Dressing black holes with Galileons

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Galileons

many faces of the same model (I)

Monge'1784, Ampère'1820

Monge-Ampère equation

$$A(u_{xx}u_{yy} - u_{xy}^2) + Bu_{xx} + Cu_{xy} + Du_{yy} + E = 0$$

- to find a surface with a prescribed Gaussian curvature
- optimizing transportation costs

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Horndeski'1974

The most generic scalar-tensor theory in 4D, whose equations of motion contain no more than second derivatives (no Ostrogradski ghost)

$$S = \int d^4x\, F\left[g, \partial g, \partial^2 g, \varphi, \partial\varphi, \partial^2\varphi\right] \qquad E[g, \partial g, \partial^2 g, \varphi, \partial\varphi, \partial^2\varphi] = 0$$

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"Universal field equations"

Fairlie et al'1991

$$\mathcal{L}_n = F_n(\partial\varphi)W_{n-1}, \quad W_0 = 1 \qquad \mathcal{L}_1 = (\partial\varphi)^2 \to W_1 = \Box\varphi \to$$

$$W_n = \mathcal{E}\mathcal{L}_n \qquad \mathcal{L}_2 = (\partial\varphi)^2\Box\varphi \to \mathcal{E}\mathcal{L}_2 = (\Box\varphi)^2 - (\nabla\nabla\varphi)^2$$

Galileons

many faces of the same model (II)

DGP: brane model of gravity

Dvali et al'00

Particular limit of the theory (decoupling limit) gives scalar field Lagrangian,

Luty et al'03

$$\mathcal{L}_{DGP} = -\frac{M_P^2}{4} h^{\mu\nu} \left(\mathcal{E}h\right)_{\mu\nu} - 3(\partial\pi)^2 - \frac{r_c^2}{M_P}(\partial\pi)^2\Box\pi + \frac{1}{2}h^{\mu\nu}T_{\mu\nu} + \frac{1}{M_P}\pi T$$

direct coupling to matter

$$(\Box\varphi)^2 - (\nabla\nabla\varphi)^2$$

Monge-Ampère type

Galileons

many faces of the same model (III)

Generalization of DGP scalar: Nicolis et al'09

$$\mathcal{L}_\pi = \sum_{i=1}^{i=5} c_i \mathcal{L}_i,$$

$$\mathcal{L}_i \sim \pi^i$$

$$\mathcal{L}_1 = \pi,$$

$$\mathcal{L}_2 = -\frac{1}{2}\partial_\mu\pi\partial^\mu\pi,$$

$$\mathcal{L}_3 = -\frac{1}{2}\left(\partial\pi\right)^2\Box\pi,$$

$$\mathcal{L}_4 = -\frac{1}{4}(\Box\pi)^2\partial_\mu\pi\partial^\mu\pi + \frac{1}{2}\Box\pi\partial_\mu\pi\partial_\nu\pi\partial^\mu\partial^\nu\pi + \ldots$$

$$\mathcal{L}_5 = -\frac{1}{5}(\Box\pi)^3\partial_\mu\pi\partial^\mu\pi + \frac{3}{5}(\Box\pi)^2\partial_\mu\pi\partial_\nu\pi\partial^\mu\partial^\nu\pi + \ldots$$

Galileons

many faces of the same model (IV)

Covariant Galileon: adding non-minimal scalar-matter coupling to flat Galileon.

Deffayet et al'09
+ many other works

Shift-symmetric version:

$$\mathcal{L}_2 = K\left(X\right)$$

$$\mathcal{L}_3 = G^{(3)}\left(X\right)\Box\varphi$$

$$\mathcal{L}_4 = G^{(4)}_{,X}(X)\left[(\Box\varphi)^2 - \left(\nabla\nabla\varphi\right)^2\right] + R\,G^{(4)}(X),$$

$$\mathcal{L}_5 = G^{(5)}_{,X}\left(X\right)\left[(\Box\varphi)^3 - 3\Box\varphi\left(\nabla\nabla\varphi\right)^2 + 2\left(\nabla\nabla\varphi\right)^3\right] - 6G_{\mu\nu}\nabla^\mu\nabla^\nu\varphi\, G^{(5)}\left(X\right)$$

Decoupling limit of de Rham-Gabadadze-Tolley model of massive gravity

De Rham,
Gabadadze'10

Black holes are bald

- Gravitational collapse…
- black holes eat or expel surrounding matter
- their stationary phase is characterized by a limited number of charges
- and no details
- black holes are bald.

No hair arguments/theorems dictate that adding degrees of freedom lead to trivial (GR) or singular solutions.
For example in the standard scalar-tensor theories black hole solutions are GR black holes with constant scalar.

Example of hairy black hole:

The BBMB solution

Conformally coupled scalar:

Bocharova et al'70, Bekenstein'74

$$S[g_{\mu\nu},\phi] = \int \sqrt{-g}\left(\frac{R}{16\pi G} - \frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - \frac{1}{12}R\phi^2\right)d^4x$$

Static spherically symmetric (non-trivial) solution:

$$ds^2 = -\left(1-\frac{m}{r}\right)^2 dt^2 + \frac{dr^2}{\left(1-\frac{m}{r}\right)^2} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

with secondary scalar hair:

$$\phi = \sqrt{\frac{3}{4\pi G}}\frac{m}{r-m}$$

Note: the geometry is that of extremal RN

the scalar field is unbounded at r=m

No hair for Galileons:

Hui & Nicolis'12

1. Take a shift-symmetric scalar coupled to gravity $L(\nabla\phi,\ \nabla\nabla\phi, \ldots)$
2. There is a Noether current J^μ
3. Equation of motion for the scalar $\nabla_\mu J^\mu = 0$
4. Static spherically symmetric metric $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$

   and time-independent scalar $\phi = \phi(r)$
5. Compute the scalar quantity $J^2 = J_\mu J^\mu = g_{rr}\left(J^r\right)^2$
6. Require J^2 to be finite, at the horizon $J^r = 0$
7. By EOM $\nabla_\alpha J^\alpha = \frac{1}{\sqrt{-g}}\partial_\alpha\left(\sqrt{-g}J^\alpha\right) = \frac{1}{\sqrt{-g}}\partial_r\left(\sqrt{-g}J^r\right) = 0$

$$\rightarrow J^r = \frac{const}{\sqrt{-g}} \quad \rightarrow J^r = 0$$ everywhere

For particular theories one can argue that $J^r = \nabla^r\phi(r) \rightarrow \phi(r) = \text{const}$

No hair for Galileons:

Hui & Nicolis'12

Indeed for canonical scalar field $$J^r = \nabla^r \phi(r) \to \phi(r) = \text{const}$$

For other Galileons the proof is more tricky…

Our idea is to use the loopholes:

1. $$J^r = \nabla^r \phi(r) \quad \overset{?}{\to} \quad \phi(r) = \text{const}$$
2. $$\phi(r) \quad \to \quad \phi = \phi(t, r)$$

Model

Action and EOMs

Action $$S = \int d^4x\sqrt{-g}\left[\zeta R - 2\Lambda - \eta\left(\partial\phi\right)^2 + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right]$$

- The action is invariant under $\phi \to \phi + \mathrm{const}$
- Scalar field equation can be written as a current conservation

$$\nabla_\mu J^\mu = 0, \quad J^\mu = \left(\eta g^{\mu\nu} - \beta G^{\mu\nu}\right)\partial_\nu\phi$$

- Metric equations of motion:

$$\zeta G_{\mu\nu} - \eta\left(\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2\right) + g_{\mu\nu}\Lambda$$
$$+\frac{\beta}{2}\left((\partial\phi)^2 G_{\mu\nu} + 2P_{\mu\alpha\nu\beta}\nabla^\alpha\phi\nabla^\beta\phi\right.$$
$$\left.+g_{\mu\alpha}\delta^{\alpha\rho\sigma}_{\nu\gamma\delta}\nabla^\gamma\nabla_\rho\phi\nabla^\delta\nabla_\sigma\phi\right) = 0$$

$$P_{\alpha\beta\mu\nu} = -\frac{1}{4}\epsilon_{\alpha\beta\rho\sigma}R^{\rho\sigma\gamma\delta}\epsilon_{\mu\nu\gamma\delta}$$

Model

the Ansatze and assumptions

Let us assume the following

- Ansatze for metric: $$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$
- Ansatze for scalar: $$\phi(t,r) = q\,t + \psi(r)$$
- Number of independent functions: 3 $\quad [\psi(r),\ h(r),\ f(r)]$
- Number of independent equations: 4 $\quad [E_{tt},\ E_{tr},\ E_{rr},\ E_{\phi}]$
- Avoiding no-hair theorem: $\beta G^{rr} - \eta g^{rr} = 0$

$$\partial_t\left(\sqrt{-g}J^t\right) + \partial_r\left(\sqrt{-g}J^r\right) = 0$$ - EOM for scalar is satisfied

Also (tr)-component of the metric equation is satisfied

Regularity?

$$\beta G^{rr} - \eta g^{rr} = 0 \;\rightarrow\; f = \frac{(\beta + \eta r^2)h}{\beta (rh)'}$$

For $q \neq 0$ the current is non-zero, so is J^2 diverging at the horizon ?

In fact no, because $$J^t = \frac{\left(2\eta r h - 2\beta h' - r(\beta + \eta r^2)h''\right) q}{r(rh)'^2}$$

$J^2 = g_{tt} J^t J^t = 0$ unless $(rh)' = 0$ (extremal black hole)

Need to solve two ODE's, the (rr) and (tt) component of the metric equation.
The hypothesis is consistent, moreover the current is ok at the horizon.

Model

Solving the remaining EOMs

- From (rr)-component we find

$$\psi' = \pm \frac{\sqrt{r}}{h(\beta + \eta r^2)} \left(q^2 \beta(\beta + \eta r^2) h' - \frac{\lambda}{2} (h^2 r^2)' \right)^{1/2}$$

$$\lambda \equiv \zeta\eta + \beta\Lambda$$

- Note that for $\eta = \Lambda = 0$ and no time-dependence, the scalar is trivial
- And finally, (tt)-component gives h(r) via,

$$h(r) = -\frac{\mu}{r} + \frac{1}{r} \int \frac{k(r)}{\beta + \eta r^2} dr$$

and

$$\boxed{q^2 \beta(\beta + \eta r^2)^2 - \left(2\zeta\beta + (2\zeta\eta - \lambda)\, r^2\right) k + C_0 k^{3/2} = 0}$$

Any solution of the algebraic equation for $k = k(r)$ gives full solution to the system !

Examples:

solution with static scalar

$$\phi = \phi(r)$$

$J^2 = 0$ everywhere

$$h = 1 - \frac{\mu}{r} + \frac{\eta}{3\beta}\frac{2\zeta\eta - \lambda}{2\zeta\eta + \lambda} r^2 + \frac{\lambda^2}{4\zeta^2\eta^2 - \lambda^2}\frac{\arctan(r\sqrt{\eta/\beta})}{r\sqrt{\eta/\beta}},$$

$$\psi'^2 = -\frac{\zeta\eta^3 r^2\left(2\zeta\beta + (2\zeta\eta - \lambda)r^2\right)^2}{\beta(4\zeta^2\eta^2 - \lambda^2)(\beta + \eta r^2)^3 h}$$

Current is ok at the horizon, but the scalar field is diverging (also its derivative)

Examples

stealth solution

- Consider $$S = \int d^4x \sqrt{-g}\, [\zeta R + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi]$$
- We obtain $$f(r) = h(r) = 1 - \frac{\mu}{r}$$

$$\phi_\pm = qt \pm q\mu \left[2\sqrt{\frac{r}{\mu}} + \log\frac{\sqrt{r}-\sqrt{\mu}}{\sqrt{r}+\sqrt{\mu}} \right] + \phi_0$$

Schwarzschild geometry with a non-trivial scalar

Regularity of the scalar ?

Examples

stealth solution

$$\phi_{\pm} = qt \pm q\mu \left[2\sqrt{\frac{r}{\mu}} + \log \frac{\sqrt{r} - \sqrt{\mu}}{\sqrt{r} + \sqrt{\mu}} \right] + \phi_0$$

looks diverging at the horizon
but the time coordinate is not good there.

Consider EF coordinates instead

$$v = t + \int (fh)^{-1/2} dr \;\rightarrow\; ds^2 = -h dv^2 + 2\sqrt{h/f}\, dv dr + r^2 d\Omega^2$$

$$\phi_+ = q \left[v - r + 2\sqrt{\mu r} - 2\mu \log \left(\sqrt{\frac{r}{\mu}} + 1 \right) \right] + \text{const}$$

The scalar is regular

Examples

Schwarzschild-Einstein black hole

$$\zeta\eta + \beta\Lambda = 0$$

$$h = 1 - \frac{\mu}{r}, \quad f = \left(1 - \frac{\mu}{r}\right)\left(1 + \frac{\eta r^2}{\beta}\right)$$

$$\psi' = \pm \frac{q}{h}\sqrt{\frac{\mu}{r(1 + \frac{\eta}{\beta} r^2)}}$$

Black hole with the Einstein static universe asymptotic

Examples

Schwarzschild-de Sitter black hole

$$S=\int d^4x\sqrt{-g}\left[\zeta R-2\Lambda-\eta\left(\partial\phi\right)^2+\beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right]$$

for $q^2=(\zeta\eta+\beta\Lambda)/(\beta\eta)$

$$f=h=1-\frac{\mu}{r}+\frac{\eta}{3\beta}r^2 \qquad \psi'=\pm\frac{q}{h}\sqrt{1-h}$$

Schwarzschild-de Sitter black hole

$\Lambda_{\rm eff}=-\zeta\eta/\beta$ The solution is valid only if $\Lambda>\Lambda_{eff}$

Solution self tunes vacuum cosmological constant but has "action induced" effective cosmological constant

CONCLUSION

- Time-dependent scalar field
- Avoid the no-hair theorems by tuning to zero the r-component of the current.
- GR and non-GR black holes with a non-trivial and regular scalar.
- The method can be applied to other Galileons [Kobayashi and Tanahashi'14]
- Stability?