

Probing primordial statistical anisotropy from WMAP and Planck data

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[arXiv:1311.3272] / Work in Progress

Quarks'14

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Planck data has come...

- Nearly scale invariant power spectrum,

$$\langle \zeta(\mathbf{k})\zeta(\mathbf{k}') \rangle = \frac{1}{4\pi k^3} \mathcal{P}_\zeta(k) \delta(\mathbf{k} + \mathbf{k}')$$

- Slight negative tilt Ade *et al.*'13

$$\frac{\partial \mathcal{P}_\zeta(k)}{\partial \ln k} = n_s - 1 \quad n_s = 0.9603 \pm 0.0073 \text{ (68% CL)}$$

- Also Gaussian and adiabatic with a high accuracy.

Strong support for single-field slow roll inflation.

Starobinsky'79 Guth'81

Linde'82 '83 Albrecht and Steinhardt'82

Statistical isotropy

Statistical isotropy: $\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_\zeta(k)$ (NB: widely believed until 2008).

Cosmic No-Hair Conjecture Wald'83

Rapid isotropisation of Universe in the presence of positive constant energy density, if the other matter fields obey the dominant and strong energy conditions $ds^2 \rightarrow dt^2 - e^{2Ht} d\mathbf{x}^2$.

- $V(\phi) \neq$ vacuum energy density.
- Quantum fluctuations of the metric with $\lambda \gg \mathcal{H}_0$ mimic anisotropies for cosmological modes.
Standard slow roll inflation \Rightarrow anisotropies $\lesssim \mathcal{O}(10^{-6})$
Shtanov and Pyatkovska'10

Conclusion: observation of statistical isotropy \Rightarrow challenge for inflation.

However...

There are a number of models predicting statistical anisotropy.

- Models of inflation with vectors.
- Exotics: Solid inflation, non-commutative geometry, p -forms.
- Alternatives to inflation

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_\zeta(k) (1 + \mathcal{Q}(\mathbf{k})) \quad \mathcal{Q}(-\mathbf{k}) = \mathcal{Q}(\mathbf{k})$$

$$\mathcal{Q}(\mathbf{k}) = \sum_{LM} q_{LM}(k) Y_{LM}(\hat{\mathbf{k}}) \quad \text{even } L \geq 2$$

Simplest case: quadrupole with axial symmetry $q_{2M} \rightarrow q_{20}$

$$\mathcal{Q}(\mathbf{k}) = g_*(\mathbf{d}\hat{\mathbf{k}})^2 - g_*/3$$

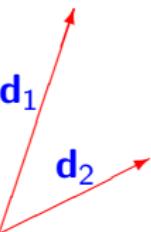
In inflation: $\langle \mathcal{A}_i \rangle \neq 0$

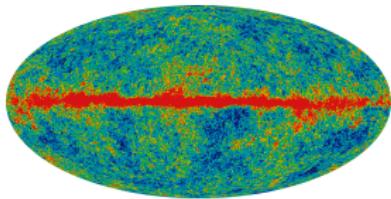
Ackerman et al.'07

$$\mathbf{d} \sim \langle \mathcal{A} \rangle$$

[3]

Generic
case
[5]





$$\mathcal{P}_\zeta(\mathbf{k}) \neq \mathcal{P}_\zeta(k) \implies$$

$$\langle \delta T(\mathbf{n}_1) \delta T(\mathbf{n}_2) \rangle \neq f(\mathbf{n}_1 \mathbf{n}_2)$$

$$a_{Im} = \int d\Omega_{\mathbf{n}} Y_{Im}^*(\mathbf{n}) \delta T(\mathbf{n})$$

Statistical anisotropy: $\langle a_{Im} a_{I'm'}^* \rangle \equiv C_{Im; I'm'} \neq C_I \delta_{II'} \delta_{mm'}$

$$C_{Im; I'm'} = 4\pi i^{I'-I} \int \frac{d\mathbf{k}}{k^3} Y_{Im}^*(\hat{\mathbf{k}}) Y_{I'm'}^*(\hat{\mathbf{k}}) \Delta_I(k) \Delta_{I'}(k) \mathcal{P}_\zeta(\mathbf{k})$$

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_\zeta(k) \left(1 + g_*(\hat{\mathbf{dk}})^2 \right) \implies \langle a_{Im} a_{I+2m'}^* \rangle \neq 0 .$$

How to measure statistical anisotropy?

$$\mathcal{P}_\zeta(\mathbf{k}) \propto \left(1 + \sum_{LM} q_{LM} Y_{LM}(\hat{\mathbf{k}})\right)$$

Consider log-likelihood of observed $\hat{a}_{lm} \neq a_{lm}$ with respect to q_{LM} .

$$\mathcal{L}(\hat{a}_{lm}|q_{LM}) = -\frac{1}{2}\hat{a}_{lm}\hat{\mathbf{C}}_{lm;l'm'}^{-1}(\mathbf{q})\hat{a}_{l'm'}^* - \frac{1}{2}\ln \det \hat{\mathbf{C}}(\mathbf{q}).$$

Assume small statistical anisotropy $q \equiv q_{LM} \ll 1$: **QML** estimators

$$\mathcal{L}(q) \approx \mathcal{L}(0) + \mathcal{L}'(0)q + \frac{1}{2}\mathcal{L}''(0)q^2 \quad \boxed{\mathcal{L}'(q) = 0}$$

Estimators for q_{LM} : unbiased and minimal varianced ([Hanson–Lewis'09](#)).

Amplitudes q_{LM} are not very convenient

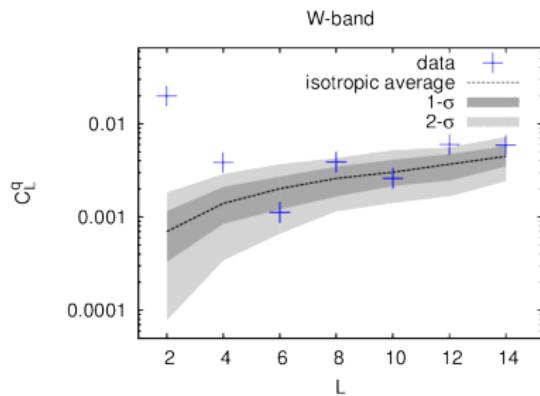
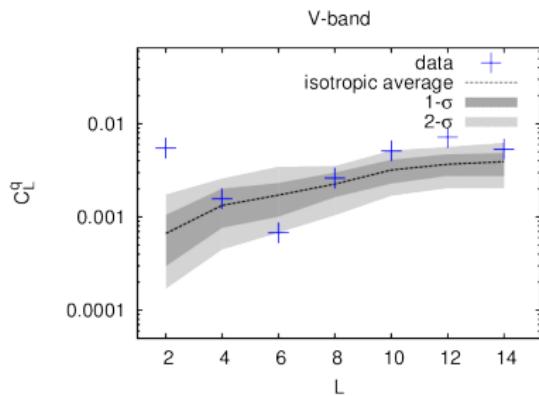
$$C_L^q = \frac{1}{2L+1} \sum_M |q_{LM}|^2.$$

Results from WMAP7 data

$$q_{LM}(k) = \text{const}$$

$l_{\max} = 400$ (otherwise, noise dominates)

Mask: $f_{\text{sky}} = 78\%$



WMAP5: Groeneboom and Eriksen'08, Hanson and Lewis'09

Strong indication of **statistical anisotropy!** $g_* = 0.29 \pm 0.031$?

Systematics?

Yes!

- Alignment with poles of ecliptic plane.
- Frequency dependence.
- Non-observation in alternative experiments ([Pullen and Hirata'10](#)).

Origin: asymmetries of beam transfer function.

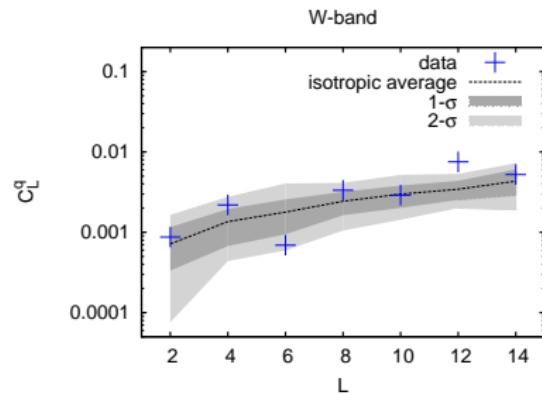
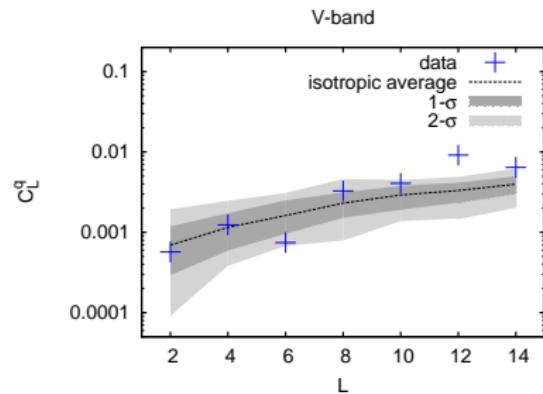
$$\hat{a}_{lm} = B_l a_{lm} + n_{lm} .$$

In fact,

$$B_l \rightarrow B_{l\text{m}}$$

Anomalous signal vanishes upon the inclusion of beam asymmetries ([Hanson et al'10](#)).

Results from WMAP9 data: systematics cured



V band: $-0.046 < g_* < 0.048$ 68% C.L.

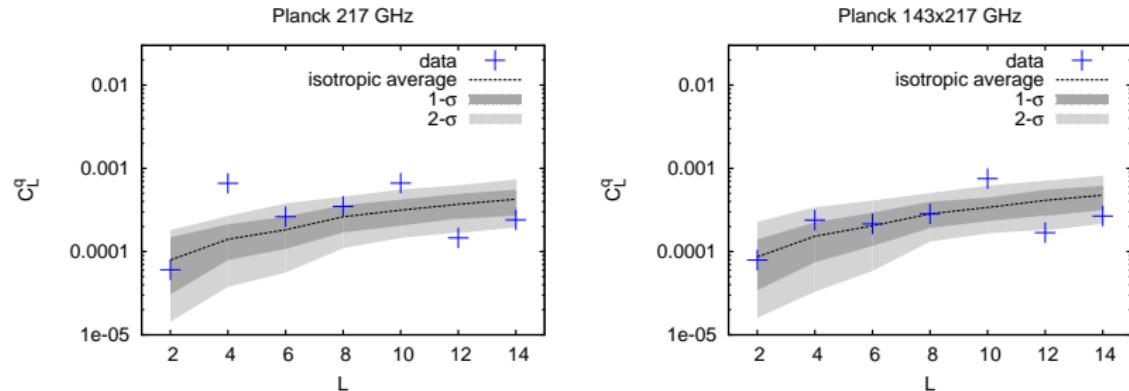
$|g_*| < 0.072$ 95% C.L.

W band:

$|g_*| < 0.085$ 95% C.L.

Results from Planck data: work in progress

$$l_{max} = 1600$$



Kim and Komatsu'13

143 GHz: $g_* = 0.002 \pm 0.016$ 68% C.L.

$|g_*| = 0.002^{+0.031}_{-0.032}$ 95% C.L.

Rubtsov *et al.*-work in progress

143×217 GHz: $|g_*| < 0.013$ 68% C.L.

$|g_*| < 0.026$ 95% C.L.

Comment 1. Hopefully, this is not the last word. We used biased estimator for g_* . Stronger constraints are expected by making use of unbiased estimators.

Comment 2. Axisymmetric quadrupole has been considered. What about general quadrupole? **Work in progress.**

Generically, g_* is a **random** quantity in
early Universe models \implies

Constraints on g_* **do not** imply constraints
on real parameters of the models

Perhaps, statistical anisotropy may originate from $\langle \mathcal{A}_i \rangle \neq 0$.

- Maxwellian field $\Rightarrow \rho_{\mathcal{A}} \propto a^{-4} \Rightarrow$ Statistical isotropy.
- Massive vectors, or general $U(1)$ -violating vectors \Rightarrow ghosts!
[Himmetoglu et al.'08 '09](#)
- Maxwellian field non-minimally coupled to inflaton [Watanabe et al.'09](#)

$$S_{\mathcal{A}} = -\frac{1}{4} \int d^4x \sqrt{-g} \cdot f^2(\phi) \cdot F_{\mu\nu} F^{\mu\nu} \quad F_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$$

Idea: $f(\phi) \propto a^{\pm 2}(\eta) \Rightarrow \rho_{\mathcal{A}} = \frac{\mathbf{E}^2 + \mathbf{B}^2}{2} = \text{const.}$

Choice $f(\phi) \propto a^{-2}(\eta) \Rightarrow$ constant “electric” field

NB:

$$\boxed{E_i = -\frac{f(\eta)}{a^2(\eta)} \mathcal{A}'_i}.$$

Statistical anisotropy from inflation

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_\zeta(k) \left(1 + g_*(\hat{\mathbf{E}} \cdot \hat{\mathbf{k}})^2\right)$$

$$g_* = -\frac{24}{\epsilon} \cdot \frac{\mathbf{E}^2}{V(\phi)} \cdot N_{min}^2$$

$\mathbf{E} = \mathbf{E}_{class} + \mathbf{E}_{IR}$ \mathbf{E}_{IR} : originates from quantum fluctuations.

Infrared modes of \mathbf{E}_{IR} which exit horizon before $N_{min} = 60$ behave as an additional classical background [Bartolo et al.'12](#)

Naturally one expects very large \mathbf{E}_{IR} . Indeed,

$$\langle \mathbf{E}_{IR}^2 \rangle = \frac{9H^4}{2\pi^2} \int_{-1/\tau_{in}}^{\mathcal{H}_0} \frac{dp}{p} = \frac{9H^4}{2\pi^2} (N_{tot} - N_{min}) \quad N_{tot} \gg N_{min} .$$

Constraints on $N_{tot} - N_{min}$

WMAP 9 (V band)

$$N_{tot} - N_{min} < 82 \quad [14] \quad 95\% \text{CL} \quad [68\% \text{CL}]$$

Planck (143 GHz \times 217 GHz)

$$N_{tot} - N_{min} < 35 \quad [6] \quad 95\% \text{CL} \quad [68\% \text{CL}]$$

Even stronger constraints are possible by making use of unbiased estimators.

$$N_{tot} - N_{min} \lesssim \mathcal{O}(1).$$

Much less tuning is expected in multi-vector models of inflation
(tomorrow's talk of F. Urban).

Alternatives to inflation \implies

Statistical anisotropy can be generated automatically.

- No tunings in parameter space.
- More diverse predictions about statistical anisotropy.

Promising “smoking gun” for alternatives.

NB: Primordial gravitational waves —“anti-smoking gun” (remember about BICEP2).

(Pseudo)conformal Universe Rubakov'09 Creminelli *et al.*'10 Hinterbichler and Khoury'11

(Pseudo)conformal Universe

- Cosmological evolution is described by nearly Minkowski metric.
- Universe is in CFT state at very early times (before the hot Big Bang).
- There are at least two scalars: χ with $\Delta \neq 0$ and θ with $\Delta = 0$.
- $SO(4|2) \rightarrow SO(4|1)$ by the time-dependent solution of field χ

Hinterbichler and Khoury'11

Scale-invariant spectrum of field θ perturbations.

If field χ dominates \Rightarrow

slow expansion/contraction with super-stiff e.o.s.

$|\mathcal{P}| \gg \rho \Rightarrow$

horizon and flatness problems are resolved.

- Conformal rolling scenario Rubakov'09

$$S_\phi = \int d^4x \sqrt{-g} (g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi^* + h^2 |\phi|^4) \quad \phi = \chi e^{i\theta/\sqrt{2}}$$

$$g_{\mu\nu} \approx \eta_{\mu\nu}$$

$$\boxed{\chi_0 = \frac{1}{h(t_* - t)}}$$

Analogy: $a(\eta) = \frac{1}{H(-\eta)}$

Perturbations θ feel like in de Sitter space-time:

$$\boxed{\mathcal{P}_\theta = \frac{h^2}{(2\pi)^2}}.$$

- Galilean Genesis Creminelli et al.'10

$$S_\pi \propto \left[-f^2 e^{2\pi} (\partial\pi)^2 + \frac{f^3}{\Lambda_G^3} (\partial\pi)^2 \square\pi + \frac{f^3}{2\Lambda_G^3} (\partial\pi)^4 + \frac{1}{2} f^2 e^{2\pi} (\partial\theta)^2 \right]$$

Analogy:

$$\chi \leftrightarrow fe^\pi$$

$$h^2 \leftrightarrow \frac{2}{3} \frac{\Lambda_G^3}{f^3}$$

Statistical anisotropy

Interaction of $\delta\theta$ with IR perturbations of the field χ sources SA.

$$\chi_0 \rightarrow \chi(t, \mathbf{x}) = \frac{1}{h(t_*(\mathbf{x}) - t)} \quad \langle \delta t_*(\mathbf{x})^2 \rangle \propto h^2 \int \frac{dp}{p^3}$$

Relevant:

$$v_i = -\partial_i t_* \text{ flat}$$

$$\partial_i \partial_j t_* \text{ blue}.$$

Case 1. Evolution of perturbations $\delta\theta$ stops after conformal rolling

$$\frac{k}{a(t_*)} < H(t_*)$$

Libanov and Rubakov'10, Creminelli et al.'12

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_\zeta(k) \left(1 - \frac{\pi}{k} \cdot \partial_i \partial_j t_* \hat{k}_i \hat{k}_j + g_*(\hat{\mathbf{v}} \hat{\mathbf{k}})^2 \right) \quad g_* = -\frac{3}{2} v^2$$

New: general quadrupole (with decreasing amplitude, however).

The field v_i is analogous to “electric” field in case of inflation:
Gaussian, zero mean, flat spectrum $g_* < 0$

$$v_i^2 = \frac{3h^2}{8\pi^2} \ln \frac{H_0}{\Lambda} \quad h^2 \ln \frac{H_0}{\Lambda} \leftrightarrow \frac{512\pi^2}{3} \mathcal{P}_\zeta N_{min}^2 (N_{tot} - N_{min})$$

WMAP 9 S. R. and G. Rubtsov'13

$$h^2 \ln \frac{H_0}{\Lambda} < 0.2 \quad 68\% \text{CL}$$

$$h^2 \ln \frac{H_0}{\Lambda} < 1.2 \quad 95\% \text{CL}$$

Planck Rubtsov et al.

$$h^2 \ln \frac{H_0}{\Lambda} < 0.09 \quad 68\% \text{CL}$$

$$h^2 \ln \frac{H_0}{\Lambda} < 0.5 \quad 95\% \text{CL}$$

Case 2. Non-trivial evolution of perturbations $\delta\theta$ after conformal rolling

and before hot Big Bang

$$\frac{k}{a(t_*)} > H(t_*)$$

Libanov et al.'11.

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_\zeta(k) \left(1 + \sum_{LM} q_{LM} Y_{LM}(\hat{\mathbf{k}}) \right)$$

$$\langle q_{LM} q_{L'M'}^* \rangle = \frac{3}{\pi} \cdot \frac{h^2}{(L-1)(L+2)} \delta_{LL'} \delta_{MM'} \quad \text{even } L$$

WMAP 9 S. R. and G. Rubtsov'13

$$h^2 < 0.006 \quad \text{at} \quad 95\% \text{CL}$$

Planck G. Rubtsov et al.

$$h^2 < 0.0013 \quad \text{at} \quad 95\% \text{CL}$$

Conclusions

- Non-observation of statistical anisotropy in WMAP9 and Planck \implies strong constraints on parameters of statistical anisotropy.
- At 95% CL $|g_*| < 0.072$ (WMAP9, V) and $|g_*| < 0.026$ (Planck, 143×217 GHz).
- Statistical anisotropy implies tuned duration of inflation in models with single Maxwellian term coupled to inflaton.
- Pseudo(conformal) Universe: strong limits in scenarios with long intermediate stage.

Thanks!