Instability of magnetic fields in electroweak plasma driven by neutrino asymmetries

Victor B. Semikoz *(IZMIRAN)*

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Outline

- Photon polarization operator in plasma and $\nu\bar{\nu}$ -gas
- Modified Faraday equation in electroweak plasma
- Lower bound on neutrino asymmetries providing magnetic field generation in early universe
- Generation of magnetic field in magnetars driven by electron neutrino asymmetry $\Delta n_{\nu_e} \neq 0$
- Conclusion

Chern-Simons term in photon polarization operator

$$\boxed{\Pi_{\mu\nu}(q) = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Pi_T + \frac{q_\mu q_\nu}{q^2}\Pi_L + i\varepsilon_{\mu\nu\alpha\beta}q^\alpha(f_L^\beta - f_R^\beta)\Pi_P,}$$

$q^\mu = (\omega, \mathbf{k})$ is the photon momentum. In an isotropic medium ($\nu\bar{\nu}$ -gas at rest frame as a whole, $\mathbf{f}_{L,R} = 0$), and $\Pi_2(q) = (f_L^0 - f_R^0)\Pi_P$ is the parity violating form-factor entering the Chern-Simons term in the effective Lagrangian for electromagnetic field A_μ:

$$\boxed{L_{CS} = \Pi_2 \mathbf{A}\cdot\mathbf{B}, \quad \mathbf{B} = \nabla\times\mathbf{A}}$$

.

$$\boxed{f_L^0 - f_R^0 = -2\sqrt{2}G_F\sum_a c_a^{(A)}\Delta n_{\nu_a}, \quad \Delta n_{\nu_a} = n_{\nu_a} - n_{\bar{\nu}_a}}$$

Here $a = e, \mu, \tau$, $c_a^{(A)} = \mp 0.5$ is the axial SM coupling, upper sign for electron neutrinos, ν_e.

CS term for $\boxed{\nu\bar{\nu} \text{ gas in vacuum of charged leptons}}$

CS term $\Pi_2(q)$ in the lowest order $\sim G_F$

$$\Pi_2^{(\nu)}(q) = -(f_L^0 - f_R^0)\left(\frac{\alpha_{em}}{\pi}\right)\left(\frac{q^2}{m^2}\right)\int_0^1 dx \left[\frac{x(1-x)}{1-\frac{q^2}{m^2}x(1-x)}\right]$$

is zero, $\Pi_2^{(\nu)} = 0$, since $q^2 = \omega^2 - \mathbf{k}^2 = 0$ for on-shell photons (Gell-Mann theorem for $\gamma\gamma \to \nu\bar{\nu}$ process, 1961). However, at the two loop level ($\sim G_F/M_W^4$) Compton $\gamma\nu \to \gamma\nu$ is possible for $q^2 = 0$ (Dicus & Repko, 1993, Carl & Novikov, 2001).

$\boxed{\nu\bar{\nu} \text{ -gas in plasma where } q^2 = \omega_p^2 \neq 0}$

Matsubara:

$$i\int \frac{dp_0}{2\pi} \to T\sum_n, \quad p_0 = i(2n+1)\pi T + \mu, \quad n = 0, \pm 1, \pm 2, \ldots.$$

Low density classical plasma, $q^2 = \omega_p^2 \ll m^2$

$$\Pi_2^{(\nu l)} = -\frac{7e^2}{6}(f_L^0 - f_R^0)\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\frac{1}{\mathcal{E}_{\mathbf{p}}^3}$$
$$\times\Big\{\frac{m^2}{\mathcal{E}_{\mathbf{p}}^2}\left[\frac{1}{\exp[\beta(\mathcal{E}_{\mathbf{p}}-\mu)]+1}+\frac{1}{\exp[\beta(\mathcal{E}_{\mathbf{p}}+\mu)]+1}\right]$$
$$+\frac{m^2\beta}{2\mathcal{E}_{\mathbf{p}}}\left[\frac{1}{\cosh[\beta(\mathcal{E}_{\mathbf{p}}-\mu)]+1}+\frac{1}{\cosh[\beta(\mathcal{E}_{\mathbf{p}}+\mu)]+1}\right]$$
$$-\frac{\beta^2\mathbf{p}^2}{6}\left[\frac{\tanh[\beta(\mathcal{E}_{\mathbf{p}}-\mu)/2]}{\cosh[\beta(\mathcal{E}_{\mathbf{p}}-\mu)]+1}+\frac{\tanh[\beta(\mathcal{E}_{\mathbf{p}}+\mu)/2]}{\cosh[\beta(\mathcal{E}_{\mathbf{p}}+\mu)]+1}\right]\Big\},$$

where $\mathcal{E}_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$, $\quad \beta = T^{-1}$

$\nu\bar{\nu}$ **gas in nonrelativistic plasma,** $T \ll m$, $n_e \ll m^3$

$$\Pi_2^{(\nu)} = -\frac{2\alpha_{em}^2}{3}(f_L^0 - f_R^0)\left(\frac{n_e}{m^3}\right) \ll \Pi_2^{(\nu l)} = -\frac{7\pi\alpha_{em}}{3}(f_L^0 - f_R^0)\left(\frac{n_e}{m^3}\right)$$

CS term of photon polarization in relativistic plasmas

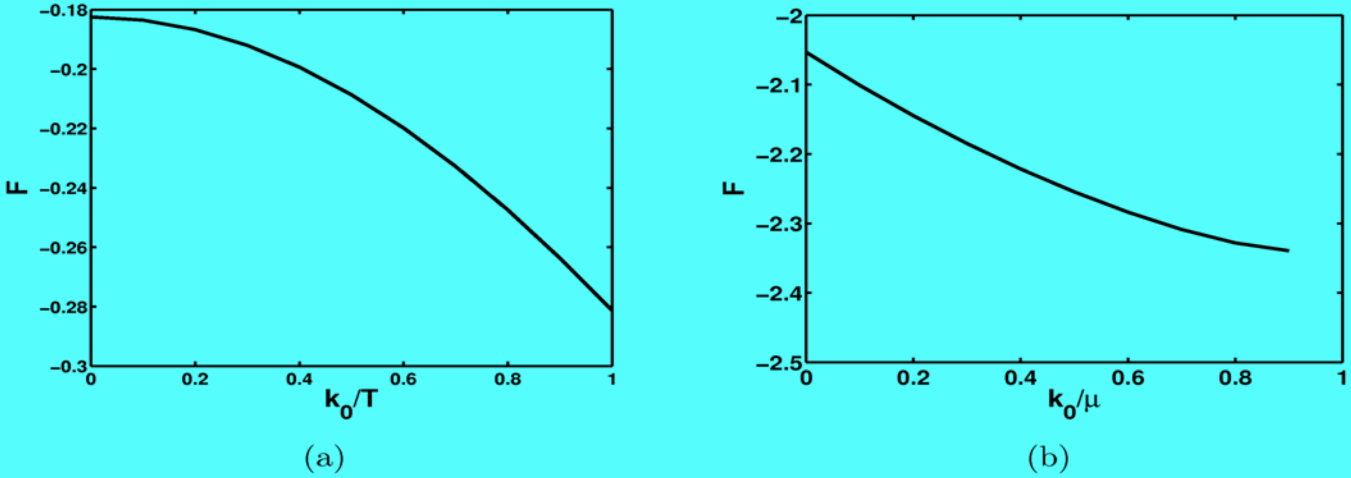

Figure 1: The function F versus k_0. (a) Hot relativistic plasma. (b) Degenerate relativistic plasma.

Relativistic plasmas, factor $F(k_0/T)$:

early universe $T \gg m_e$, (left panel)

a supernova $\mu \gg T, m_e$ (right panel)

$$\boxed{\Pi_2(\omega, 0) = \frac{\alpha_{em}}{\pi}(f_L^0 - f_R^0) F\left(\frac{\omega}{T}\right),}$$

where $k_0 \equiv \omega$, $\alpha_{em} = (137)^{-1}$, and $f_L^0 - f_R^0 = -2\sqrt{2}G_F \sum_a c_a^{(A)} \Delta n_{\nu_a}$

Faraday equation modified in SM

Maxwell equation:

$$\boxed{i\mathbf{k} \times \mathbf{B}(\omega, \mathbf{k}) + i\omega \mathbf{E}(\omega, \mathbf{k}) = \mathbf{j}(\omega, \mathbf{k}) + \mathbf{j}_5(\omega, \mathbf{k}),}$$

where neglecting spatial dispersion, $\omega \gg k\langle v\rangle$, if $k \to 0$:

$$\boxed{\mathbf{j}(\omega, \mathbf{k}) = \sigma_{cond}\mathbf{E}(\omega, \mathbf{k}), \quad \mathbf{j}_5(\omega, \mathbf{k}) = \Pi_2(\omega, 0)\mathbf{B}(\omega, \mathbf{k})}$$

MagnetoHydroDynamics=MHD

$\boxed{\sigma_{cond} \gg \omega,}$ $\varepsilon_{tr}(\omega) = 1 + i\sigma_{cond}/\omega$, then for $\boxed{\mathbf{kB} = 0:}$

appropriate dispersion equation $\boxed{\omega^2\varepsilon_{tr}(\omega) = k^2c^2 \Longrightarrow}$

$\omega^2 + i\omega\sigma_{cond} = k^2c^2 \approx i\omega\sigma_{cond} \Longrightarrow \boxed{\lambda_{skin} = (Im\ k)^{-1}}$

substituting $\boxed{\sigma_{cond} = \omega_p^2/\nu_{coll}}$ one gets skin layer

$$\boxed{\lambda_{skin} = \frac{c}{\omega_p}\sqrt{\frac{\nu_{coll}}{\omega}} \to \infty, \quad \text{if} \ \ \omega \to 0}$$

Modified Faraday equation in SM

Substituting $\mathbf{E}(\omega, \mathbf{k})$ from the Maxwell equation modified due to pseudovector current $\mathbf{j}_5(\omega, \mathbf{k}) = \Pi_2 \mathbf{B}(\omega, \mathbf{k})$:

$$i\mathbf{k} \times \mathbf{B}(\omega, \mathbf{k}) + i\omega \mathbf{E}(\omega, \mathbf{k}) = \sigma_{cond} \mathbf{E}(\omega, \mathbf{k}) + \Pi_2 \mathbf{B}(\omega, \mathbf{k}),$$

into Bianchi identity $\omega \mathbf{B}(\omega, \mathbf{k}) = \mathbf{k} \times \mathbf{E}(\omega, \mathbf{k})$, one gets for low MHD frequencies, $\omega \ll \sigma_{cond}$:

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = \alpha \nabla \times \mathbf{B} + \beta \nabla^2 \mathbf{B},}$$

where $\alpha = \Pi_2/\sigma_{cond}$ is the magnetic helicity parameter, $\beta = (\sigma_{cond})^{-1}$ is the magnetic diffusion coefficent. Stress!!! α is *scalar* instead of *pseudoscalar*

$\alpha_{MHD} \simeq \langle \mathbf{v} \cdot (\nabla \times \mathbf{v}) \rangle / 3$ in standard MHD.

Instability of magnetic fields driven by neutrino asymmetry

Keeping in mind

$$\alpha = \frac{\Pi_2(0)}{\sigma_{cond}} = -\frac{\alpha_{em} 2\sqrt{2} G_F F(0) (\sum_a c_a^{(A)} \Delta n_{\nu_a})}{\pi \sigma_{cond}},$$

one gets in conformal variables, $\eta = M_0/T$ for the maximum helical field, $h(k,\eta) = 2\rho_B(k,\eta)/k$,

$$\boxed{B(k,\eta) = B_0(k,\eta_0) \exp\left(\int_{\eta_0}^{\eta} \left[|\alpha(\eta')| \, k - k^2 \beta(\eta')\right] d\eta'\right)}$$

In hot universe plasma, $\sigma_{cond} = \sigma_c T$, $\quad \sigma_c \simeq 100$;
$\Delta n_{\nu_a} = \xi_{\nu_a} T^3/6$, where $\xi_{\nu_a} = \mu_{\nu_a}/T$, and

$$\alpha(T) = \frac{\alpha_{em} G_F \sqrt{2} T^2 F(0)}{6\pi\sigma_c} \left[\xi_{\nu_e}(T) - \xi_{\nu_\mu}(T) - \xi_{\nu_\tau}(T)\right]$$

Lower bound on neutrino asymmetries

In causal scenario magnetic field scale

$\Lambda_B = \frac{\beta}{\mid \alpha \mid} < l_H = H^{-1}$, that leads to the **lower bound**

$$\boxed{\mid \xi_{\nu_e} - \xi_{\nu_\mu} - \xi_{\nu_\tau} \mid > \frac{1.1 \times 10^{-6}\sqrt{g^*/106.75}}{(T/MeV)} ,}$$

where for hot plasma $T \gg m_e$ factor $\mid F(\omega = 0) \mid = 0.2$ was substituted in $\alpha = \Pi_2/\sigma_{cond}$. This bound is in agreement with the **upper BBN limit**, $\mid \xi_{\nu_e} \mid < 0.07$ (Dolgov et al, 2002), based on neutrino oscillations,

$$\xi_{\nu_e} \sim \xi_{\nu_\mu} \sim \xi_{\nu_\tau} \text{ at } T \leq T_{dec} \sim 3\ MeV, \quad g^* = 10.75.$$

Generation of magnetic fields in magnetars

One neglects $\nu_{\mu,\tau}$ -emission since $\Delta n_{\nu_\mu} = \Delta n_{\nu_\tau} = 0$, while just after collapse $\Delta n_{\nu_e} \neq 0$, even $n_{\nu_e} \gg n_{\bar{\nu}_e}$ because urca-process $\boxed{p + e^- = n + \nu_e}$ prevails decay $n \to p + e^- + \bar{\nu}_e$ during $t < 0.01$ sec. Then:

$$\boxed{\Pi_2(\omega, 0) = \left[\frac{\sqrt{2}\alpha_{em}G_F(n_{\nu_e} - n_{\bar{\nu}_e})}{\pi}\right] F(\omega/\mu)}$$

Then magnetic field scale $\Lambda_B = \beta / \mid \alpha \mid = (\mid \Pi_2 \mid)^{-1}$ reaches the core radius $R_{core} = 10$ km, if

$$\boxed{n_{\nu_e} - n_{\bar{\nu}_e} = \frac{\pi}{R_{core}\alpha_{em}G_F\sqrt{2} \mid F(0) \mid} = 5 \times 10^{27}\ \text{cm}^{-3}.}$$

Generation of magnetic fields in magnetars

The magnetic field itself grows exponentially through dynamo for a small wave number $k <\mid \alpha \mid /\beta =\mid \Pi_2 \mid$:

$$\boxed{B(k,t) = B_0 \exp\left(\int_{t_0}^{t}\left[\mid \alpha \mid k - k^2\beta)\right] dt'\right),}$$

driven here by a non-zero electron neutrino asymmetry $\alpha \sim \Delta n_{\nu_e} \neq 0$ during ν_e burst (first milliseconds).

Here a seed field B_0 is given by the growth of a small protostar field $B_{proto} = 1 \div 10^2$ G during collapse,

$$\boxed{B_0 = B_{proto}\left(\frac{R_{proto}}{R_{core}}\right)^2 = 10^{10}{\div}10^{12}\ \mathrm{G}}, \text{ if } \boxed{R_{proto} = 10^6\,\mathrm{km} \sim R_{\odot}}$$

Thus, parity violation in SM due to νe-interactions in the photon self-energy $\sim \Pi_2$ can provide amplification of B_0 up to $\boxed{B_{magnetar} = 10^{14} \div 10^{15}\ \mathrm{G}.}$

Comparison with chiral mechanisms

- Boyarsky, Frölich & Ruchayskiy (PRL 2012)

$$\boxed{\alpha = \frac{\alpha_{em}\Delta\mu}{\pi\sigma_{cond}}, \quad \Delta\mu = \mu_{eL} - \mu_{eR}, \quad \alpha_{em} = \frac{1}{137}}$$

via Adler anomaly, $\partial(j^\mu_L - j^\mu_R)/\partial x^\mu = (2\alpha_{em}/\pi)\mathbf{E}\cdot\mathbf{B}$ is not efficent to generate $\mathbf{B}$ since $\Delta\mu \to 0$ due to chirality flip with increasing rate $\Gamma_f \sim (m_e^2/T^2)$ in cooling universe. In contrary to that for neutrinos there are no anomalies, moreover, asymmetries ξ_{ν_a} remain constant after neutrino decoupling, or $\alpha \sim \xi_{\nu_a}$ is not zero and drives evolution of magnetic field:

$$\boxed{\frac{\partial\mathbf{B}}{\partial t} = \alpha\nabla\times\mathbf{B} + \beta\nabla^2\mathbf{B}.}$$

Comparison with chiral mechanisms

• Akamatsu and Yamamoto ("Chiral plasma instabilities", PRL 2013)

Chiral instability for electrons $\mu_5 = (\mu_R - \mu_L) \neq 0$ and generation of **B** in magnetars (Ohnishi , Yamamoto, 2014) gives huge $B \sim 10^{18}\ G$ for $\mu_5 = 200\ MeV$ where

$$\boxed{k \sim \alpha_{em}\mu_5, \quad \mid \mathbf{A} \mid \sim \frac{\mu_5}{\alpha_{em}}, \quad B \sim k \mid \mathbf{A} \mid \sim \mu_5^2}$$

Here k is too big, $\Lambda_B = k^{-1} \sim (MeV)^{-1} \sim 10^{-11}\ cm$ is microscopic, this is far from MHD fields. Moreover, authors assume that an initial imbalance $\mu_L \neq \mu_R$ comes from a non-equilibrium stage.

Conclusion

- Basing on FTFT methods we calculated the CS term Π_2 in the photon self energy (PSE) for $\nu\bar{\nu}$ gas embedded in plasma accounting for νl interaction in SM.

- The CS term $\Pi_2(\omega, 0)$ for a long-range magnetic fields should be also static $\omega \to 0$ that leads to penetration of $\mathbf{B}$ deeply the medium obeying the MHD approximation, $\omega \ll \sigma_{cond}$. Such a parameter is master for helicity coefficent $\alpha = \Pi_2/\sigma_{cond}$ and provides a growth of magnetic field.

- Since $\alpha \sim (\xi_{\nu_e} - \xi_{\nu_\mu} - \xi_{\nu_\tau})$ in causal scenario $(\Lambda_B < l_H)$ generation of $\mathbf{B}$ is provided for $\boxed{|\xi_{\nu_e}| > 10^{-7}}$ at $T \geq O(MeV)$ (**lower bound** consistent with the BBN upper one, $\boxed{|\xi_{\nu_e}| < 0.07}$)

Conclusion

• The presence of a non-zero electron neutrino asymmetry, $\Delta n_{\nu_e} \neq 0$, at first milliseconds of SN neutrino burst can provide $\Pi_2 \neq 0$ and a fast amplification of a seed $\boxed{B \sim 10^{10} \div 10^{12}\ G \text{ up to } B_{magnetar} \sim 10^{15}\ G.}$

• Our mechanism based on a neutrino asymmetry is preferable versus chiral instability mechanisms in plasma in cases:

• early universe plasma with vanishing chiral asymmetry parameter $\Delta\mu = \mu_L - \mu_R \to 0$ due to increasing chirality flip; and

• in degenerate electron gas of a supernova with huge magnetic fields (magnetars) which are small-scale ones.