Effects of Lorentz violation in superluminal theories

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Outline

- Motivation: modified gravity theories
- A toy model with superluminality
- Instabilities and Lorentz violation
- Pathologies in stress-energy tensor
- Conclusions

cf. Alex Vikman talk on Wednesday

Modified gravity theories: examples

Covariant non-linear theory of massive gravity

$$S = M_{Pl}^2 \int d^4x \sqrt{-g} \left[\frac{R}{2} + m_g^2 (\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4) \right] \; ;$$

$$\mathcal{L}_i = \mathcal{L}_i(\delta^\mu_\nu - \sqrt{g^{\mu\rho}\eta_{ab}\partial_\rho\phi^a\partial_\nu\phi^b})$$

de Rham, Gabadadze, 2010
de Rham, Gabadadze, Tolley, 2010

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- Free of Boulware-Deser ghosts

Hassan, Rosen, 2011

- Provides self-accelerated cosmological solutions, $\Lambda = c(\alpha_i) m_g^2$

Gumrukcuoglu, Lin, Mukohyama, 2011
de Felice et al., 2013

Modified gravity theories: examples

Decoupling limit of covariant massive gravity:

$M_{Pl} \to \infty$; $m_g \to 0$; $M_{Pl} m_g^2 = \text{const}$

$$\Downarrow$$

Galileon theory

$$S = \int d^4x \left[\frac{1}{2} (\partial_\mu \pi)^2 - \partial_\mu^2 \pi (\partial_\mu \pi)^2 + ... \right] \ ; \quad \partial_\mu \pi \to \partial_\mu \pi + c_\mu \quad \text{invariance}$$

Nicolis, Rattazzi, Trincherini, 2009

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Features

- Ghost-free
- Late time cosmic acceleration
- Alternative to inflation (Galilean Genesis)

Creminelli, Nicolis, Trincherini, 2010
Creminelli et al., 2012

Modified gravity theories: problems

One and the same feature of all these theories — presence of superluminal modes on nontrivial backgrounds

- Galileon

Nicolis, Rattazzi, Trincherini, 2009

- Massive Gravity

Burrage et al., 2011
de Rham, Gabadadze, Tolley, 2011

Superluminality itself looks not so bad...

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$\Downarrow$

Possible problems with causality

$\Downarrow$

Chronology protection conjecture — there are no closed timelike curves.
Proven for some cases.

Mukhanov, Babichev, Vikman, 2007
de Rham et al., 2011

Another problem — UV-completion

Adams et al., 2006

Key moment: number of derivatives in self-coupling should be greater than 2. Take for example Galileon second term.

$$\mathcal{L} = \partial_\mu \pi \, \partial^\mu \pi + \frac{c}{\Lambda^4} \partial_\mu^2 \pi (\partial_\nu \pi)^2 + \ldots \, ; \quad c < 0$$

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- Problems with Lorentz notion of causality
- Scattering amplitudes do not satisfy S-matrix analyticity axioms

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$$\Downarrow$$

Superluminal effective theories could not be UV-completed to local QFT or unitary string theory!

Lorentz breaking: toy model

Plan

- Explicit demonstration on Lorentz invariance violation by quantum effects
- Calculation of stress-energy tensor VEV

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Toy model with superluminality in D=2

$$S = \int d^2x \left[\frac{1}{2}(\partial_\mu \phi \, \partial^\mu \phi) - \frac{m^2}{2}\phi^2 - \frac{1}{4\Lambda^2}(\partial_\mu \phi \, \partial^\mu \phi)^2 + M\delta(x)\phi \right]$$

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Consider classical perturbations on a background of a static solution. It is enough to consider EOM without non-linear ϕ term.

$$\phi'' - m^2\phi = -M\delta(x) \;\Rightarrow\; \tilde{\phi} = \frac{M}{2m}\,\mathrm{e}^{-m|x|}$$

Consider quadratic action for classical perturbations $\phi = \tilde{\phi} + \xi$;
collect terms up to the order $\frac{1}{\Lambda^2}$

$$S_\xi = \int d^2x \left[\frac{1}{2} Z^{\mu\nu} \partial_\mu \xi \, \partial_\nu \xi - \frac{m^2}{2} \xi^2 \right] ; \; Z^{\mu\nu} = \begin{pmatrix} 1 + \frac{M^2}{4\Lambda^2} \, \mathrm{e}^{-2m|x|} & 0 \\ 0 & -1 - \frac{3M^2}{4\Lambda^2} \, \mathrm{e}^{-2m|x|} \end{pmatrix}$$

Superluminality & classical stability

Consider quadratic action for classical perturbations $\phi = \tilde{\phi} + \xi$;
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Dispersion relation: $\omega_\pm = \pm \sqrt{-\frac{Z^{11}}{Z^{00}}}\, k \quad \Rightarrow \quad \left| \frac{d\omega}{dk} \right| > 1$

Group velocity exceeds the speed of light.

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Looking for a physical difference between static and boosted classical solutions

Classical instabilities = tachions arise as $\det Z^{\mu\nu}$ change its sign

$$\det Z^{\mu\nu}_{boost} = \det \Lambda^\mu_\rho \Lambda^\nu_\sigma Z^{\rho\sigma}_{static} = \det Z^{\mu\nu}_{static} < 0$$

$$\Downarrow$$

The solution $\tilde{\phi}$ is classically stable in any frame.

Quantum instability

Looking for a quantum instability

As Z^{00} change its sign in a boosted frame $\Rightarrow$ Ghosts appear!

$$Z^{00} = 1 - \frac{M^2}{4\Lambda^2}\gamma^2(3\beta^2 - 1)\,\mathrm{e}^{-2m\gamma|x+\beta t|} < 0$$

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Critical boost factor $\gamma > \frac{\Lambda\sqrt{2}}{M}\,\mathrm{e}^{m\gamma|x+\beta t|}$

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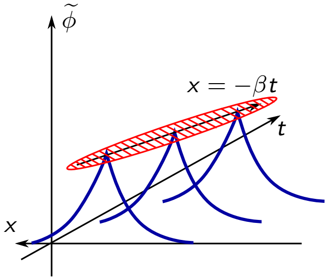
The trajectory of classical solution $\tilde{\phi}$ should be close to $x = -\beta t$



Quantize excitations $S_\xi = \int d^2x \left[\frac{1}{2} Z^{\mu\nu} \partial_\mu \xi \, \partial_\nu \xi\right]$ on the background $Z^{\mu\nu}_{boost}$

High frequency regime: $m_\xi = 0$; $Z^{\mu\nu}_{boost} = \text{const}$

Looking for evolution of negative energy modes

Add interaction with scalar field χ: $S_\chi = \int d^2x \left[\frac{1}{2}(\partial_\mu \chi \, \partial^\mu \chi) - \frac{m_\chi^2}{2}\chi^2 + \frac{g}{2}\xi\chi\chi\right]$

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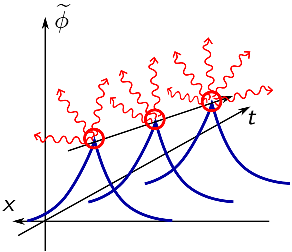
Triples $\xi\chi\chi$ are created from vacuum as $Z^{00} < 0$

Required hierarchy of parameters:

$m_\chi < m_\xi \gamma \ll k \ll \Lambda$; $m_\xi \ll M$; $g \frac{M}{m}_\xi \ll m_\chi^2$



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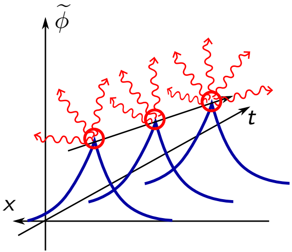

Background decay rate & energy loss rate

$$\Gamma \sim \frac{dE}{dt} \sim \frac{g^2}{4\pi m_\xi^2 \gamma^2} \lesssim \frac{m_\chi^4}{\Lambda^2}$$

Integrals saturated in IR

Lorentz invariance is violated!

Quantum instability

Naive estimation for $4D$

$$S_\chi = \int d^2x \left[\frac{1}{2}(\partial_\mu \chi \, \partial^\mu \chi) - \frac{m_\chi^2}{2}\chi^2 + \frac{g'}{2}\xi(\partial_\mu \chi \partial^\mu \chi) \right]$$

This simulates coupling $\frac{g'}{2}\xi^2\chi^2$ in $4D$ as it have the same dimensionality.

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Background decay rate

$$\Gamma \sim \frac{g'^2}{32\pi}\Lambda^2$$

Energy loss rate

$$\frac{dE}{dt} \sim \frac{g'^2}{32\pi}\frac{\Lambda^3}{m\gamma}$$

Integrals saturated in UV

This can be very high and probably could destroy the classical solution.

Stress-energy tensor pathology

Expect a divergence of $T^{\tilde{\phi}}_{\mu\nu}(\xi)$ as $\gamma \to \gamma_{critical}$

Below $\gamma_{critical}$: $T^{\tilde{\phi}^{boosted}}_{\mu\nu}(\xi) = T^{\tilde{\phi}^{static}}_{\mu\nu}(\xi) =$ *finite* , because of Lorentz covariance

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Looking for accelerated solution $\tilde{\phi}$

No need for external force, all we need is a coupling to dynamical source

$$\mathcal{L}_{int} = M\delta(x)\phi \quad \to \quad \mathcal{L}_{int} = \frac{G}{4}\left(\psi^2_{kink} - \upsilon^2\right)^2 \phi$$

Kink-antikink pairs are producing from vacuum and accelerating

$$\mathcal{L}_{\psi} = \frac{1}{2}(\partial_\mu \psi)^2 - \frac{\lambda}{4}\left(\psi^2 - \upsilon^2\right)^2 - \varepsilon(\psi - \upsilon)$$

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Take effective metric in a form

$$Z^{\mu\nu} = \begin{pmatrix} 1 - t^2/\alpha^2 & -t^2/\alpha^2 \\ -t^2/\alpha^2 & -1 - t^2/\alpha^2 \end{pmatrix}; \quad S_\xi = \int d^2x\, Z^{\mu\nu} \partial_\mu \xi\, \partial_\nu \xi$$

Solve the EOM $\partial_\mu[Z^{\mu\nu}\partial_\nu \xi] = 0$ with an ansatz $\xi = \xi(t)\, \mathrm{e}^{ikx}$

$$\xi(t,x) = c_1\, \mathrm{e}^{-ik(t-x)} + c_2\, \mathrm{e}^{-ik(t-x-\alpha \log \frac{\alpha+t}{\alpha-t})}$$

Quantize ξ regarding $Z^{\mu\nu}$ as "curved" background

Now we are able to use curved space QFT methods!

Renormalize $T_{\mu\nu}$ vacuum expectation value using point splitting method and Schwinger–DeWitt technique

$$\langle 0| T^{ren.}_{\mu\nu} |0\rangle = \frac{1}{4} \lim_{x \to x'} \mathcal{D}_{\mu\nu}(x, x') \left(G^{(1)}_{ex.}(x, x') - G^{(1)}_{pert.}(x, x') \right) ;$$

where we use Hadamar function

$$G^{(1)}(x, x') = \langle 0| \{\xi(x), \xi(x')\} |0\rangle$$

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An exact function

$$G^{(1)}_{ex.}(x,x') = \int\limits_0^\infty \frac{dk}{2\pi\sqrt{k^2+m^2}} \left(\mathrm{e}^{-ik(t-t'-x+x')} + \mathrm{e}^{-ik(t-t'-x+x'-\alpha \log \frac{(\alpha+t)(\alpha-t')}{(\alpha-t)(\alpha+t')})} \right)$$

And a perturbative one

$$G^{(1)}_{pert.}(x,x') = \int\limits_0^\infty \frac{dk}{2\pi\sqrt{k^2+m^2}} \left(\mathrm{e}^{-ik(\bar{t}-\bar{x})} + \mathrm{e}^{-ik(\bar{t}+\bar{x})} \right)$$

The perturbation theory by $(\bar{x}-\bar{x}')$ with respect to riemannian normal coordinates $\bar{x}_\mu$

$$Z_{\mu\nu}(\bar{x}) = \eta_{\mu\nu} + \frac{1}{3} R_{\mu\nu\rho\sigma}(\bar{x}')\bar{x}^\rho \bar{x}^\sigma + \ldots$$

A direct calculation gives

$$G^{(1)}_{ren.} = \lim_{x \to x'} [G^{(1)}_{ex.}(x,x') - G^{(1)}_{pert.}(x,x')] = \log\left[\frac{(\alpha^2 - t'^2)^2}{\alpha^2(\alpha^2 + t'^2)}\right] + \text{regular terms}$$

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For $\langle T_{\mu\nu} \rangle$ the divergence is even worse, for example

$$\langle 0 | T^{ren.}_{00}(x') | 0 \rangle = \frac{t'^6 - t'^4\alpha^2 + 6t'^2\alpha^4 - 5\alpha^6}{24\alpha^6(\alpha^2 - t'^2)} + \text{regular terms}$$

The indication for pathology as $t' \to \alpha$.

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As $\langle T_{\mu\nu}\rangle$ exceeds Λ^2 the effective theory breaks. Nevertheless, it is clear, that the theory

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is incomplete not only in UV but at any energy scale!

We may guess this conclusion to be true for any theory with superluminal excitations.

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How to make a full description of this kind of theories is a point of discussion...

Conclusions & Discussion

In the toy model with superluminality:

- Lorentz invariant action together with superluminal excitations leads to Lorentz non-invariant behaviour of physical solutions.
- Stress-energy tensor of the excitations shows pathology on top of the physical solution.

$\Longrightarrow$

- Theory is pathological and we don't need to consider it.
- As the theory is non-Lorentzian we need to complete a Lagrangian with other Lorentz-braking terms.
- Consider it as a spontaneous breaking of Lorentz invariance. We need to develop a mechanism of this breaking to understand which terms can be generated.

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This analysis seem to be suitable for other theories that have superluminal propagation on classical backgrounds.

Thank you!