Cosmic Antigravity

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Outline

- Dark Energy or Modifed Gravity ?
- Curvature Oscillations in $F(R)$ Theories in Systems with Rising Energy Density
- Spherically Symmetric Solutions in $F(R)$ Gravity and Gravitational Repulsion
- Conclusions

Cosmological Acceleration

The data in favor of accelerated expansion:

- observation of the large scale structure of the universe
- measurements of the angular fluctuations of the CMBR
- determination of the universe age
- discovery of the dimming of distant Supernovae

With cosmological inflation, at the very beginning, the picture would be:

- first acceleration (initial push)
- then normal deceleration
- and lastly (today) surprising acceleration again

Cosmological Equations

Universe expansion is described by scale factor $\mathbf{a(t)}$, which satisfies the Friedmann equations. In particular, cosmological acceleration is given by:

$$\frac{\ddot{\mathbf{a}}}{\mathbf{a}} = -\frac{4\pi\, \mathbf{G_N}}{3}\left(\varrho + 3\mathbf{P}\right)$$

NB: Pressure gravitates!

- Source of gravitational force $\varrho + 3\mathbf{P}$, not only ϱ.
- Negative pressure is a source of the cosmological expansion, cosmic antigravity.

Equation of State

- There are two independent cosmological equations for three functions $\mathbf{a(t)}$, $\varrho\mathbf{(t)}$, $\mathbf{P(t)}$.
- These equations should be supplemented by the equation of state $\mathbf{P = P(\varrho)}$, which is determined by physical properties of matter.

Usually matter is described by linear equation of state:

$$\mathbf{P = w\varrho}$$

- Non-relativistic matter: $\mathbf{w_{nr} = 0}$
- Relativistic matter: $\mathbf{w_{rel} = 1/3}$
- Dark energy: $\mathbf{w_{DE} = -1.13^{+0.13}_{-0.10}}$

Possible source of the cosmic acceleration?

Dark Energy: P $< -\varrho/3$

- small vacuum energy, which is identical to cosmological constant
- energy density associated with an unknown, presumably scalar field, which slowly varies in the course of the cosmological evolution

Modification of Gravity, which is considered below.

Gravity Modification

Action in $f(R)$ theories:

$$
\begin{aligned}
S &= \frac{m_{Pl}^2}{16\pi}\int d^4x\sqrt{-g}f(R) + S_m \\
&= \frac{m_{Pl}^2}{16\pi}\int d^4x\sqrt{-g}[R + F(R)] + S_m
\end{aligned}
$$

Here $m_{Pl} = 1.22 \cdot 10^{19}\text{GeV}$ is the Planck mass and S_m is the matter action.

- Usual Einstein gravity: $f(R) = R$
- Modified gravity: $f(R) = R + F(R)$

Pioneering Works

The pioneering suggestion:

- *S. Capozziello, S. Carloni, A. Troisi,* Recent Res. Develop. Astron. Astrophys.1(2003)625; astro-ph/0303041.
- *S.M. Carroll, V. Duvvuri, M. Trodden, M.S. Turner,* Phys. Rev. D70 (2004) 043528, astro-ph/0306438.

$$\mathbf{F(R) = -\mu^4/R}$$

$\mu^2 \sim \mathbf{R_c} \sim \mathbf{1/t_u^2}$ is a small parameter with dimension of mass squared.

- Agreement with Newtonian limit for sufficiently small μ.
- Strong instability in presence of matter

Can modified gravity explain accelerated cosmic expansion?
A.D. Dolgov, M. Kawasaki, Phys.Lett. B573 (2003) 1.

Modified modified gravity: free from exponential instability

W.Hu, I. Sawicki, Phys. Rev. D **76**, 064004 (2007).

$$F_{HS}(R) = -\frac{R_{vac}}{2}\frac{c\left(\frac{R}{R_{vac}}\right)^{2n}}{1+c\left(\frac{R}{R_{vac}}\right)^{2n}},$$

A.Appleby, R. Battye,Phys. Lett. B **654**, 7 (2007).

$$F_{AB}(R) = \frac{\epsilon}{2}\log\left[\frac{\cosh\left(\frac{R}{\epsilon}-b\right)}{\cosh b}\right]-\frac{R}{2},$$

A.A. Starobinsky, JETP Lett. **86**, 157 (2007).

$$F_S(R) = \lambda R_0\left[\left(1+\frac{R^2}{R_0^2}\right)^{-n}-1\right].$$

Curvature Oscillations in Modified Gravity

- *E. V. Arbuzova, A. D. Dolgov, L. Reverberi*, Curvature Oscillations in Modified Gravity and High Energy Cosmic Rays, Eur.Phys.J.**C**(2012) 72:2247, arXiv:1211.5011; Particle Production in $f(R)$ Gravity during Structure Formation, Phys.Rev.**D** 88, 024035 (2013), arXiv:1305.5668.

Starobinsky model with $\mathbf{R^2}$ term:

$$\mathbf{F(R) = -\lambda R_0 \left[1 - \left(1 + \frac{R^2}{R_0^2} \right)^{-n} \right] - \frac{R^2}{6m^2}}$$

- Parameter $\mathbf{m}$ is bounded by $\mathbf{m \gtrsim 10^5}$ GeV to preserve successful predictions of BBN.
- $\mathbf{R^2}$-term is included to prevent curvature singularities in the presence of contracting bodies.
- *E.V. Arbuzova, A.D. Dolgov,* Explosive phenomena in modified gravity. Phys.Lett.**B**700(2011)289: $\mathbf{R^2}$ prevents from hitting infinity but still the maximum amplitude of $\mathbf{R}$ reaches a large value much larger than in GR

Basic Equations

The evolution of $\mathbf{R}$ is determined from the equation:

$$3\mathcal{D}^2 F'_R - R + RF'_R - 2F = T$$

- $\mathcal{D}^2$ is the covariant D'Alambertian operator, $F'_R \equiv dF/dR$
- $T \equiv 8\pi T^{\mu}_{\mu}/m^2_{Pl}$ and $T_{\mu\nu}$ is energy-momentum tensor of matter.

We are interested in the regime $|R_0| \ll |R| \ll m^2$, in which:

$$F(R) \simeq -\lambda R_0 \left[1 - \left(\frac{R_0}{R}\right)^{2n}\right] - \frac{R^2}{6m^2}\,.$$

We study the evolution of R in a contracting astrophysical system with rising energy density:

$$\varrho = \varrho_0(1 + t/t_{contr})$$

We assume that the gravity of matter is not strong and thus the background metric is flat: $3\partial_t^2 F'_R - R - T = 0.$

New Notations: Oscillator Equation

With the dimensionless quantities:

$$z \equiv \frac{T(t)}{T(t_{in})} \equiv \frac{T}{T_0} = \frac{\varrho_m(t)}{\varrho_{m0}}, \qquad y \equiv -\frac{R}{T_0},$$

$$g = \frac{1}{6\lambda n (m t_U)^2} \left(\frac{\varrho_{m0}}{\varrho_c}\right)^{2n+2}, \quad \tau \equiv m\sqrt{g}\, t$$

and new function, proportional to $F'(R)$:

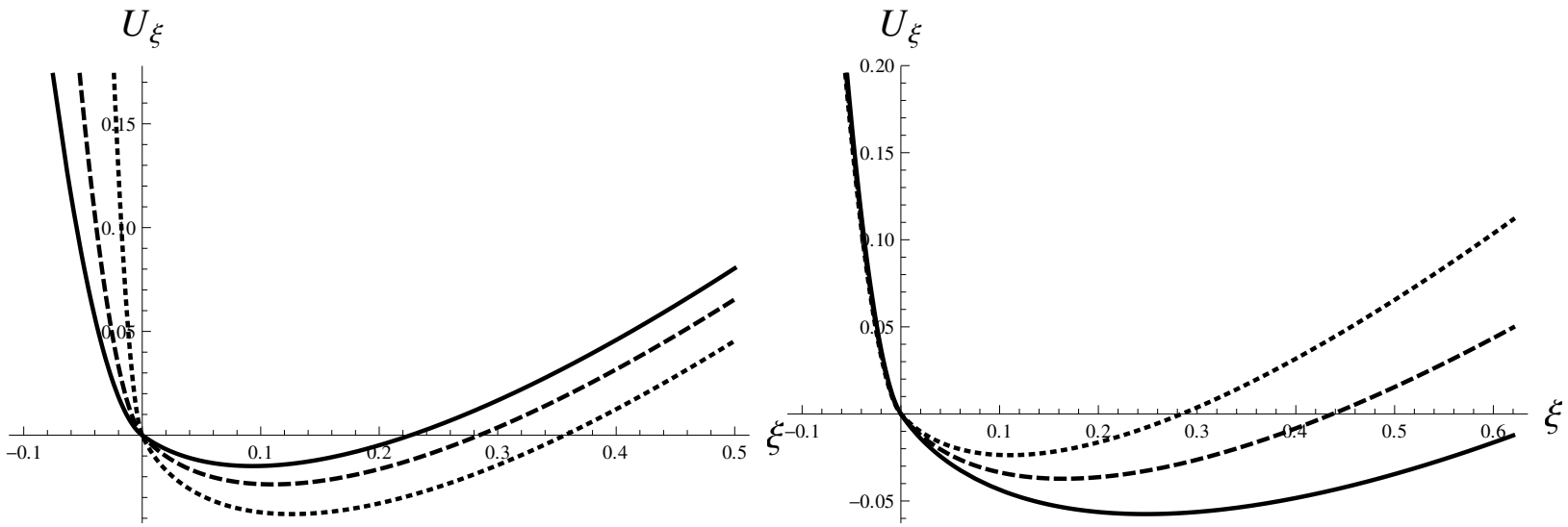
$$\xi \equiv \frac{1}{2\lambda n}\left(\frac{T_0}{R_0}\right)^{2n+1} F'_R = \frac{1}{y^{2n+1}} - gy$$

The equation of motion for ξ takes the simple oscillator form:

$$\xi'' + dU/d\xi = 0\,, \quad \text{where} \quad dU/d\xi = z - y(\xi).$$

y cannot be expressed through ξ analytically so we have to use different approximate expressions in different ranges of ξ.

Potential: $\mathbf{U}(\xi) = \mathbf{U}_+(\xi)\Theta(\xi) + \mathbf{U}_-(\xi)\Theta(-\xi)$



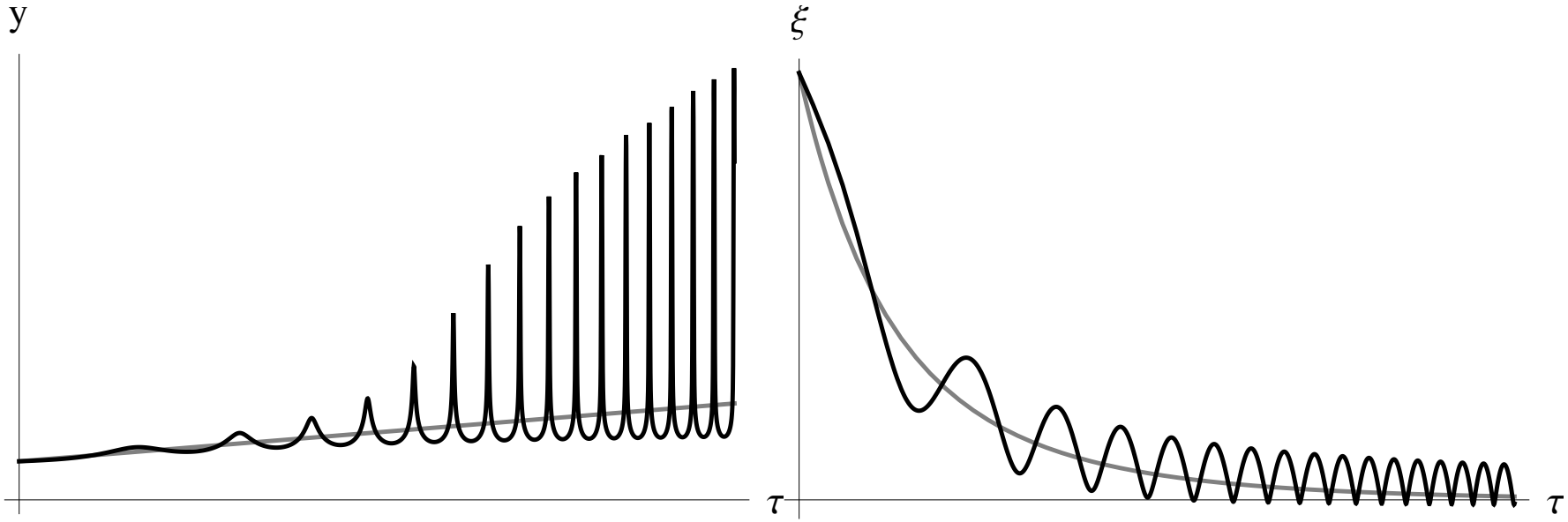
- *Left panel* ($\mathbf{n = 2,\ z = 1.5}$): solid line: $\mathbf{g = 0.02}$, dashed line: $\mathbf{g = 0.01}$, dotted line: $\mathbf{g = 0.002}$. *Right panel* ($\mathbf{n = 2,\ g = 0.01}$): solid line: $\mathbf{z = 1.3}$, dashed line: $\mathbf{z = 1.4}$, dotted line: $\mathbf{z = 1.5}$.

$$\mathbf{U}_+(\xi) = \mathbf{z}\xi - \frac{\mathbf{2n+1}}{\mathbf{2n}}\left[\left(\xi + \mathbf{g}^{(2n+1)/(2n+2)}\right)^{2n/(2n+1)} - \mathbf{g}^{2n/(2n+2)}\right],$$

$$\mathbf{U}_-(\xi) = \left(\mathbf{z} - \mathbf{g}^{-1/(2n+2)}\right)\xi + \frac{\xi^2}{\mathbf{2g}}.$$

Oscillations of Curvature. "Spike-like" Solutions

In contrast to ξ the oscillations of $\mathbf{y}$ are strongly unharmonic.
For negative and even very small $|\xi|$ the amplitude of $\mathbf{y}$ may be very large because $\mathbf{y} \approx -\xi/\mathbf{g}$, according to $\xi = \mathbf{1}/\mathbf{y}^{2n+1} - \mathbf{g}\mathbf{y}$.



"Spikes" in the solutions. $\mathbf{n} = \mathbf{2}$, $\mathbf{g} = \mathbf{0.001}$, and $\mathbf{y}_0' = \mathbf{0.2}$.

If the energy density rises with time, fast oscillations of the scalar curvature are induced, with an amplitude possibly much larger than usual GR value $\mathbf{R} = -\tilde{\mathbf{T}}$.

Frequency of Oscillations

For negative ξ potential behaves as $\mathbf{U} \approx \xi^2/(2\mathbf{g})$, so the characteristic frequency of oscillations in the spike region:

$$\Omega \sim 1/\sqrt{\mathbf{g}} \text{ in dimensionless time or } \omega \approx \mathbf{m} \text{ in physical time.}$$

Evidently frequency of oscillations of $\mathbf{y}$ in this region is the same.

NB

- The presented values correspond to maximum value of the frequency. Anyhow, $\omega \leq \mathbf{m}$.

Spike-like Solutions

Solution of modified gravity equations for finite-size astronomical objects with rising energy density:

$$R = R_{GR}(r)y(t), \quad R_{GR} = -\tilde{T}(r) = -\frac{8\pi T^{\mu}_{\mu}(r)}{m_{Pl}^2}$$

The maximum value of y in the spike region is:

$$y_{max}(t) \sim 6n(2n+1)mt_u \left(\frac{t_u}{t_{contr}}\right) \left[\frac{\varrho_m(t)}{\varrho_{m0}}\right]^{(n+1)/2} \left(\frac{\varrho_c}{\varrho_{m0}}\right)^{2n+2}$$

The energy density of the contracting cloud behaves as

$$\varrho_m(t) = \varrho_{m0}(1 + t/t_{contr})$$

- The mass $m \geq 10^5$ GeV to avoid a conflict with BBN.
- $mt_u \geq 10^{47}$ and y can reach a very high value

Spike Region

Spikes of high amplitude are formed if:

$$6n^2(2n+1)^2\left(\frac{t_u}{t_{contr}}\right)^2\left[\frac{\varrho_m(t)}{\varrho_{m0}}\right]^{3n+1}\left(\frac{\varrho_c}{\varrho_{m0}}\right)^{2n+2} > 1$$

- Formation of galaxies or their clusters: $\varrho_{m0}/\varrho_c = 1 - 10^3$ and $\varrho_m(t)/\varrho_{m0}$ varying in the range $1 - 10^5$.
- Formation of stellar or planetary type objects from the intergalactic gas with the initial density 10^{-24} g/cm^3: $\varrho_{m0}/\varrho_c = 10^5$ and $\varrho_m(t)/\varrho_{m0}$ can vary in the range $1 - 10^{24}$ or more.

The oscillations of curvature in such systems are excited if their mass density started to rise with time.

Spherically Symmetric Solutions in $F(R)$ Gravity

Assumption: the background space-time is nearly flat and so the background metric is almost Minkowsky.
Is this approximation valid for large deviation of curvature from its GR value?

- *E.V. Arbuzova, A.D. Dolgov, L. Reverberi*, Spherically Symmetric Solutions in F(R) Gravity and Gravitational Repulsion. Astropart.Phys.54(2014)44-47.

We consider a spherically symmetric bubble of matter of radius $\mathbf{r_m}$, and study spherically symmetric solution of modified EoM

$$\left(1+F'_R\right)R_{\mu\nu}-\frac{1}{2}\left(R+F\right)g_{\mu\nu}+\left(g_{\mu\nu}D_\alpha D^\alpha-D_\mu D_\nu\right)F'_R=\tilde{T}_{\mu\nu}$$

$$3D^2F'_R-R+RF'_R-2F=\tilde{T}$$

We use the Schwarzschild metric and assume that the metric is close to the flat one:

$$ds^2=A(r,t)dt^2-B(r,t)dr^2-r^2(d\theta^2+\sin^2\theta\, d\phi^2)$$

$$A_1=A-1\ll 1 \text{ and } B_1=B-1\ll 1$$

Equations of Motion

It is convenient to use EoM in the following form:

$$R_{00} - R/2 = \frac{\tilde{T}_{00} + \Delta F'_R + F/2 - RF'_R/2}{1 + F'_R}$$

$$R_{rr} + R/2 = \frac{\tilde{T}_{rr} + (\partial_t^2 + \partial_r^2 - \Delta)F'_R - F/2 + RF'_R/2}{1 + F'_R}$$

In the weak field limit:

$$R_{00} \approx \frac{A'' - \ddot{B}}{2} + \frac{A'}{r}, \quad R_{rr} \approx \frac{\ddot{B} - A''}{2} + \frac{B'}{r}$$

$$R \approx A'' - \ddot{B} + \frac{2A'}{r} - \frac{2B'}{r} + \frac{2(1-B)}{r^2}$$

If the energy density of matter inside the the cloud, i.e. for $r < r_m$, is much larger than the cosmological energy density, then:

$$F'_R \ll 1 \quad \text{and} \quad F \ll R$$

General Solutions

We assume that spatial derivatives of $\mathbf{F}'_{\mathbf{R}}$ are small and we find:

$$\begin{aligned} B_1' + \frac{B_1}{r} &= r\tilde{T}_{00} \\ A_1'' - \frac{A_1'}{r} &= -\frac{3B_1}{r^2} + \ddot{B}_1 + \tilde{T}_{00} - 2\tilde{T}_{rr} + \frac{\tilde{T}_{\theta\theta}}{r^2} + \frac{\tilde{T}_{\varphi\varphi}}{r^2 \sin^2\theta} \equiv S_A \end{aligned}$$

Equation for $\mathbf{B_1}$ has the solution:

$$B_1(r,t) = \frac{C_B(t)}{r} + \frac{1}{r}\int_0^r dr' r'^2 \tilde{T}_{00}(r',t)$$

To avoid a singularity at $\mathbf{r = 0}$ we have to assume that $\mathbf{C_B(t) \equiv 0}$.

$$A_1(r,t) = C_{1A}(t)r^2 + C_{2A}(t) + \int_r^{r_m} dr_1\, r_1 \int_{r_1}^{r_m} \frac{dr_2}{r_2} S_A(r_2,t)$$

The Schwarzschild Limit

The mass of matter inside a radius $\mathbf{r}$ is defined in the usual way:

$$\mathbf{M(r,t) = \int_0^r d^3r\, T_{00}(r,t) = 4\pi \int_0^r dr\, r^2\, T_{00}(r,t)}$$

If all matter is confined inside a radius $\mathbf{r_m}$, the total mass is $\mathbf{M \equiv M(r_m)}$ and it does not depend on time.

Since $\mathbf{\tilde{T}_{00} = 8\pi T_{00}/m_{Pl}^2}$, we obtain for $\mathbf{r > r_m}$:

$$\mathbf{B_1 = r_g/r}, \text{ where } \mathbf{r_g = 2M/m_{Pl}^2}$$

The metric coefficient $\mathbf{A_1}$ outside the source is:

$$\mathbf{A_1 = -\frac{r_g}{r} + \left[C_{1A}(t) - \frac{r_g}{2r_m^3}\right] r^2 + \left[C_{2A}(t) + \frac{3r_g}{2r_m}\right]}$$

Modified Gravity Solutions

The coefficient $C_{1A}(t)$ can be found from equation for R:

$$R \approx A'' + \frac{2A'}{r} - \ddot{B} - \frac{2B'}{r} + \frac{2(1-B)}{r^2}$$

- In systems with rising energy density the curvature scalar may be much larger than its value in GR.

Using eqs. for A_1 and B_1 and comparing them to expression for R, we get:

$$C_{1A}(t) = R(t)/6$$

$$R_{max}(t) \sim -6n(2n+1)mt_u \left(\frac{t_u}{t_{contr}}\right) \left[\frac{\varrho_m(t)}{\varrho_{m0}}\right]^{(n+1)/2} \left(\frac{\varrho_c}{\varrho_{m0}}\right)^{2n+2} \tilde{T}$$

The difference between modified and standard solutions in vacuum:

- In the standard case the term proportional to r^2 appears both at $r < r_m$ and $r > r_m$ with the same coefficient and we have to choose the arbitrary constant so that this term vanishes.
- For modified gravity such condition is not applicable and the $C_{1A}r^2$-term may be present at $r < r_m$ and absent at $r \gg r_m$.

Metric Functions inside a Cloud

The metric functions inside the cloud are equal to:

$$\mathbf{B(r,t)} = 1 + \frac{2M(r,t)}{m_{Pl}^2 r} \equiv 1 + B_1^{(Sch)}$$

$$\mathbf{A(r,t)} = 1 + \frac{R(t)\, r^2}{6} + A_1^{(Sch)}(r,t)$$

For the Schwarzschild part of the solution we find:

$$A_1^{(Sch)}(r,t) = \frac{r_g r^2}{2r_m^3} - \frac{3r_g}{2r_m} + \frac{\pi \ddot{\varrho}_m}{3m_{Pl}^2}\,(r_m^2 - r^2)^2$$

The oscillating part $R(t)r^2/6$ gives the dominant contribution into A_1:

- $r^2 R(t) \sim r^2 y(t)\, R_{GR}$ with $y > 1$, $|R_{GR}| = 8\pi\varrho_m/m_{Pl}^2$.
- the canonical Schwarzschild terms: $r_g/r_m \sim \varrho_m r_m^2/m_{Pl}^2 \sim r_m^2 R_{GR}$.

Applicability of the Approximation

- Assumption: the background metric weakly deviates from the flat Minkowsky one.
- Though it is certainly true for the Schwarzschild part of solution, this may be questioned for the $\mathbf{r^2 R(t)/6}$ - term.

The flat background metric is not noticeably distorted if

$$\mathbf{r^2 < 6/R(t)}$$

If the initial energy density of the cloud is of the order of the cosmological energy density

$$\mathbf{R_{GR} \sim 1/t_u^2},$$

the metric would deviate from the Minkowsky one for clouds with:

$$\mathbf{r_m > t_u/\sqrt{y}}.$$

At the stage of the rising $\mathbf{R(t)}$, when $\mathbf{y > 1}$ but not huge, the flat space approximation would be valid over all the volume of the collapsing cloud.

Anti-gravity inside a Cloud

In the lowest order in the gravitational interaction the motion of a non-relativistic test particle is governed by the equation:

$$\ddot{r} = -\frac{A'}{2} = -\frac{1}{2}\left[\frac{R(t)r}{3} + \frac{r_g r}{r_m^3}\right]$$

- Since $R(t)$ is always negative and large, the modifications of GR considered here lead to anti-gravity inside a cloud with energy density exceeding the cosmological one.

Gravitational repulsion dominates over the usual attraction if:

$$\frac{|R| r_m^3}{3 r_g} = \frac{|R| r_m^3 m_{Pl}^2}{6M} = \frac{|R| r_m^3 m_{Pl}^2}{8\pi \varrho\, r_m^3} = \frac{|R|}{\tilde{T}_{00}} > 1$$

so basically whenever **oscillations of R start rising**, regardless of the initial value of ϱ and to some extent of the specific $F(R)$ considered.

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- Structure formation in modified gravity would be very much different from that in the standard GR.
- Sufficiently large primordial clouds might not shrink down to smaller and smaller bodies with more or less uniform density but form thin shells empty (or almost empty) inside.
- This anti-gravitating behavior may also be a possible driving force for the creation of cosmic voids.

THE END

THANK YOU FOR YOUR ATTENTION!