Neutrino photoproduction on the electron in dense magnetized medium

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Outline

- Introduction
- Photon dispersion in the magnetized medium
- Neutrino emissivity
- Non-resonance case in the process $\gamma e \to e \nu \bar{\nu}$
- Resonance in the process $\gamma e \to e \nu \bar{\nu}$
- Conclusion

Introduction

Based on the papers by N. Mikheev, D. R. and M. Chistyakov *JETP V.146. 2014, in press* and A. Kuznetsov, D. R. and D. Shlenev *in preparation*

New view on the old thing

One interesting process

$e\gamma \to e\nu\bar{\nu}$ (V. Skobelev 2000, D. R. and M. Chistyakov 2008, A. Borisov et al 2012)

Where? The outer crust of magnetar, $B \sim 10^{14} - 10^{16}$ G.,

$B \gg B_e$, $B_e = m^2/e \simeq 4.41 \times 10^{13}$ G,

$T \sim 10^8 - 10^9$ K, $T \ll m$,

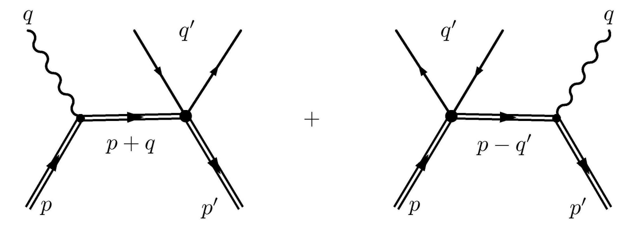
$$\frac{p_F}{m} \simeq 0.34 \frac{B_e}{B} \frac{\rho}{\rho_6}, \quad \rho_6 = 10^6 \text{g/cm}^3, \quad \rho_6 \leq \rho \leq 10^3 \rho_6$$

.

The photon dispersion properties were taken into account inaccurately!

We have corrected this defect!

Feynman diagrams for the process $\gamma e \to e\nu\bar{\nu}$.



Some notations

p^μ (p'^μ) are the momenta of initial (final) electrons,
q^μ and q'^μ are the momenta of initial photon and neutrino pair,
$(ab)_\perp = a_x b_x + a_y b_y$, $(ab)_\parallel = a_0 b_0 - a_z b_z$, $(a\varphi b) = a_y b_x - a_x b_y$.
$\varphi_{\alpha\beta} = F_{\alpha\beta}/B$ and $\tilde{\varphi}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\mu\nu}\varphi_{\mu\nu}$ are the dimensionless field tensor and dual field tensor correspondingly.

Photon dispersion in the magnetized medium

We begin to consider the process $e\gamma \to e\nu\bar{\nu}$ with investigation of the photon dispersion properties.

It is convenient to describe the propagation of the electromagnetic radiation in any active medium in terms of normal modes (eigenmodes). In turn, the polarization and dispersion properties of normal modes are connected with eigenvectors $r_\alpha^{(\lambda)}(q)$ and eigenvalues of polarization operator $\varkappa^{(\lambda)}(q)$ correspondingly.

Photon dispersion in the magnetized medium

- Magnetic field without plasma.

  In this case the eigenvectors are $r_\mu^{(\lambda)} = b_\mu^{(\lambda)}$, where

$$b_\mu^{(1)} = \frac{(\varphi q)_\mu}{\sqrt{q_\perp^2}}, \qquad b_\mu^{(2)} = \frac{(\tilde{\varphi} q)_\mu}{\sqrt{q_\parallel^2}},$$

$$b_\mu^{(3)} = \frac{q^2 \, (\Lambda q)_\mu - q_\mu \, q_\perp^2}{\sqrt{q^2 q_\parallel^2 q_\perp^2}}, \qquad b_\mu^{(4)} = \frac{q_\mu}{\sqrt{q^2}}.$$

The photon has the linear polarization.

Photon dispersion in the magnetized medium

- Strongly magnetized plasma

$$r_\mu^{(\lambda)} = \sum_{\lambda'=1}^{3} A_{\lambda'}^{\lambda}(q)\, b_\mu^{(\lambda')} .$$

Here $A_{\lambda'}^{\lambda}(q)$ are some complex coefficients, and the photon has the elliptical polarization.

The photon polarization operator in this case can be presented in the following form

$$\mathcal{P}_{\alpha\beta} = \sum_\lambda \varkappa^{(\lambda)} \frac{r_\alpha^{(\lambda)} (r_\beta^{(\lambda)})^*}{(r^{(\lambda)})^2} \simeq \varkappa^{(2)} \frac{r_\alpha^{(2)} (r_\beta^{(2)})^*}{(r^{(2)})^2} + \dots$$

Photon dispersion in the magnetized medium

Strong magnetic field $B \gg B_e$. The leading contribution to the amplitude is provided by the photon of mode $\lambda = 2$, $r_\alpha^{(2)} \simeq b_\alpha^{(2)}$, just as in the magnetic field without plasma.
In the case of cold plasma, $\omega, T \ll \mu - m$, we obtain

$$\varkappa^{(2)} \simeq \frac{\omega_p^2 \, q_\parallel^2}{\omega^2 - v_F^2 k_z^2}\,, \quad v_F = \sqrt{1 - \frac{m^2}{\mu^2}}\,,$$

where v_F is the Fermi velocity,
$\omega_p^2 = (2\alpha e B/\pi) v_F$ is the plasma frequency.

Photon dispersion in the magnetized medium

The dispersion equation $q^2 - \varkappa^{(2)} = 0$, in two particular cases:

- nonrelativistic plasma, $v_F \ll 1$:

$$\omega^2 \simeq \frac{1}{2}\left[k^2 + \omega_p^2\right] + \frac{1}{2}\sqrt{\left[k^2 + \omega_p^2\right]^2 - 4\omega_p^2 k^2 \cos^2\theta}\,,$$

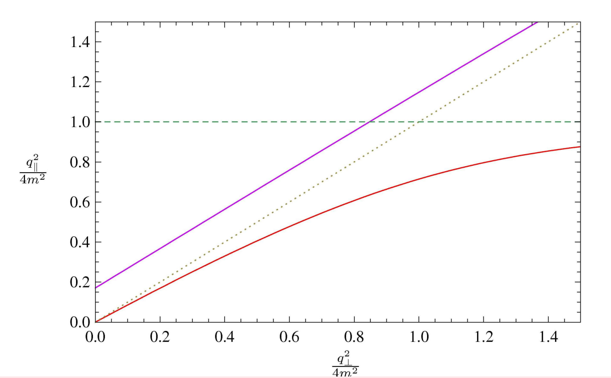
  where θ is an angle between the photon momentum and the magnetic field direction.
  Anisotropy exists in this case!

- relativistic plasma, $v_F \sim 1$:

$$\omega^2 \simeq k^2 + \omega_p^2\,.$$

Photon dispersion in the magnetized medium

Photon dispersion law for the mode 2 in strong magnetic field $B = 10^{16}$ G and $k_z = 0$. The red curve corresponds to the case without plasma. The purple curve corresponds to the case of plasma density $\rho = 7 \cdot 10^8 \text{g/cm}^3$.



Neutrino emissivity

A general expression for the neutrino emissivity (the loss of energy from a unit volume per unit time due to the neutrino escape) can be defined as follows:

$$Q = \frac{1}{V} \int \prod_i \mathrm{d}\Gamma_i f_i \prod_f \mathrm{d}\Gamma_f (1 \pm f_f)\, q_0' \,\frac{|S_{if}|^2}{\tau},$$

where $\mathrm{d}\Gamma_i$ ($\mathrm{d}\Gamma_f$) are the number of states of initial (final) particles; f_i (f_f) are the corresponding distribution functions, the sign $+$ $(-)$ corresponds to final bosons (fermions); q_0' is the neutrino pair energy; V is the plasma volume, τ is the interaction time, S_{if} is the S-matrix element.

Neutrino emissivity

When calculating S-matrix element we will consider the case of relatively low momentum transfers $|q'^2| \ll m_W^2$. Under this condition, the weak interaction of neutrinos with electrons can be considered in the local limit by using the effective Lagrangian

$$\mathcal{L} = -\frac{G_F}{\sqrt{2}} \left[\bar{e}\gamma_\alpha (C_V + C_A\gamma_5) e\right] j_\alpha \,,$$

where $C_V = \pm 1/2 + 2\sin^2\theta_W$, $C_A = \pm 1/2$, $j_\alpha = \bar{\nu}\gamma_\alpha(1+\gamma_5)\nu$ – is the neutrino current.

"$+$" – ν_e,

"$-$" – ν_μ and ν_τ

Neutrino emissivity

All electrons are on the ground Landau level

The process amplitude

$$\mathcal{M}_{\gamma e \to e \nu \bar{\nu}} = \frac{G_F}{e\sqrt{2q_{\parallel}'^2}} \left[C_V \, (q' \tilde{\varphi} j) - C_A \, (q' \tilde{\varphi} \tilde{\varphi} j) \right] \mathcal{M}_{2\to 2} \, .$$

Here $\mathcal{M}_{2\to 2}$ is the amplitude of the Compton scattering (D. R., M. Chistyakov 2009):

$$\mathcal{M}_{2\to 2} = 16\pi\alpha m \, \frac{\sqrt{q_{\parallel}^2 q_{\parallel}'^2} \, \sqrt{(-Q_{\parallel}^2)} \, \varkappa}{(qq')_{\parallel}^2 - \varkappa^2 (q\tilde{\varphi}q')^2},$$

$$\varkappa = \sqrt{1 - 4m^2/Q_{\parallel}^2}, \; Q^{\mu} = (q - q')^{\mu}.$$

Two partial cases

Nonrelativistic strongly magnetized plasma

$eB \gg p_F^2 \gg Tm, \; p_F \ll m$

$$Q = Q_s \; F\left(\frac{\omega_p}{T}\right) .$$

$$Q_s = \frac{8\pi^2 \alpha G_F^2 eB T^9}{4725 m p_F} \left[\overline{C_V^2} + \overline{C_A^2}\right] \simeq 1.3 \cdot 10^6 B_{15}^2 \rho_6^{-1} T_8^9 \frac{\text{erg}}{\text{cm}^3 \; \text{s}} .$$

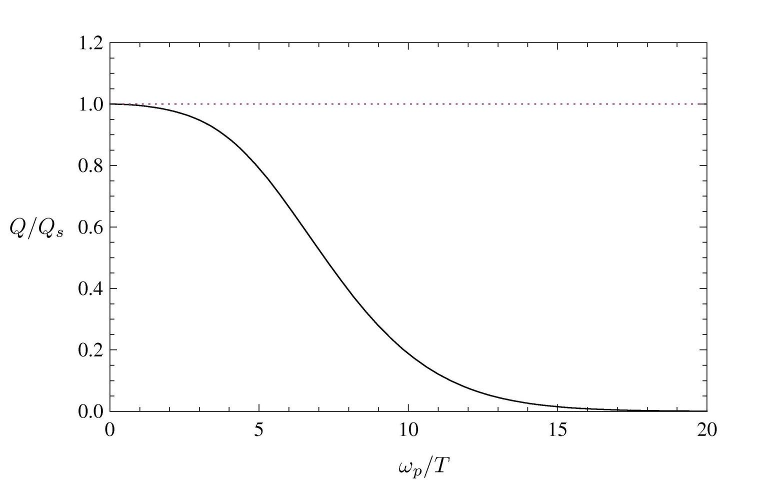
$$B_{15} = B/(10^{15}\,\text{G}), \; \rho_6 = \rho/(10^6\,\text{g/cm}^3), \; T_8 = T/(10^8\,\text{K})$$

(V.V. Skobelev 2000)

We obtained the following approximation for $F(y)$ with an accuracy of 0.5%

$$F(y) \simeq \frac{3\mathrm{e}^{-y}}{4\pi^8}\big(2y^7 + 15y^6 + 95y^5 + 495y^4 + 2040y^3 + 6240y^2 + 12600y + 12600\big) .$$

Non-resonance case in the process $\gamma e \to e\nu\bar{\nu}$



$$\rho = 10\rho_6,\ \omega_p/T_8 \simeq 7,\ Q \simeq 0.5 Q_s$$

Non-resonance case in the process $\gamma e \to e \nu \bar{\nu}$

The case of relativistic plasma, $\mu \gg m$

$$Q \simeq Q_b R\left(\frac{\omega_p}{2T}\right)$$

$$Q_b = \frac{G_F^2 \alpha(\overline{C_V^2} + \overline{C_A^2})}{576(2\pi)^{11/2}} \frac{B}{B_e} \left(\frac{m}{\mu}\right)^6 \omega_p^{15/2} T^{3/2} \mathrm{e}^{-\omega_p/T}$$

$$\simeq 10^{11} B_{16}^{43/4} \rho_9^{-6} T_9^{3/2} \exp\left(-6 B_{16}^{1/2} T_9^{-1}\right) \frac{\text{erg}}{\text{cm}^3\ \text{s}}$$

(A.V. Borisov et al. 2012)

$B_{16} = B/(10^{16}\,\text{G})$, $\rho_9 = \rho/(10^9\,\text{g/cm}^3)$, $T_9 = T/(10^9\,\text{K})$,

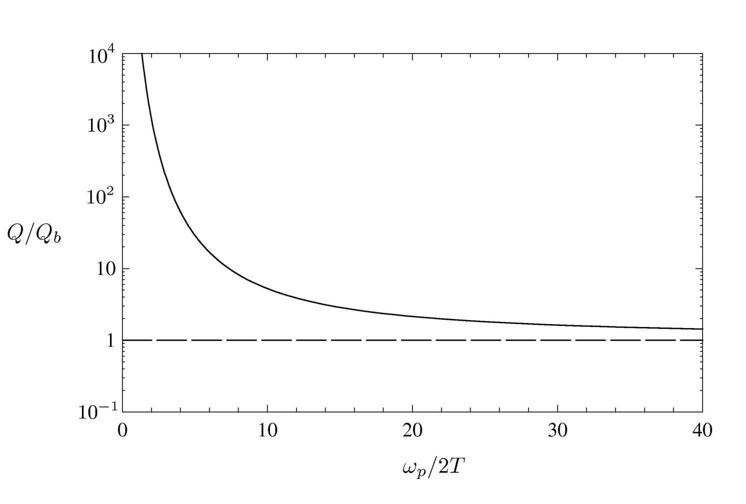
Non-resonance case in the process $\gamma e \to e \nu \bar{\nu}$

We obtained the following approximation for $R(z)$ with an accuracy of 0.8%

$$R(z) \simeq 1 + \frac{0.7627}{z^{1/2}} + \frac{66.875}{z^{3/2}} + \frac{271.654}{z^{5/2}} + \frac{2509.36}{z^{7/2}} + \frac{6754.62}{z^{9/2}} + \frac{16612.9}{z^{11/2}} + \frac{19843.8}{z^{13/2}} + \frac{10188.5}{z^{15/2}} .$$

Non-resonance case in the process $\gamma e \to e \nu \bar{\nu}$

The result for Q_b was understated by several orders of magnitude



$$\rho = \rho_9,\ B = B_{16},\ \omega_p/(2T_9) \simeq 3,\ Q \simeq 2 \cdot 10^2 Q_b$$

At the electron density $\rho \gtrsim \rho_9$, the higher Landau levels of virtual electron are excited.

The denominator of the electron propagator $P_\parallel^2 - m^2 - 2eBn$ can be equal to zero and this singularity is not removed by renormalization of the photon wave function.

Solving: we added the imaginary part to the electron mass: $m \to m - \mathrm{i}\Gamma_n/2$, to obtain

$$\frac{1}{P_\parallel^2 - m^2 - 2eBn} \to \frac{1}{P_\parallel^2 - m^2 - 2eBn + \mathrm{i}P_0\Gamma_n}.$$

Γ_n is the full width of the change of the electron state, $P_\mu = (p+q)_\mu$.

The amplitude squared, averaged over polarizations of initial photon, is factorized

$$|\mathcal{M}_{\gamma e \to e\nu\bar{\nu}}|^2 \simeq \sum_{n=1}^{\infty} \frac{\pi}{P_0 \Gamma_n} \delta(P_\parallel^2 - m^2 - 2eBn) \, |\mathcal{M}_{e_0\gamma \to e_n}|^2 \, |\mathcal{M}_{e_n \to e_0 \nu\bar{\nu}}|^2 \, .$$

We can present Γ_n in the following way (Weldon, 1983).

$$\Gamma_n = \Gamma^{abs} + \Gamma^{cr} \simeq \Gamma^{cr}_{e_0\gamma \to e_n} \left[1 + \mathrm{e}^{(P_0 - \mu)/T} \right] \, .$$

Here $\Gamma^{cr}_{e_0\gamma \to e_n}$ is the width of the electron production in the n-th Landau level.

Resonance in the process $\gamma e \to e\nu\bar{\nu}$

The neutrino emissivity due to the process $\gamma e_0 \to e_0\nu\bar{\nu}$ can be written as

$$Q_{\gamma e_0 \to e_0\nu\bar{\nu}} = \sum_{n=1}^{\infty} Q_{e_n \to e_0\nu\bar{\nu}} \,,$$

where $Q_{e_n \to e_0\nu\bar{\nu}}$ is the neutrino emissivity due to the process $e_n \to e_0\nu\bar{\nu}$.

Conclusion

- We have considered the neutrino photoproduction on an electron, $e\gamma \to e\nu\bar{\nu}$, in dense magnetized medium in both resonance and non-resonance cases.
- The changes of the photon dispersion properties in a magnetized medium are investigated. It has been shown, that taking into account of the photon dispersion anisotropy in the limit of non-relativistic plasma leads to substantial modification of the neutrino emissivity due to the process $\gamma e \to e\nu\bar{\nu}$, if compared with the previously obtained results.
- We have obtained the most general expression for the neutrino emissivity due to the process $\gamma e \to e\nu\bar{\nu}$ in relativistic and non-relativistic plasma at an arbitrary relation between the plasma frequency and temperature.

Conclusion

- It has been shown that the result known in the literature for the contribution of the photoneutrino process to the neutrino emissivity in the limit of a relativistic plasma was understated by several orders of magnitude.
- It has been shown that in the case of resonance on the virtual electron, the neutrino emissivity due to the process $\gamma e_0 \to e_0 \nu \bar{\nu}$ can be expressed in terms of the neutrino emissivity due to the process $e_n \to e_0 \nu \bar{\nu}$.

Thank you!!!