Multisoliton solutions in CGHS model with a boundary

M. Fitkevich, D. Levkov, S. Sibiryakov, Y. Zenkevich

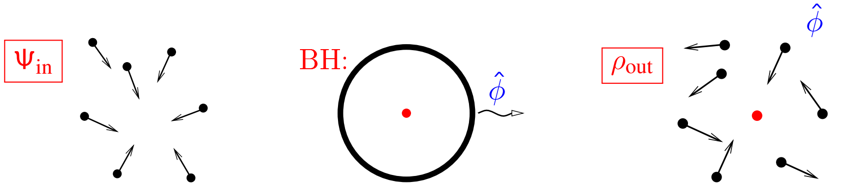
Institute for Nuclear Research RAS

Quarks-2014

Suzdal, June 7, 2014

Motivation: Information paradox

Information paradox: Hawking '75



Heuristic explanations: complementarity, remnants, Page & scrambling times, etc.

We need solvable models!

Boundary CGHS

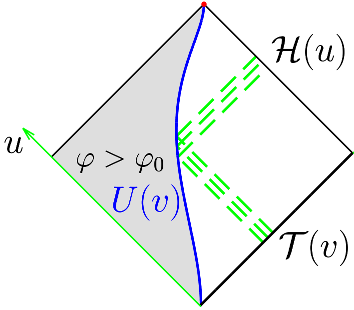
Callan, Giddings, Harvey, Strominger'92

dilaton gravity — conformal matter

$$\mathcal{S} = \int_{\varphi>0} d^2x\sqrt{|g|}e^{-2\varphi}\left(R + 4(\nabla\varphi)^2 + 4\lambda^2\right) - \frac{1}{2}\int_{\varphi>0} d^2x\sqrt{|g|}\,(\nabla f)^2$$

$$+2\int_{\varphi=\varphi_0} d\sigma\sqrt{|h|}e^{-2\varphi}\,(K+2\lambda)$$

Gibbons-Hawking term



In the bulk: $g_{\mu\nu} = e^{2\varphi}\eta_{\mu\nu}$ (on shell)

$$\boxed{f = f^{in}(v) + f^{out}(u)}$$

$$\mathcal{T}(v) = -(\partial_v f^{in})^2/2$$

$$\mathcal{H}(u) = -(\partial_u f^{out})^2/2$$

Boundary condition: $\boxed{\partial_v^2\psi = \mathcal{T}\psi}$

$$\boxed{\psi(v) = e^{\mu\tau(v)}}$$

$$\mu \equiv \lambda e^{\varphi_0}$$

$$U(v) = -\partial_v \ln\psi/\mu^2 + \int^v \mathcal{T}, \qquad \mathcal{H} = \mathcal{T}\left(\frac{dU}{dv}\right)^{-2}$$

Solitons

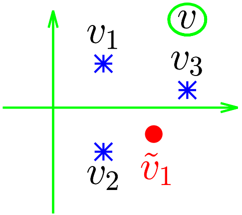
$$\partial_u^2 \psi = \mathcal{T}\psi$$

General solution: $\mathcal{T} = \partial_v^2 \psi / \psi$

Not so trivial!

$\rightarrow \mathcal{T}(v) \neq \infty$ at $\psi = 0$

$\rightarrow \mathcal{T}(v) \leq 0$ — positivity condition



Kovalevskaya idea:

$\mathcal{T}(v) \longleftrightarrow$ simple analytic structure of general solution $\psi(v)$

$$\psi(v) = \prod_i \overset{\text{poles}}{(v - v_i)^{-s_i}} \cdot \overset{\text{polynomial}}{P(v)}$$

Soliton solutions

Equations for coefficients

$$\psi(v) = N\prod_i (v - v_i)^{-s_i} \cdot \overbrace{\prod_m (v - \tilde{v}_m)}^{P(v)}$$

- $\mathcal{T} = \partial_v^2 \psi/\psi$ — Not singular at $v = \tilde{v}_m$

$$\Rightarrow \boxed{\sum_i \frac{s_i}{\tilde{v}_m - v_i} = \sum_{m' \neq m} \frac{1}{\tilde{v}_m - \tilde{v}_{m'}}} \Rightarrow \text{find } \tilde{v}_m$$

$$\Rightarrow \mathcal{T}(v) = \sum_i \left[\frac{s_i(s_i+1)}{(v-v_i)^2} + \frac{\mathcal{T}_i}{v - v_i} \right], \quad \mathcal{T}_i = \mathcal{T}_i(\tilde{v})$$

- ψ is a general solution $\Rightarrow$ $\boxed{s_i \in \mathbb{N}/2}$

Soliton properties

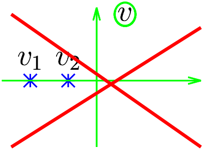
Parameters: v_i, s_i

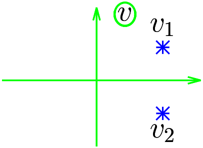
Positivity condition $\mathcal{T}(v) \leq 0$ should be imposed!

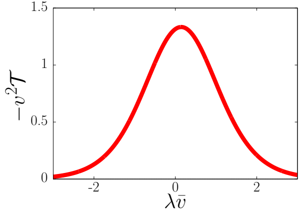
2–pole solution with $s_i = 1$

$$\left.\begin{aligned} \mathcal{T}(v) &= \sum_i \left[\frac{2}{(v-v_i)^2} + \frac{\mathcal{T}_i}{v-v_i}\right] \\ \psi(v) &= \sum_i \frac{\psi_i}{v-v_i} + Cv + 1 \end{aligned}\right\} \qquad \mathcal{T}_{1,2}, \psi_{1,2}, C \;-\; ?$$

- Eqs. for coefficients $\Rightarrow$ $\mathcal{T}_1 = -\mathcal{T}_2 = \dfrac{4}{v_2 - v_1}$
- Positivity condition: $\mathcal{T}(v) \leq 0$ at $v \geq 0$





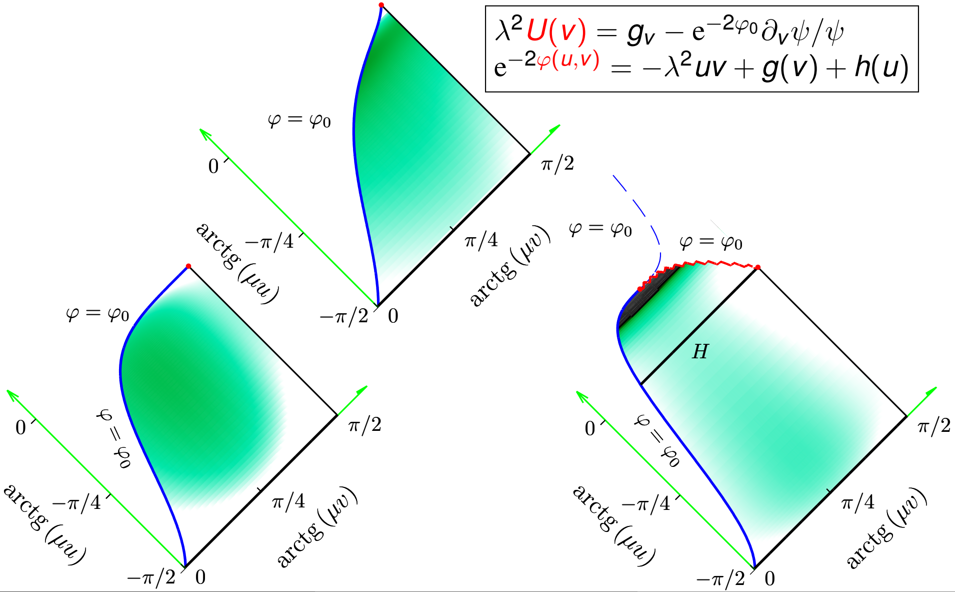


- Solution for ψ $\Rightarrow$

$$\psi_1 = \tfrac{1}{3}(v_1 - v_2)\left(\tfrac{1}{2} + C(2v_1 - v_2)\right)$$
$$\psi_2 = \tfrac{1}{3}(v_2 - v_1)\left(\tfrac{1}{2} + C(2v_2 - v_1)\right)$$
$$C = \frac{v_1^3 - v_2^3}{2(v_1+v_2)(v_1-v_2)^3}$$

Penrose diagrams for 2—pole solution with $s_i = 1$

$$\lambda^2 U(v) = g_v - e^{-2\varphi_0} \partial_v \psi / \psi$$
$$e^{-2\varphi(u,v)} = -\lambda^2 uv + g(v) + h(u)$$



Conformal symmetry

$$\partial_v^2 \psi = \mathcal{T}(v)\psi$$

$$v \to w(v) \left\{ \begin{array}{ll} \psi \to \psi'(w) = \left(\frac{dw}{dv}\right)^{1/2} \psi(v)\,, & h = -\frac{1}{2} \\ \mathcal{T} \to \mathcal{T}'(w) = \left(\frac{dw}{dv}\right)^{-2} \mathcal{T}(v) - \frac{1}{2}\{v;\, w\}\,, & h = 2 \end{array} \right.$$

$\frac{1}{2}$ ≈ $c/12$; $\{v;\, w\}$ ← Schwartz derivative

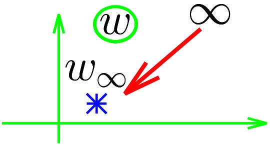
Transformations of a CFT with $c \neq 0$?!

Soliton ↔ soliton ⇒ keep univaluedness

$$\Rightarrow \boxed{z = \alpha \frac{w - w_0}{w - w_\infty}} \in SL(2, \mathbb{C})$$ — global conformal transformations

Asymptotics at $v \to \infty$

$$v = \alpha \frac{w - w_0}{w - w_\infty}$$



$$\mathcal{T}'(w) = N \cdot \frac{T(v)}{(w - w_\infty)^4} \to \frac{\text{const}}{w^4} \quad \text{at} \quad w \to \infty$$

$$\Downarrow$$

Asymptotics for solitons:

$$\mathcal{T}(v) = \sum_i \left[\frac{s_i(s_i+1)}{(v - v_i)^2} + \frac{\mathcal{T}_i}{v - v_i} \right] \to \frac{\text{const}}{v^4}$$

$$\Downarrow$$

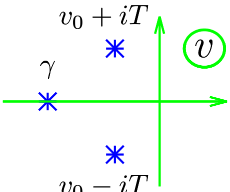
v^{-1}, v^{-2}, v^{-3}:

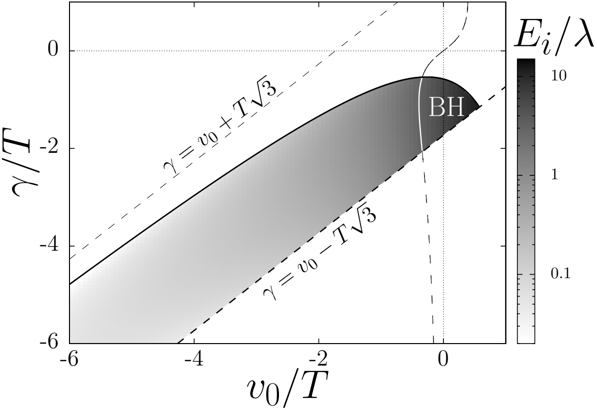
$$\sum \mathcal{T}_i = \sum [s_i(s_i+1) + \mathcal{T}_i v_i] = \sum \left[2s_i(s_i+1)v_i + \mathcal{T}_i v_i^2 \right] = 0$$

How many soluitions with N poles?

3–pole solution with $s_i = 1$

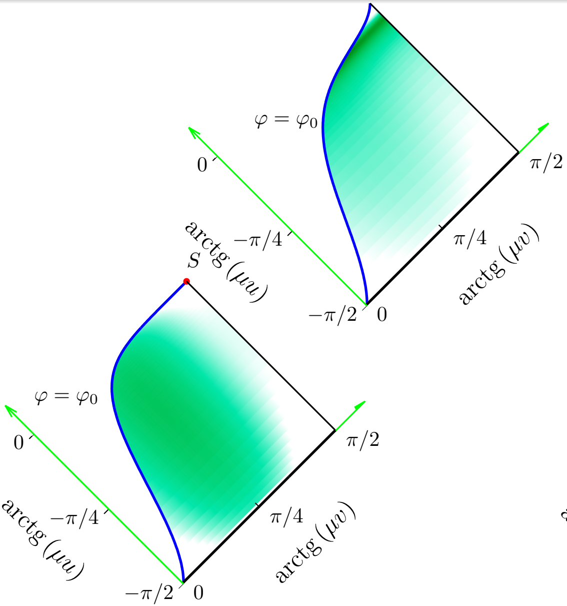
$$\mathcal{T}(v) = -\frac{8T^2}{\big((v-v_0)^2+T^2\big)^2} + \frac{2\big((v_0-\gamma)^2+T^2\big)}{(v-\gamma)^2\big((v-v_0)^2+T^2\big)}$$





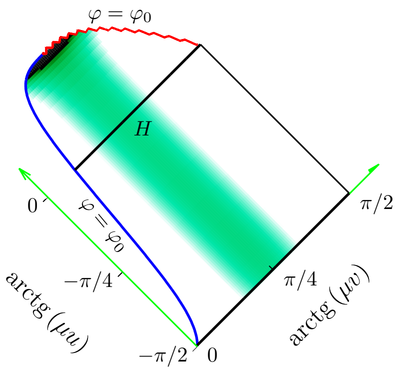
$\mathcal{T}(v) \leq 0$ at $v \geq 0$

3–pole solution with $s_i = 1$

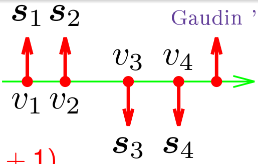


$$\psi(v) = \sum_i \frac{\psi_i}{v - v_i} + Cv + 1$$

$$\psi_i = \psi_i(v_j), \quad C = C(v_j)$$



Rational Gaudin model



Gaudin '76

Gaudin problem:

$$H_i = \sum_{j\neq i} \frac{(\boldsymbol{s}_i \boldsymbol{s}_j)}{v_i - v_j}, \qquad [H_i, H_j] = 0$$

Find eigenspectrum of H_i

$$s_i(s_i+1)$$
$$||$$

$$\boldsymbol{s}(v) = \sum_i \frac{\boldsymbol{s}_i}{v - v_i}, \qquad \hat{\mathcal{T}}(v) = \boldsymbol{s}^2(v) = \sum_i \left(\frac{\boldsymbol{s}_i^2}{(v-v_i)^2} + \frac{2H_i}{v-v_i} \right)$$

$|0\rangle = |\downarrow\downarrow \dots \downarrow\rangle$ — vacuum

Solution: Bethe Ansatz

$$s^+(v) = s_x(v) + i s_y(v)$$

$s^+(\tilde{v}_1) \dots s^+(\tilde{v}_M)|0\rangle$ — solution if

$$\sum_i \frac{s_i}{\tilde{v}_m - v_i} = \sum_{m' \neq m} \frac{1}{\tilde{v}_m - \tilde{v}_{m'}}$$

Conclusion: $\mathcal{T}(v)$ = eigenvalue of $\hat{\mathcal{T}}(v)$

$\psi(v)$ has poles and zeroes at $v = v_i, \ \tilde{v}_i$

Selection rules

Total number of states: $\boxed{\prod (2s_i + 1)}$

$$v \to \infty : \quad \begin{cases} \boldsymbol{s}(v) = \frac{1}{v} \overbrace{\sum_i \boldsymbol{s}_i}^{\boldsymbol{S}} \\ \mathcal{T}(v) \equiv \boldsymbol{s}^2(v) \to \frac{1}{v^2} \boldsymbol{S}^2 \end{cases}$$

BUT: $\mathcal{T}(v) \to O(v^{-4})$!

$$\Downarrow$$

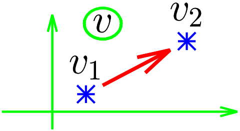
Solitons have $\boxed{\boldsymbol{S} = \sum \boldsymbol{s}_i = 0}$

Spins	Number of solitons
$1 \otimes 1 = 0 \oplus 1 \oplus 2$	1
$1 \otimes 1 \otimes 1 = 1 \oplus 0 \oplus 1 \oplus 2 \oplus 1 \oplus 2 \oplus 3$	1
$\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} \otimes = 0 \oplus 1 \oplus 1 \oplus 0 \oplus 1 \oplus 2$	2

Coalescence of singularities

$$\boxed{v_1 \to v_2}$$

$$\boldsymbol{s}(v) = \frac{\boldsymbol{s}_1}{v - v_1} + \frac{\boldsymbol{s}_2}{v - v_2} + \ldots \to \frac{\boldsymbol{s}_1 + \boldsymbol{s}_2}{v - v_2} + \ldots$$

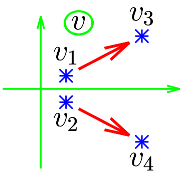


$$\Downarrow$$

Spins sum up! $\underbrace{(\frac{1}{2} \otimes \frac{1}{2})}_{2\times 2 \text{ solutions}} \otimes \ldots \to \underbrace{(0 \oplus 1)}_{1+3 \text{ solutions}} \otimes \ldots$

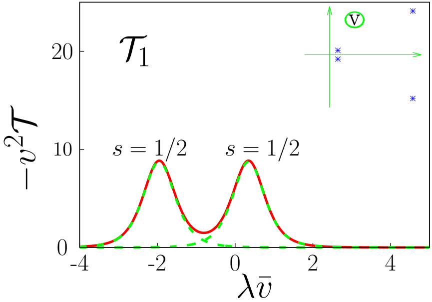
Example: 4–pole solution, $s_i = 1/2$

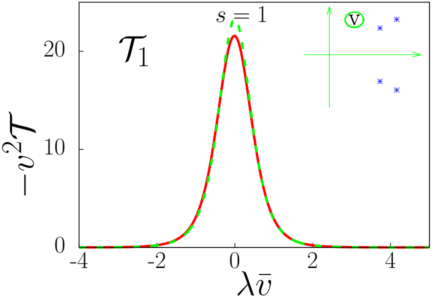
$$\mathcal{T}_{1,2}(v) = \frac{3(v_2-v_1)^2}{4(v-v_1)^2(v-v_2)^2} + \frac{3(v_4-v_3)^2}{4(v-v_3)^2(v-v_4)^2} + \frac{C_T \mp \sqrt{\Delta}}{(v-v1)(v-v2)(v-v3)(v-v4)}$$



$\mathcal{T}_1(v) \to \mathcal{T}^{(1,1)}(v)$

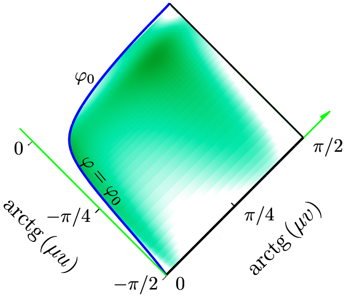
$\mathcal{T}_2(v) \to 0$





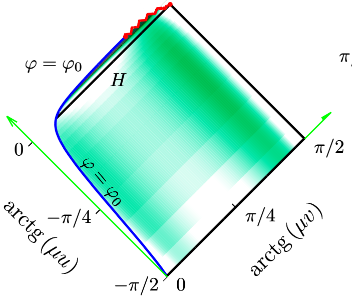
+:

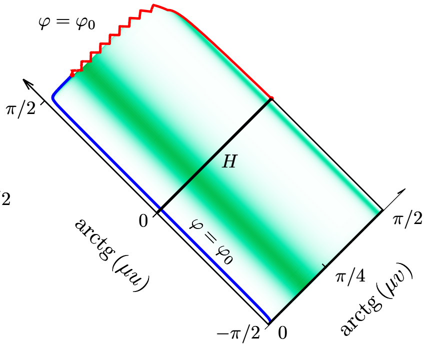
4–pole solution with $s_i = 1/2$



$$\psi(v) = \left(\sum_i \frac{\psi_i}{v - v_i}\right) \cdot \prod_j \sqrt{v - v_j}$$

$$\psi_i = \psi_i(v_j)$$



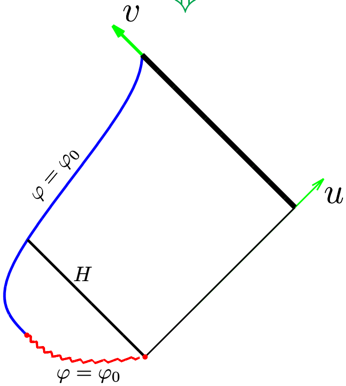


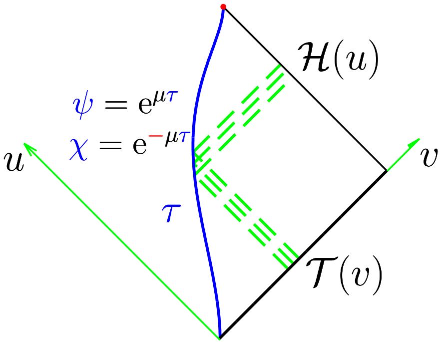
T–symmetry

in $\mathcal{T}(v)$, $\psi(v)$: $\partial_v^2 \psi = \mathcal{T}\psi$

out $\mathcal{H}(u)$, $\chi(u)$: $\partial_u^2 \chi = \mathcal{H}\chi$

$\Downarrow$





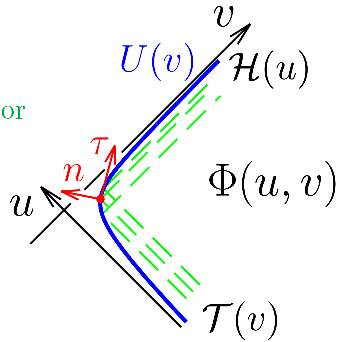
Two sets of solitons!

From boundary CGHS to Liouville

Conformal symmetry $\Rightarrow$ CFT?

Liouville equation: $\boxed{\Box\Phi + 8e^{\Phi} = 0}$

$$\left.\begin{aligned}\mathcal{T}(v) &= 2\pi T_{vv}\\ \mathcal{H}(u) &= 2\pi T_{uu}\end{aligned}\right\}$$ Energy–momentum tensor



Solution

$\psi_{1,2}(v)$: $\partial_v^2\psi_i = \mathcal{T}(v)\psi_i$

$\chi_{1,2}(u)$: $\partial_u^2\chi_i = \mathcal{H}(u)\chi_i$

$$\boxed{e^{-\Phi/2} = \psi_1(v)\chi_1(u) + \psi_2(v)\chi_2(u)}$$

Boundary CGHS	Liouville
$\mathcal{T}, \mathcal{H}$	$\mathcal{T}, \mathcal{H}$
$\psi = e^{\mu\tau(v)}$	$\psi_1(v) \Rightarrow \psi_2(v)$
$\chi = e^{-\mu\tau(u)}$	$\chi_1(u) \Rightarrow \chi_2(u)$

BCs in Liouville: $(\mathcal{R}|_\Gamma = 0)$

$K = -\mu + \frac{\pi}{\mu}(\mathcal{T}_{\mu\nu}\tau^\mu\tau^\nu)$

$\mathcal{T}_{\mu\nu}\tau^\mu n^\nu = 0$

May help with quantization!

Outlook

Boundary CGHS is an exactly solvable model!

Possible applications:

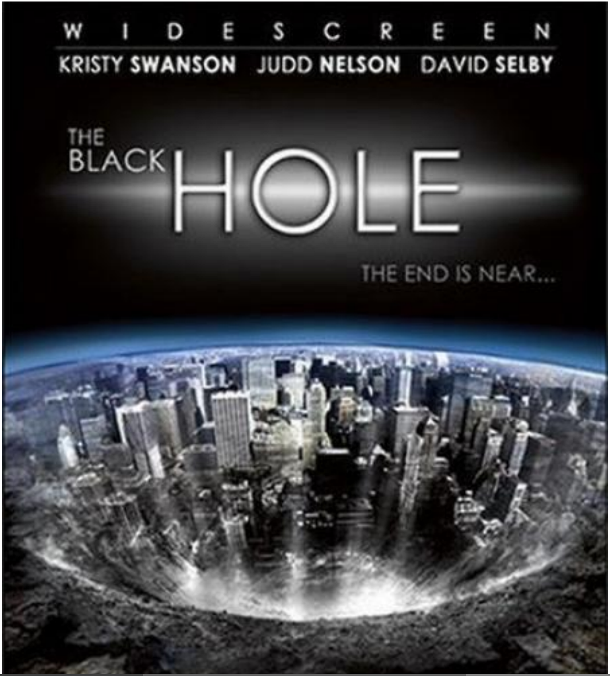
- Semiclassical calculations
  $\rightarrow$ S–matrix approach: $\langle i|S|f\rangle \leftrightarrow$ complex classical solutions

  t'Hooft et al

  $\rightarrow$ One–loop corrections: back reaction
  Simplificaltion: singularities behind the boundary!

  Callan, Giddings, Harvey, Strominger

- Exact solvability at quantum level?
  Relation to Liouville theory.


WIDESCREEN
KRISTY SWANSON JUDD NELSON DAVID SELBY
THE BLACK HOLE
THE END IS NEAR...