Critical Phenomena in Two-dimensional Dilaton Gravity with a Boundary

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Introduction

- Open problems of BH physics.
  - Information paradox. Violation of unitarity via black holes evaporation.

    Hawking et al.
  - Firewall proposal. Actual singularity at horizon which prevents penetration into black holes.

    Almheiri, Marolf, Polchinski, Sully
- Hypothesis: Hawking's calculation is not consistent. Considering of processes with black holes as a processes of scattering.

  t'Hooft et al.
- Our approach. Simple model for studying quantum gravity effects.
- Critical phenomena. Nontrivial dynamics at the threshold of black hole formation.

  Choptuik

CGHS model
Spherical reduction

S-wave sector of EH gravity.

Metric

$$ds^2 = g^{(2)}_{\mu\nu} dx^\mu dx^\nu + \frac{e^{-2\phi}}{\lambda^2} d^2\Omega$$

$g^{(2)}_{\mu\nu}$ — 1+1 metric tensor and ϕ — dilaton field.
In Schwarzschild gauge:

$$r = e^{-\phi}/\lambda \, .$$

So, the dilaton corresponds to an area of a sphere:

$$S = 4\pi r^2 = 4\pi e^{-2\phi}/\lambda^2 \, .$$

Action

Einstein-Hilbert 1+1 action

$$S = \int d^2x\sqrt{-g}e^{-2\phi}\left(R + 2(\nabla\phi)^2 + 2\lambda^2 e^{2\phi}\right) - \frac{4\pi}{\lambda^2}\int d^2x\sqrt{-g}e^{-2\phi}(\nabla f)^2$$

Action

Einstein-Hilbert 1+1 action

$$S = \int d^2x\sqrt{-g}e^{-2\phi}\left(R + 2(\nabla\phi)^2 + 2\lambda^2 e^{2\phi}\right) - \frac{4\pi}{\lambda^2}\int d^2x\sqrt{-g}e^{-2\phi}(\nabla f)^2$$

Action

CGHS model action

$$S = \int d^2x\sqrt{-g}e^{-2\phi}\left(R + 4(\nabla\phi)^2 + 4\lambda^2\right) - \frac{1}{2}\int d^2x\sqrt{-g}(\nabla f)^2$$

Callan, Giddings, Harvey and Strominger, 1992

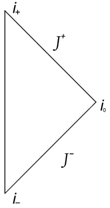
Comparison with 1+1 gravity

EH 1+1

Logarithmic dilaton vacuum:

$$\phi = -\ln(\lambda r) \ .$$

Physical domain — $r \in (0, +\infty)$.
Origin at $r = 0$.

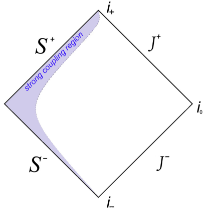


CGHS

Linear dilaton vacuum:

$$\phi = -\lambda r \ .$$

Physical domain — $r \in (-\infty, +\infty)$.
New feature: strong coupling region $S^{\pm}$.
Gravitational coupling: $g = e^{\phi}$.



Model with the boundary

Action with Gibbons-Hawking term

Neumann condition

$$\nabla_n f = 0$$

at the timelike hypersurface $\phi = \phi_0$ (or more clearly $r = const$)

Action with Gibbons-Hawking term

$$S = S_{CGHS} + 2 \oint_{\phi=\phi_0} d\sigma \sqrt{|h|} e^{-2\phi} \left(K + 2\lambda \right),$$

In addition, we obtain another Neumann condition on the dilaton

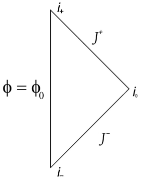
$$\nabla_n \phi = \lambda$$

Solution in the bulk:

$$f(v,u) = f_1(v) + f_2(u) \, ,$$

$$e^{-2\phi} = -\lambda^2 vu + g(v) + h(u) \, ,$$

$$f_v^2 = -2g_{vv} \, , \qquad f_u^2 = -2h_{uu} \, .$$



Equation for the boundary

Neumann conditions are equivalent to equation

$$U_v = \frac{e^{2\phi_0}}{\lambda^2}(g_v - \lambda^2 U)^2$$

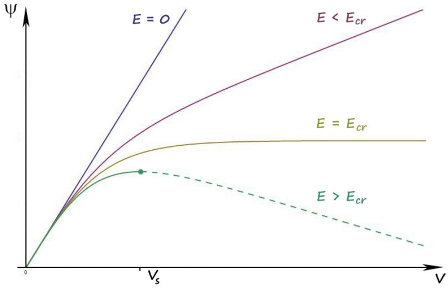
Changing variables $g_v - e^{-2\phi_0}\frac{\psi_v}{\psi} = \lambda^2 U$ we get

"Schrodinger"equation

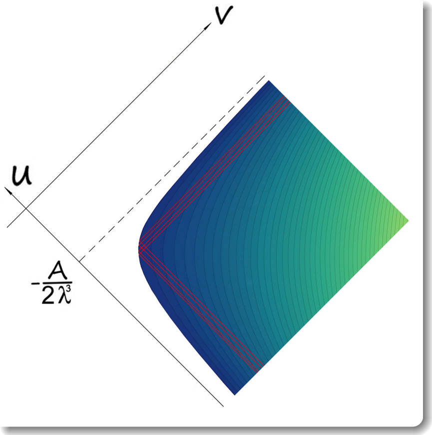
$$\psi''(v) = e^{2\phi_0} g_{vv}\psi(v), \quad \psi(0) = 0$$

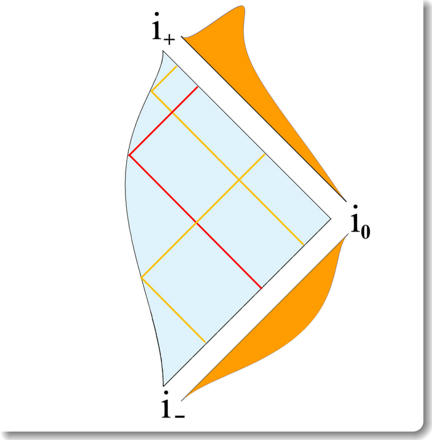
Physical meaning of ψ:

$$d\tau \;\propto\; d\psi/\psi\ .$$



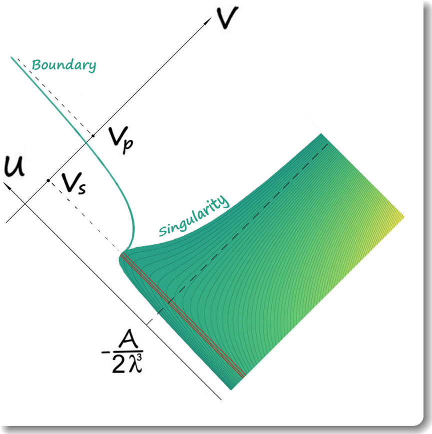
Reflection $0 < E < E_{cr}$

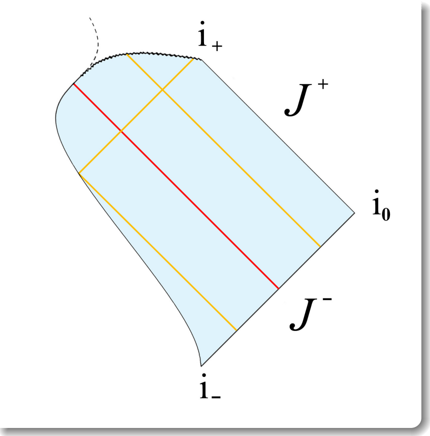




- $u \mapsto u + g_v(\infty)/\lambda^2$.
- $E_f = E_i$.

Collapse $E > E_{cr}$

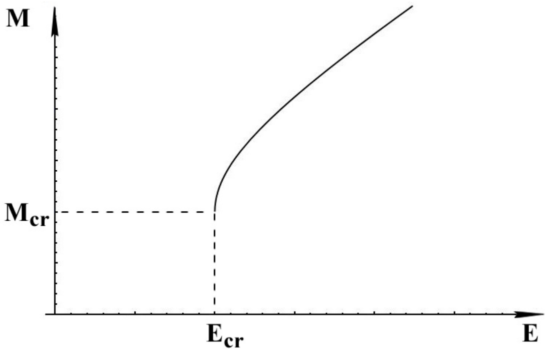




- $u_{hor} = g_v(\infty)/\lambda^2$.

- $E_f = E_i - M$.

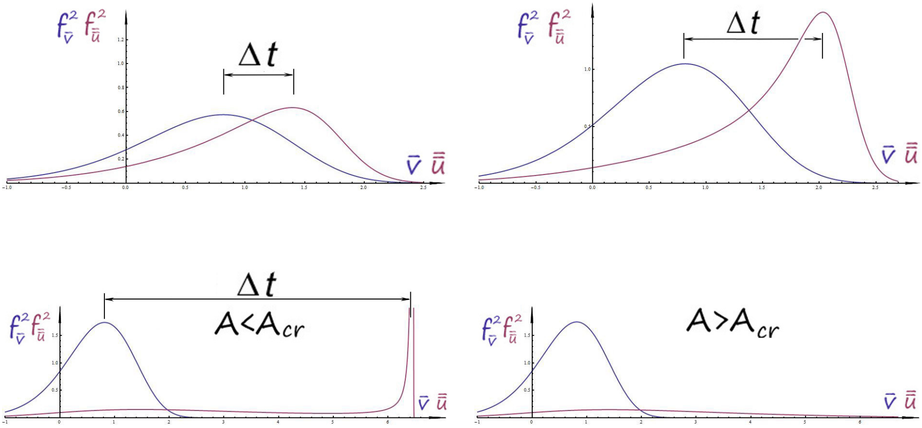
1st order phase transition



- $M_{cr} = 2\lambda e^{-2\phi_0}$.
- E_{cr} depends on wave packet parameters (shape).

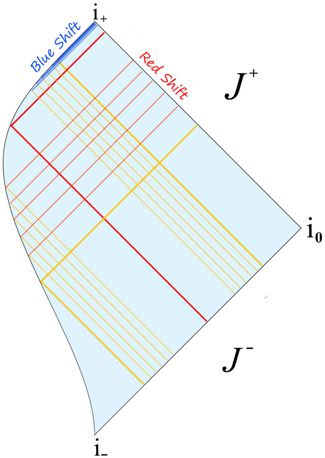
Flux at infinity

$\bar{v}$, $\bar{u}$ — null asymptotic coordinates at J^-, J^+.
$\Delta\ t$ — delay in Schwarzschild time.

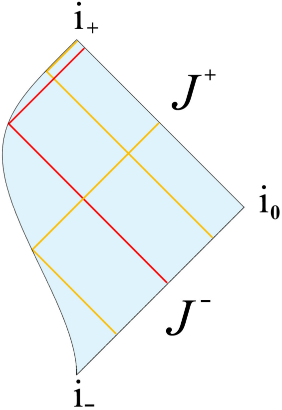


Subcritical and critical

Subcritical reflection



Critical solution



Classical firewall

Small parameter: $C \propto E_{cr} - E$.

Blue-shifted packet

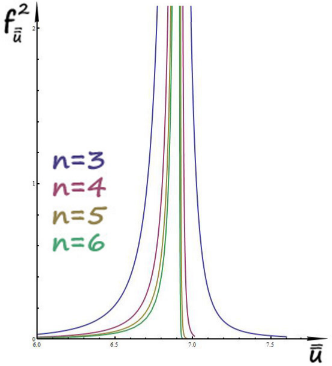
$$f_{\bar{u}}^2(\bar{u}) = -\frac{2\lambda^2 C^2 g_{vv}(v(\bar{u}))}{(C - e^{2\phi_0} g_v(v(\bar{u})))^4} \propto C^{-2} ,$$

$g_{vv} = -\frac{1}{2} f_v^2(v)$.

- Width goes to zero.
- Peak energy density $\to \infty$ at $C \to 0$.

Classical firewall?

$g_{vv} \sim v^{-n}$



Universal properties

Classical firewall energy

$$E = M_{cr}\left(1 + \frac{C}{e^{2\phi_0} g_v(0)} + \mathcal{O}(C^2)\right) .$$

Classical firewall is a delta-functional wave packet:

$$\boxed{f_{\bar{u}}^2(\bar{u}) = M_{cr}\delta(\bar{u} - \bar{u}_{delay})} .$$

Perturbative solution for equation of the boundary

$$\lambda e^{-\lambda\bar{u}(v)+2\phi_0} = C - C^2 v - 2Ce^{2\phi_0} g(v)|_v^{\infty} + e^{4\phi_0}\int_v^{+\infty} dv' g_{v'}^2 .$$

Time delay

$$\Delta\, t(C) \propto -\frac{1}{\lambda}\ln\frac{C}{\lambda} \qquad \Delta\, t \to \infty \quad \text{at} \quad E \to E_{cr} .$$

Quantum information retrieval

Xerox paradox

No-cloning theorem

A process

$$|\psi\rangle \mapsto |\psi\rangle \otimes |\psi\rangle$$

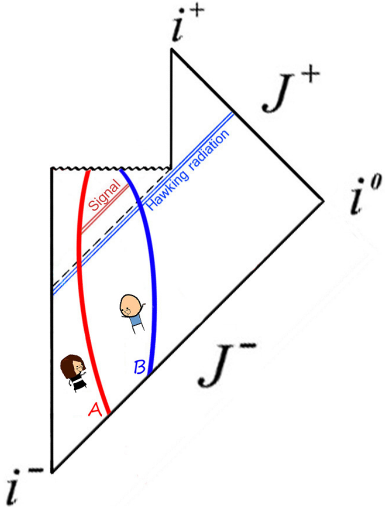
is forbidden.

Two copies of infalling object:

- Encoded in Hawking radiation.
- Under horizon.

If we do not add a new property of Hawking radiation, we can construct a quantum black hole xerox. Superposition principle is violated.

Xerox paradox



Quantum information retrieval
Scrambling time

Uncertainty principle:

$$\delta t \delta E \sim \hbar \mapsto \delta u \sim M^{-1} .$$

Singularity in the model with boundary at ϕ_0:

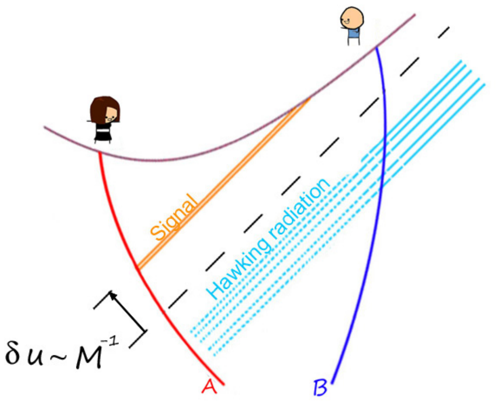
$$\frac{M}{2\lambda} - e^{-2\phi_0} = \lambda^2 v \delta u \propto e^{\lambda t} ,$$

We obtain a scrambling time

$$\boxed{t_{scr} \approx \frac{1}{\lambda} \ln \frac{|C|}{\lambda} ,}$$

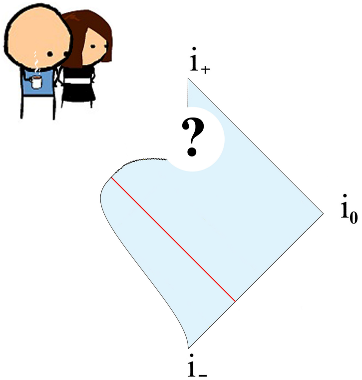
where $C \propto M_{cr} - M \propto E_{cr} - E$.

Scrambling time



Outlook

- We proposed a modification of CGHS model with the boundary which allows to study critical collapse.
- We found the classical firewall with properties:
  - Delta-functional.
  - With universal energy.
  - With universal time delay.
- The model is exactly solvable at the classical level. Stay for the next talk!



RST model: failed

Idea: effective action with first order correction.

Russo, Susskind and Thorlacius, 1992

$$\delta S = -\frac{1}{48}\int d^2x\sqrt{-g}R\frac{1}{\Box}R$$

Why failed: a thunderbolt singularity with infinite energy at the end of evaporation!