Towards creating a universe in the laboratory

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Best guess for pre-hot comsological epoch: inflation

Starobinsky' 1979, Guth' 1981,
Linde' 1982, Albrecht, Steinhardt' 1982

Homogeneous and isotropic Universe:
Friedmann–Lemaitre–Robertson–Walker (FLRW) metric

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 \ .$$

Inflation: nearly exponential expansion

$$a(t) = \mathrm{e}^{\int H dt}$$

$H = \dot{a}/a \approx \text{const}$ (Hubble parameter).

Exponential expansion: tiny region of space expands to huge size in, say, $\Delta t = 100 H^{-1}$.

Large, spatially flat, homogeneous Universe out of tiny region of space.

Can one in principle create a universe in the laboratory?

- Question raised in mid-80's, right after invention of inflationary theory

Berezin, Kuzmin, Tkachev' 1984;
Guth, Farhi' 1986

Idea: create, in a finite region of space, the conditions such that this region enters inflationary regime $\Longrightarrow$ this region will inflate to enormous size and in the end will look like our Universe.

- Do not need much energy: pour little more than Planckian energy into little more than Planckian volume. But
At that time: negaive answer!
[In the framework of classical General Relativity]

Guth, Farhi' 1986;
Berezin, Kuzmin, Tkachev' 1987

- Need inflationary conditions in a region larger than Hubble volume, $R > H^{-1}$

- Penrose theorem:

Penrose' 1965

There must be singularity in the past

Assumption of the theorem: Null Energy Condition, NEC

$$T_{\mu\nu} n^{\mu} n^{\nu} > 0$$

for any null vector n^{μ}, such that $n_{\mu} n^{\mu} = 0$.

$T_{\mu\nu}$ = energy-momentum tensor

Trapped surface:

a closed surface on which outward-pointing light rays actually converge (move inwards)

Spherically symmetric examples:

$$ds^2 = g_{00}dt^2 + 2g_{0R}dt\,dR + g_{RR}dR^2 - R^2 d\Omega^2$$

$4\pi R^2$: area of a sphere of constant t, R.

Trapped surface: R decreases along all light rays.

- Sphere inside horizon of Schwarzschild black hole
- Hubble sphere in contracting Universe $\Longrightarrow$

  Hubble sphere in expanding Universe = anti-trapped surface $\Longrightarrow$ singularity in the past.

Meaning:

A. Vikman's talk

- Homogeneous and isotropic region of space: metric

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 \; .$$

Local Hubble parameter $H = \dot{a}/a$.
Wish to create region whose size is larger than H^{-1}

This is the definition of a universe

Hubble size regions evolve independently of each other

$\Longrightarrow$ Legitimate to use eqs. for FLRW universe

- A combination of Einstein equations:

$$\frac{dH}{dt} = -4\pi G(\rho + p)$$

$\rho = T_{00}$ = energy density
$p = T_{11} = T_{22} = T_{33}$ = effective pressure.

- Null Energy Condition:
$T_{\mu\nu} n^\mu n^\nu \geq 0$, $n^\mu = (1, 1, 0, 0) \Longrightarrow \rho + p > 0 \Longrightarrow dH/dt < 0$,

Hubble parameter was greater early on.

At some moment in the past, there was a singularity, $H = \infty$.

Side remark

- Null Energy Condition, $\rho + p > 0 \Longrightarrow dH/dt < 0 \Longrightarrow$
  impossibility of a bounce in cosmology,
  transition from collapse ($H < 0$)
  to expansion ($H > 0$)

- Another side of the NEC
  Covariant energy-momentum conservation:

$$\frac{d\rho}{dt} = -3H(\rho + p)$$

  NEC: energy density decreases during expansion,
  except for $p = -\rho$, cosmological constant.

Ways out so far:

- Give up General Relativity $\Longleftrightarrow$ possible, but at strong gravitational fields only

Mukhanov, Brandenberger' 1992

- Give up *classical* field theory

Frolov, Markov, Mukhanov' 1987

$\Longleftrightarrow$ creation of a universe as tunneling event, possible but rare and not under full control.

Berezin, Kuzmin, Tkachev' 1988
Farhi, Guth, Guven' 1990
Fischler, Morgan, Polchinski' 1990

Can Null Energy Condition be violated?

A. Vikman's talk

- Folklore until recently: NO!

  Pathologies:

  - Ghosts:

$$E = -\sqrt{p^2 + m^2}$$

  Example: theory with wrong sign of kinetic term,

$$\mathscr{L} = -(\partial\phi)^2 \quad \Longrightarrow \quad \rho = -\dot{\phi}^2 - (\nabla\phi)^2 \,, \qquad p = -\dot{\phi}^2 + (\nabla\phi)^2$$

$$\rho + p = -2\dot{\phi}^2 < 0$$

  Catastrophic vacuum instability

Other pathologies

- Gradient instabilities:

$$E^2 = -(p^2 + m^2) \implies \varphi \propto \mathrm{e}^{|E|t}$$

- Superluminal propagation of excitations

No-go theorem for theories with Lagrangians involving first derivatives of fields only

Dubovsky, Gregoire, Nicolis, Rattazzi' 2006

NEC violation today: YES,

Null Energy Condition can be violated without obvious pathologies

Senatore' 2004;
V.R.' 2006;
Creminelli, Luty, Nicolis, Senatore' 2006

- General properties of non-pathological NEC-violating field theories:

  - Non-standard kinetic terms
  - Non-trivial background

    - Non-standard kinetic terms: second derivative Lagrangians yielding second derivative field equations

Horndeski' 1974

Fairlie, Govaerts, Morozov' 1992

Luty, Porrati, Rattazzi' 2004

Nicolis, Rattazzi, Trincherini' 2009

Single scalar field π; $X = \partial_\mu \pi \partial^\mu \pi$

$$L_n = K_n(X, \pi) \partial^{\mu_1} \partial_{[\mu_1} \pi \cdot \cdots \cdot \partial^{\mu_n} \partial_{\mu_n]} \pi$$

Five Lagrangians in 4D, including K_0

Sivanesan's talk

Generalization to GR: L_0, L_1 trivial, $L_{n>1}$ non-trivial

Deffayet, Esposito-Farese, Vikman' 09

Playground:

$$L = F(Y) \cdot \mathrm{e}^{4\pi} + K(Y) \cdot \Box\pi \cdot \mathrm{e}^{2\pi}$$

$$\Box\pi \equiv \partial_\mu \partial^\mu \pi \,, \qquad Y = \mathrm{e}^{-2\pi} \cdot (\partial_\mu \pi)^2$$

Deffayet, Pujolas, Sawicki, Vikman' 2010

Kobayashi, Yamaguchi, Yokoyama' 2010

- Second order equations of motion
- Scale invariance: $\pi(x) \to \pi'(x) = \pi(\lambda x) + \ln\lambda$.

(technically convenient)

Homogeneous solution in Minkowski space (attractor)

$$e^{\pi_c} = \frac{1}{\sqrt{Y_*}(t_* - t)}$$

- $Y \equiv e^{-2\pi_c} \cdot (\partial_\mu \pi_c)^2 = Y_* = \text{const}$, a solution to

$$Z(Y_*) \equiv -F + 2Y_* F' - 2Y_* K + 2Y_*^2 K' = 0$$

$$' = d/dY \; .$$

Energy density

$$\rho = e^{4\pi_c} Z = 0$$

Effective pressure T_{11}:

$$p = e^{4\pi_c} (F - 2Y_* K)$$

Can be made negative by suitable choice of $F(Y)$ and $K(Y)$
$\Longrightarrow$ $\rho + p < 0$, violation of Null Energy Condition.

Switching on gravity

$$p = \mathrm{e}^{4\pi_c}(F - 2Y_* K) = -\frac{M^4}{Y_*^2 (t_* - t)^4}\,, \qquad \rho = 0$$

M: mass scale characteristic of π

- Use $\dot{H} = -4\pi G(p+\rho) \Longrightarrow$

$$H = \frac{4\pi}{3}\frac{M^4}{M_{Pl}^2 Y_*^2 (t_* - t)^3}$$

NB:

$$\rho \sim M_{Pl}^2 H^2 \sim \frac{1}{M_{Pl}^2 (t_* - t)^6}$$

Early times $\Longrightarrow$ weak gravity, $\rho \ll p$, expansion irrelevant for dynamics of π

Perturbations about homogeneous solution Minkowski

$$\pi(x^\mu) = \pi_c(t) + \delta\pi(x^\mu)$$

- Quadratic Lagrangian for perturbations:

$$L^{(2)} = \mathrm{e}^{2\pi_c} Z' (\partial_t \delta\pi)^2 - V(\vec{\nabla}\delta\pi)^2 + W(\delta\pi)^2$$

$V = V[Y; F, K, F', K', K'']$. Absence of ghosts:

$$Z' \equiv dZ/dY > 0$$

Absence of gradient instabilities and of superluminal propagation

$$V > 0\,; \qquad V < \mathrm{e}^{2\pi_c} Z'$$

Can be arranged.

NB: Useful for constructing alternatives to inflation, Vikman's talk

Creating a universe: first attempt

- Prepare quasi-homogeneous initial configuration.

  Large sphere $Y = Y_*$ inside, $\pi =$ const (Minkowski) outside,

  smooth interpolation in between.

  Spatial derivatives small compared with time derivatives.

- Initial state: energy density and pressure small everywhere, geometry nearly Minkowskian. No antitrapped surface. Possible to create.

- Evolution: Genesis inside the sphere, Minkowski outside

Done?

Not quite!

Obstruction

- Energy density:

$$\rho = \mathrm{e}^{4\pi_c} Z$$

$Z = 0$ both outside the sphere and inside the sphere $\Longrightarrow$ dZ/dY is negative somewhere in between.

- On the other hand: absence of ghosts requires

$$dZ/dY > 0$$

Hence, there are ghosts somewhere in space $\equiv$ instability

- This is a general property of theories of one scalar field with
  - Second order field equations
  - Scale invariance: $\pi(x) \to \pi'(x) = \pi(\lambda x) + \ln \lambda$.

Proof

- Equation for homogeneous field always coincides with energy conservation (Noether theorem)

$$\frac{\delta S}{\delta \pi} \propto -\dot{\rho} = 0$$

This is second order equation, hence ρ contains first derivatives only, hence by scale invariance

$$\rho = \mathrm{e}^{4\pi} \cdot Z[\mathrm{e}^{-2\pi}(\partial\pi)^2]$$

- Write $\pi = \pi_c + \delta\pi$, then eqn. for $\delta\pi$ is

$$-Z' \; \partial_t^2 \delta\pi + \text{lower} \;\; \text{time derivatives} = 0$$

Hence

$$\mathscr{L}(\delta\pi) \propto Z'(\partial_t \delta\pi)^2 + \dots$$

QED

Possible ways out

- Give up Genesis inside the sphere, take $\rho \neq 0$ there.

  Hardly works. $Z = 0$ (Minkowski) is attractor.

- Give up scale invariance.

  A lot more technically demanding.

- Take initial data such that gravity is important

  Even more technically demanding.

- Give up single field, make model more complicated.

  But keep dynamics simple.

Second – and successful (?) attempt

- Make the Lagrangian for π explicitly dependent on radial coordinate r.

  To this end, introduce a new field whose background configuration is $\varphi(r)$

- Example:

$$F = a(\varphi) + b(\varphi)(Y - \varphi) + \frac{c(\varphi)}{2}(Y - \varphi)^2$$

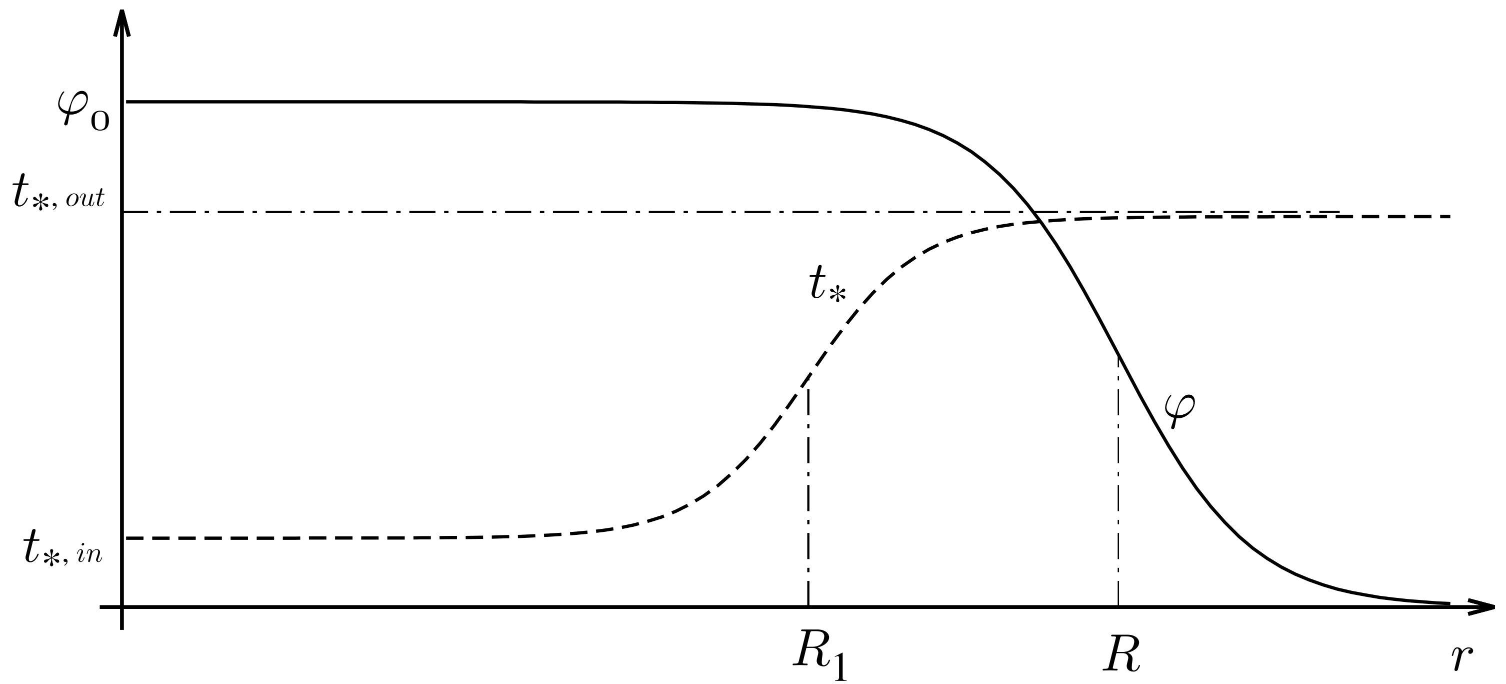
$$K = \kappa(\varphi) + \beta(\varphi)(Y - \varphi) + \frac{\gamma(\varphi)}{2}(Y - \varphi)^2$$

  Choose functions $a(\varphi)$, ... in such a way that quasi-homogeneous solution is

$$\mathrm{e}^{\pi} = \frac{1}{\sqrt{\varphi_0} t_*(r) - \sqrt{\varphi(r)} t}$$

  Make sure that there are no pathologies about this solution.

- Interior: $Y = \varphi_0 \Longrightarrow$ Genesis $\quad t_{*,in}$ small $\Longrightarrow$ quick start
  Exterior $\dot{\pi} = 0 \Longrightarrow Y = 0 \Longrightarrow$ Minkowski



Initial conditions, $t = 0$: at $r < R$ pressure

$$p_{in} = \frac{M^4}{Y_0^2 t_{*,in}^4}$$

Require $p_{in} R^3 / M_{Pl}^2 \ll R \Longrightarrow$ weak gravity, gravitational potentals small everywhere.

Together with $t_{*,in} \ll R$ this guarantees

$$H_{in} = \frac{4\pi M^4}{3 M_{Pl}^2 Y_0^2 t_{*,in}^3} \ll R^{-1}$$

No antitrapped surfaces initially. Anti-trapped surface (Hubble size) gets formed when

$$(t_{*,in} - t_1) \sim \left(\frac{M^4 R}{M_{Pl}^2 Y_0^2} \right)^{1/3}$$

Gravity is still weak at that time. No black hole (yet?).

Creation of a universe in controlled, weak gravity regime

Why question mark?

- What do spatial gradients do?
- Where does the system evolve once gravity is turned on?

  What is the global geometry?

  Does a black hole get formed?
- Explicit (numerical) solution needed

To conclude

- There exist field theory models with healthy violation of the Null Energy Condition
- This removes obstruction for creating a universe in the laboratory
- A concrete scenario is fairly straightforward to design.
- Are there appropriate fields in Nature?

  Hardly. Still, we may learn at some point that our Universe went through Genesis or bounce phase. This will mean that Null Energy Condition was violated in the past by some exotic fields. In that case one may try to use the these fields for creating a universe in the laboratory.

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