_5\,+\,\frac{\beta}{a}\,=\,\frac{1}{a}\,. \tag{35}$$

Here we introduce parameter β, which describes wouldbe additional contributions. We consider as physical solutions those with very small β. Now we look for solutions of set (34, 35) for variables x_i, a, β. Of course, there is the trivial solution: all $x_i\,=\,0$, $\beta\,=\,1$. However there are also non-trivial solutions. Namely, there are the the following two ones with $x_1\,=\,1$

$$x_2\,=\,0\,;\;x_3\,=\,0.729625\,;\;x_4\,=\,0\,;\;x_5\,=\,0\,;\;\beta_1\,=\,1\,;\;\beta_2\,=\,\frac{0.729625\,(a-1)}{a}\,; \tag{36}$$

for any a, and the following three ones with $x_1\,=\,0$

$$\begin{array}{l} x_2=0,\;x_3=3.070337,\;x_4=0,\;x_5=3.61378\,,\;a=0.8504594,\;\beta\,=\,-\,5.06\cdot 10^{-16};\\ x_2=0.48772,\;x_3=0,\;x_4=1.2654,\;x_5=0,\;a=0.33801,\;\beta=-\,1.2\cdot 10^{-5};\\ x_2=0.5,\;x_3=1.09555,\;x_4=0,\;x_5=0,\;a=-\,0.75556,\;\beta=1. \end{array} \tag{37}$$

Very small β are appropriate for the first solution of (37) with $\beta\simeq-\,5\cdot 10^{-16}$ and for the second one with $\beta\simeq-\,1.2\cdot 10^{-5}$. Note, that for solutions (36) smallness of β is achieved only for the second one with $a\,\to\,1$, that is in an absence of the mixing. The solution with the smallest β gives for the mixing parameter

$$\sin^2\theta_W(z_0)\,=\,1-a^2\,=\,0.27672\,. \tag{38}$$

This value corresponds to scale Λ (4), which is defined by parameter z_0. At this scale we have according to (3, 38) the following relations

$$\alpha_{ew}(z_0)=\frac{g(z_0)^2}{4\,\pi}=0.028999;\quad \alpha(z_0)\,=\,\alpha_{ew}(z_0)\,\sin^2\theta_W(z_0)\,=\,0.0080244\,. \tag{39}$$

With the well-known evolution expression for electromagnetic coupling we have ($\Lambda\gg M_W$)

$$\alpha(z_0)\,=\,\frac{\alpha(M_Z)}{1-\frac{5\alpha(M_Z)}{6\,\pi}\,\ln\big[\frac{\Lambda^2}{M_Z^2}\big]}=0.0080244;\quad \alpha(M_Z)\,=\,0.00772\,. \tag{40}$$

in an almost indecent agreement with experimental value [11]

$$\alpha(M_Z)\,=\,0.0077562\pm 0.0000012. \tag{41}$$

Provided we take the value of boundary momentum Λ being an order of magnitude up and down of that defined by relations (3, 4), we have the following variation of results

$$\alpha(M_Z)_{up}\,=\,0.00765\,;\quad \alpha(M_Z)_{down}\,=\,0.00779\,. \tag{42}$$

The results being demonstrated can not be regarded as finally decisive ones and are rather indications of how things might occur. However in view of a fundamental importance of a possibility to define parameters of the Standard Model, we do present these considerations. We would also draw attention to an appearance of very small numbers in solutions being considered. *E.g.* solution (37) contains parameter $\beta\simeq-\,5\cdot 10^{-16}$. This might be useful in application to hierarchy problems [15, 16].

Let us try to proceed to the next approximation, that means inclusion to the analysis of up quarks also. This means consideration of the following effective interaction to be added to expressions (5, 20)

$$
\begin{aligned}
\Delta' L = \frac{8\pi^2}{\Lambda^2}\Big(& t_{21}(\bar u_L u_R \bar s'_R s'_L + \bar u_R u_L \bar s'_L s'_R) + t_{22}(\bar u_L u_R \bar c_R c_L + \bar u_R u_L \bar c_L c_R) + \\
& y_{22}\big(\bar u_L u_R(\bar s'_R d'_L + \bar d'_R s'_L) + \bar u_R u_L(\bar s'_L d'_R + \bar s'_R d'_L) + h.c.\big) + y_{11}\bar c_L c_R \bar c_R c_L + \\
& y_{21}\bar s'_L s'_R \bar s'_R s'_L + y_{42}\big(\bar c_L c_R(\bar s'_R d'_L + \bar d'_R s'_L) + \bar c_R c_L(\bar s'_L d'_R + \bar s'_R d'_L) + h.c.\big) + \\
& t_{31}(\bar s'_L s'_R \bar c_R c_L + \bar s'_R s'_L \bar c_L c_R)\Big)\,. \qquad (43)
\end{aligned}
$$

Bearing in mind the stability property of solutions (17, 18, 23) in respect to $A \to 0$, we put $A = 0$ (that simplifies the hunting for solutions), and using for additional interaction (43) the same rules as previously, we obtain the following set of equations

$$
\begin{aligned}
& -y_1 + 3(y_1^2 + y_3^2 + t_{21}^2 + t_{22}^2 + y_{22}^2) = 0;\; -y_2 + 3(y_2^2 + y_3^2 + t_{31}^2 + t_{32}^2 + y_{12}^2) = 0;\\
& -y_3 + 3(y_3(y_1 + y_2) + t_{21}t_{31} + t_{22}t_{32} + y_{22}y_{12});\\
& -y_{12} + 3(y_{12}y_2 + y_{22}y_3 + y_{32}t_{32} + y_{42}t_{31} + 2\,y_{52}y_{12}) = 0;\\
& -y_{22} + 3(y_{22}y_1 + y_{12}y_3 + y_{32}t_{22} + y_{42}t_{31} + 2\,y_{52}y_{22}) = 0;\\
& -y_{32} + 3(y_{12}t_{32} + y_{22}t_{22} + y_{32}y_{21} + y_{42}y_{31} + 2\,y_{52}y_{32}) = 0;\\
& -y_{42} + 3(y_{12}t_{31} + y_{22}t_{21} + y_{32}y_{31} + y_{42}y_{11} + 2\,y_{52}y_{42}) = 0;\\
& -y_{52} + 3(y_{12}^2 + y_{22}^2 + y_{32}^2 + y_{42}^2 + 2\,y_{52}^2) = 0; \qquad (44)\\
& -t_{21} + 3(y_1 t_{21} + y_3 t_{31} + t_{21}y_{11} + t_{22}y_{31} + y_{22}y_{42}) = 0;\\
& -t_{22} + 3(y_1 t_{22} + y_3 t_{32} + t_{21}y_{31} + t_{22}y_{21} + y_{22}y_{32}) = 0;\\
& -t_{31} + 3(y_3 t_{21} + y_2 t_{31} + t_{31}y_{11} + t_{32}y_{31} + y_{12}y_{42}) = 0;\\
& -t_{32} + 3(y_3 t_{22} + y_2 t_{32} + t_{31}y_{31} + t_{32}y_{21} + y_{12}y_{32}) = 0;\\
& 3(t_{21}^2 + t_{31}^2 + 2y_{42}^2 + y_{31}^2 + y_{11}^2) = y_{11};\; 3(y_{21}^2 + y_{31}^2 + 2y_{32}^2 + t_{22}^2 + t_{32}^2) = y_{21};\\
& -y_{31} + 3(y_{11}y_{31} + y_{21}y_{31} + 2y_{32}y_{42} + t_{21}t_{22} + t_{31}t_{32}) = 0;\\
& 1 = 3B(y_1 + y_3\xi_1 + 2y_{22}\xi_6 + t_{21}\xi_3 + t_{22}\xi_4);\\
& \xi_1 = 3B(y_3 + y2\xi_1 + 2y_{12}\xi_6 + t_{31}\xi_3 + t_{32}\xi_4);\\
& \xi_3 = 3B(t_{21} + t_{31}\xi_1 + y_{11}\xi_3 + y_{31}\xi_4 + 2y_{42}\xi_6);\\
& \xi_4 = 3B(t_{22} + t_{32}\xi_1 + y_{31}\xi_3 + y_{21}\xi_4 + 2y_{32}\xi_6);\\
& \xi_6 = 3B(y_{22} + y_{12}\xi_1 + y_{42}\xi_3 + y_{32}\xi_4 + 2\,y_{52}\xi_6).
\end{aligned}
$$

Here B, which has to be equal to unity, is the same as in (22). Additional mass parameters are defined in the following way by extending (21) to the following expression

$$
-m_u\big(\bar u u\,+\,\xi_1 \bar d' d'\,+\,\xi_3 \bar c c\,+\,\xi_4 \bar s' s'\,+\,\xi_6(\bar s' d'\,+\,\bar d' s')\big)\,; \qquad (45)
$$

There is a solution of set (44), which is close to previous one (25). Namely it looks like for A=0

$$
\begin{aligned}
& y_1 = 0.1773,\; y_2 = 0.1571,\; y_3 = 0.16583,\; y_{11} = 0.3329,\; y_{21} = 0.3327,\\
& y_{31} = 0.00052098,\; y_{12} = y_{22} = y_{32} = y_{42} = 0,\; y_{52} = 0.166667, \qquad (46)\\
& t_{21} = 0.0082035,\; t_{22} = -0.0099095,\; t_{31} = -0.0087183,\; t_{32} = 0.010531,\\
& \xi_1 = 1.190304,\; \xi_3 = 9.97278,\; \xi_4 = 12.42852,\; \xi_6 = 2.68897.
\end{aligned}
$$

Solution (46) gives the following results for parameters (24)

$$
s\,=\,0.221, \quad R\,=\,22.43. \qquad (47)
$$

We see, that this result agrees actual values (27) even better than result (25). That is we may state the improvement of results in the course of successive approximations.

As a matter of fact solution (46) gives the wouldbe c-quark mass only ten times more than that of the u-quark. However, one may expect strong influence on this relation of a mixing with the heavy t-quark. Thus the approximation, which we demonstrate here is applied just for consideration of the $d\,s$ mixing.

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