Post-Newtonian limits for brane-world model

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QUARKS-2014

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07.06.2014

Parameterized post-Newtonian formalism

PPN purpose – compare metric theories of gravity.

Post-Newtonian limit:

- weak field
- asymptotically flat
- perfect fluid

Experimental values of PPN-parameters 1

PPN-parameter	Physical meaning	Current value	Effects
$\gamma - 1$	measure of space curvature produced by unit mass	2.3×10^{-5}	time delay, light deflection
$\beta - 1$	measure of non-linearity in gravitational superposition	1.1×10^{-4}	Nordtvedt effect, perihelion shift
ξ	measure of existence of preferred location effects	1×10^{-3}	Earth tides

Experimental values of PPN-parameters 2

PPN-parameter	Physical meaning	Current value	Effects
α_1	measure the existence of preferred frame effects	1×10^{-4}	orbit polarization
α_2		4×10^{-7}	spin precession
α_3		4×10^{-20}	self-acceleration
ς_1	measure of the failure of conservation laws of energy, momentum and angular momentum	2×10^{-2}	—
ς_2		4×10^{-5}	binary pulsar acceleration
ς_3		1×10^{-8}	Newton's 3rd law
ς_4		6×10^{-3}	—

Post-Newtonian limit

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$\eta_{\mu\nu}$ – Minkowski space,

1PN:

$$h_{00}(1/r) \sim O(1) + O(2)$$
$$h_{0j}(1/r) \sim O(3)$$
$$h_{ij}(1/r) \sim O(1)$$

2PN:

$$h_{00}(1/r) \sim O(3)$$
$$h_{0j}(1/r) \sim O(5)$$
$$h_{ij}(1/r) \sim O(2)$$

1981 Will "Theory and experiment in gravitational physics"

Parameterized post-Newtonian formalism. Field equations

$$R_{\mu\nu} = 8\pi \left(T_{\mu\nu} - \frac{g_{\mu\nu}}{2} T \right)$$

$$R_{00} = -\frac{1}{2} \nabla^2 h_{00} - \frac{1}{2} \left(h_{jj,00} - 2h_{j0,j0} \right) +$$

$$+ \frac{1}{2} h_{00,j} \left(h_{jk,k} - \frac{1}{2} h_{kk,j} \right) - \frac{1}{4} |\nabla h_{00}|^2 + \frac{1}{2} h_{jk} h_{00,jk},$$

$$R_{0j} = -\frac{1}{2} \left(\nabla^2 h_{0j} - h_{k0,jk} + h_{kk,0j} - h_{kj,0k} \right),$$

$$R_{ij} = -\frac{1}{2} \left(\nabla^2 h_{ij} - h_{00,ij} + h_{kk,ij} - h_{ki,kj} - h_{kj,k,i} \right).$$

1981 Will "Theory and experiment in gravitational physics"

Parameterized post-Newtonian formalism. Stress-energy tensor

$$T^{00} = \rho \ \left(1 + \Pi + v^2 + 2U\right)$$

$$T^{0i} = \rho \ \left(1 + \Pi + v^2 + 2U + \frac{p}{\rho}\right) \ v^i$$

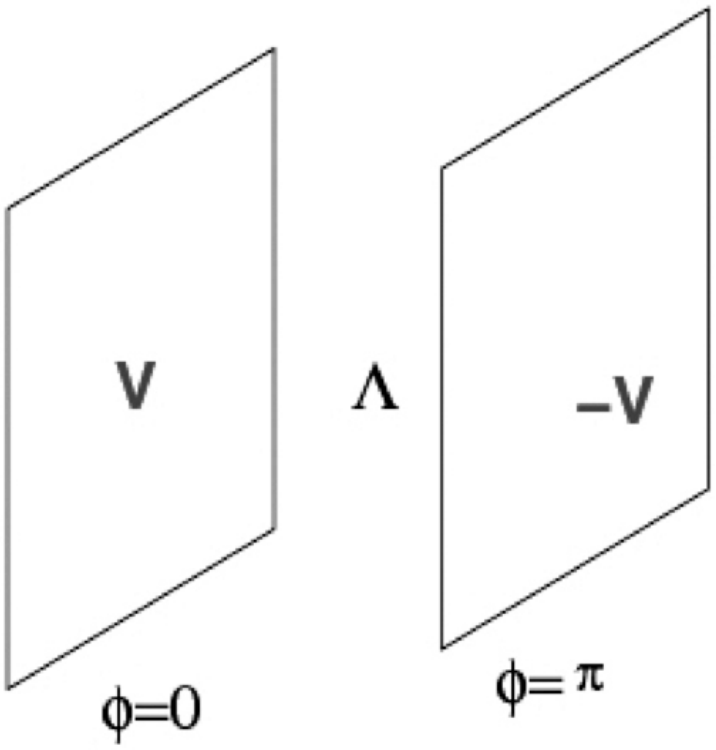
$$T^{ij} = \rho \ \left(1 + \Pi + v^2 + 2U + \frac{p}{\rho}\right) \ v^i v^j + p\delta^{ij}(1 - 2\gamma U)$$

$$U \sim v^2 \sim p/\rho \sim \Pi \sim 1/r$$

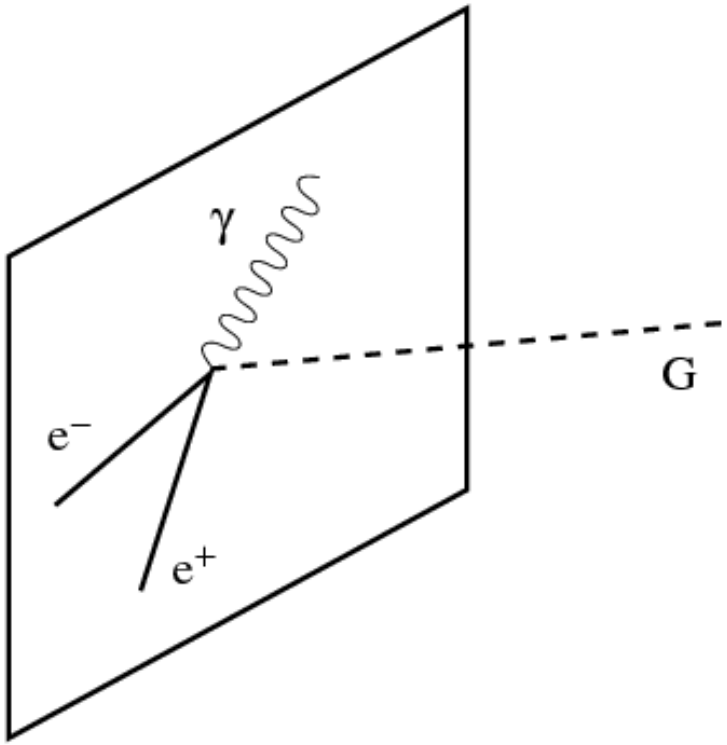
1981 Will "Theory and experiment in gravitational physics"

Randall-Sundrum gravity

Randall-Sundrum I



Randall-Sundrum II



1999 Randall, Sundrum, *PRL* **83** 3370, *PRL* **83** 4690

Black holes in Randall-Sundrum

First BH solutions

2000 Hawking, Chamblin, Reall, PRD **61** 065007,
Dadhich, Maartens, Papadopoulos, Rezania, PLB **487** 1

No BH with $r_h > l$ AdS-length in RS II

2002 Tanaka, PTPS **148** 307, 2003 Fabbri, Emparan, Kaloper, JHEP **08** 043,
Garcia-Bellido, Emparan, Kaloper, JHEP **01** 079

BH with $r_h \sim 0.2l$ in 5D RS II

2003 Wiseman, PRD **65** 124007, CQG **20** 1137

BH with $r_h \sim 2.0l$ in 6D RS II

2003 Tanaka, Kudoh, Nakamura, PRD **68** 024035, 2003 Kudoh, PTP **110** 1059,
2004 Kudoh, PRD **69** 104019

No BH in RS II

2009 Yoshino, JHEP **01** 068, 2011 Kleihaus, Kunz, Senkbeil, PRD **83** 104050

RS-II. Figueras-Wiseman solution
Field equations

$$
\begin{aligned}
G_{\mu\nu} = \; & 8\pi G_4 T^{brane}_{\mu\nu} + \\
+ \; & \epsilon^2 \left\{ 16\pi G_4 \langle T^{CFT}_{\mu\nu}[g] \rangle + a_{\mu\nu}[g] + \log\epsilon \; b_{\mu\nu}[g] \right\} + \\
+ \; & O(\epsilon^4 \log\epsilon),
\end{aligned}
$$

$$\epsilon \to 0$$

$$\langle T^{CFT}_{\mu\nu}[g] \rangle, \; a_{\mu\nu}[g], \; b_{\mu\nu}[g] = f(g_{\mu\nu}, R, R_{\mu\nu}, R_{\mu\nu\sigma\tau})$$

2011 Figueras, Wiseman, *Phys. Rev. Lett.* **107** 081101

RS-II. Figueras-Wiseman solution PPN limits

$$\mathbf{h_{00}^{FW} = \frac{121}{27}\,\frac{\epsilon^2}{l^2}\,\frac{M^4}{r^2}}$$

$$\downarrow$$

$$\beta = \mathbf{1} - \frac{\mathbf{121}}{\mathbf{108}}\,\frac{\epsilon^2}{l^2}\,M^2$$

RS-II. Abdolrahimi-Page solution

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left[r^2 + \frac{F}{-\Lambda_5}\right] d\Omega^2 + \\ + \left[1 - \frac{1}{-\Lambda_5 r^2} \frac{r - 2M}{r - 1.5M} \left(F - r\,\frac{dF}{dr}\right)\right] \left(1 - \frac{2M}{r}\right)^{-1} dr^2,$$

$$F = 1 - 1.1241 \left(\frac{2M}{r}\right) + 1.956 \left(\frac{2M}{r}\right)^2 + \cdots + 2.900 \left(\frac{2M}{r}\right)^{11}$$

2013 Abdolrahimi, Cattoen, Page, Yaghoobpour-Tari, *JCAP* **06** 39

RS-II. Field equations
Generalized form

$$G_{\mu\nu} = -\,\Lambda_4\, g_{\mu\nu} + 8\pi G_N T_{\mu\nu} + \kappa_5^4\, \pi_{\mu\nu} - E_{\mu\nu},$$

$$\pi_{\mu\nu} = -\,\frac{1}{4}\, T_{\mu\alpha} T^{\alpha}_{\nu} + \frac{1}{12}\, T T_{\mu\nu} + \frac{1}{8}\, g_{\mu\nu} T_{\alpha\beta} T^{\alpha\beta} - \frac{1}{24}\, g_{\mu\nu} T^2,$$

$$\Lambda_4 = \frac{1}{2}\, \kappa_5^2 \left(\Lambda_5 + \frac{1}{6}\, \kappa_5^2 \lambda^2 \right),$$

$$G_N = \frac{\kappa_5^4 \lambda}{48\pi}, \qquad \kappa_5^2 = 8\pi G_5,$$

$$E_{\mu\nu} = \tilde{C}_{\mu\alpha\nu\beta} n^{\alpha} n^{\beta}$$

2000 Sasaki, Shiromizu, Maeda, *Phys. Rev.* **D62** 024008

RS-II. Abdolrahimi-Page solution PPN limits

$$g_{\mu\nu} = g^{Sch}_{\mu\nu} \quad \text{till} \quad \frac{1}{r^2},$$

$$E_{\mu\nu} = 0 \text{ for } \mathrm{AdS}_5$$

2000 Maartens, *Phys. Rev.* **D62** 084023

$$\pi_{\mu\nu} \sim T^2_{\mu\nu} \sim \frac{1}{r^6} \rightarrow h^{AP}_{00} \sim \frac{1}{r^4}$$

$$\downarrow$$

$$\beta = \gamma = 1 \rightarrow \varepsilon(\beta, \gamma) = \zeta(\beta, \gamma) = 0$$

Conclusions

- Possibility of negative non-linearity in gravitational superposition for Figueras-Wiseman solution in RS II is shown.
- The Randall-Sundrum II model seems to be well consistent with GR at the Solar System scale. For obtaining the limits on it's parameters other tests are needed.

Thank you
for your attention

Parameterized post-Newtonian formalism. Metrics

$$
\begin{aligned}
g_{00} = &- \ 1 + 2U - 2\beta U^2 - 2\xi\Phi_W + (2\gamma + 2 + \alpha_3 + \varsigma_1 - 2\xi)\ \Phi_1 + \\
&+ \ 2\ (3\gamma - 2\beta + 1 + \varsigma_2 + \xi)\ \Phi_2 + 2\ (1 + \varsigma_3)\ \Phi_3 + \\
&+ 2\ (3\gamma + 3\varsigma_4 - 2\xi)\ \Phi_4 - (\varsigma_1 - 2\xi)\ \mathcal{A} - (\alpha_1 - \alpha_2 - \alpha_3)\ w^2 U + \\
&+ \ \alpha_2 w^i w^j U_{ij} + (2\alpha_3 - \alpha_1)\ w^i V_i
\end{aligned}
$$

$$
\begin{aligned}
g_{0i} = &- \ \frac{1}{2}\,(4\gamma + 3 + \alpha_1 - \alpha_2 + \varsigma_1 - 2\xi)\ V_i - \\
&- \ \frac{1}{2}\,(1 + \alpha_2 - \varsigma_1 + 2\xi)\ W_i - \frac{1}{2}\,(\alpha_1 - 2\alpha_2)\ w^i U - \alpha_2 w^i U_{ij}
\end{aligned}
$$

$$
g_{ij} = (1 + 2\gamma U)\ \delta_{ij}
$$

1981 Will "Theory and experiment in gravitational physics"

RS-II. Figueras-Wiseman solution
Field equations

$$
\begin{aligned}
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+ \; & \epsilon^2 \left\{ 16\pi G_4 \langle T^{CFT}_{\mu\nu}[g] \rangle + a_{\mu\nu}[g] + \log\epsilon \; b_{\mu\nu}[g] \right\} + \\
+ \; & O(\epsilon^4 \log\epsilon),
\end{aligned}
$$

$$
\begin{aligned}
a_{\mu\nu}[g] = & -\frac{1}{4}\nabla^2 R_{\mu\nu} + \frac{1}{12}\nabla_\mu\nabla_\nu R + \frac{1}{24}\nabla^2 R g_{\mu\nu} + \frac{1}{6} R R_{\mu\nu} + \\
& + \frac{1}{8} R_{\alpha\beta}R^{\alpha\beta} g_{\mu\nu} - \frac{1}{24} R^2_{\mu\nu} - \frac{1}{2} R_{\mu\alpha\nu\beta}R^{\alpha\beta},
\end{aligned}
$$

$$
\begin{aligned}
b_{\mu\nu}[g] = & -\frac{1}{2}\nabla^2 R_{\mu\nu} + \frac{1}{6}\nabla_\mu\nabla_\nu R + \frac{1}{12}\nabla^2 R g_{\mu\nu} + \frac{1}{3} R R_{\mu\nu} + \\
& + \frac{1}{4} R_{\alpha\beta}R^{\alpha\beta} g_{\mu\nu} - \frac{1}{12} R^2_{\mu\nu} - R_{\mu\alpha\nu\beta}R^{\alpha\beta}
\end{aligned}
$$

2011 Figueras, Wiseman, *PRL* **107** 081101

RS-II. Figueras-Wiseman solution Fefferman-Graham expansion

$$\langle T^{CFT}_{\mu\nu}\rangle = \frac{1}{4\pi G_5}\left\{-\frac{1}{4}\,(g^{(2)})^2_{\mu\nu} + \frac{1}{4}\,g^{(2)}_{\mu\nu}\,\mathrm{Tr}\,g^{(2)} - \right.$$

$$\left. - \frac{1}{8}\,g^{(0)}_{\mu\nu}\left[(\mathrm{Tr}\,g^{(2)})^2 - \mathrm{Tr}(g^{(2)})^2\right]\right\},$$

$$g^{(0)}_{\mu\nu} = g_{\mu\nu} + \frac{\epsilon^2}{2}\left(R_{\mu\nu} - \frac{1}{6}\,g_{\mu\nu}R\right) + O(\epsilon^4 log\epsilon),$$

$$g^{(2)}_{\mu\nu} = \frac{1}{d-2}\left(R_{\mu\nu} - \frac{1}{2\,(d-1)}\,R\,g^{(0)}_{\mu\nu}\right)$$

2001 Haro, Skenderis, Solodukhin, *CMP* **217** 595