Effect of intermediate Minkowskian evolution on CMB bispectrum

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Suzdal, June, 7th, 2014

based on arXiv:1312.7808 (S.M., S.Ramazanov, V.Rubakov)

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- every model has specific features (tilt, statistical anisotropy, non-gaussianity)

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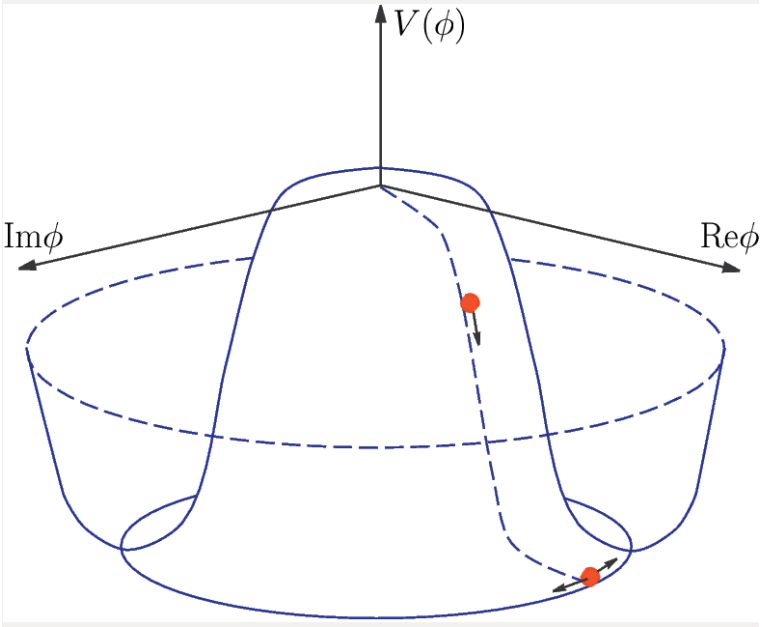

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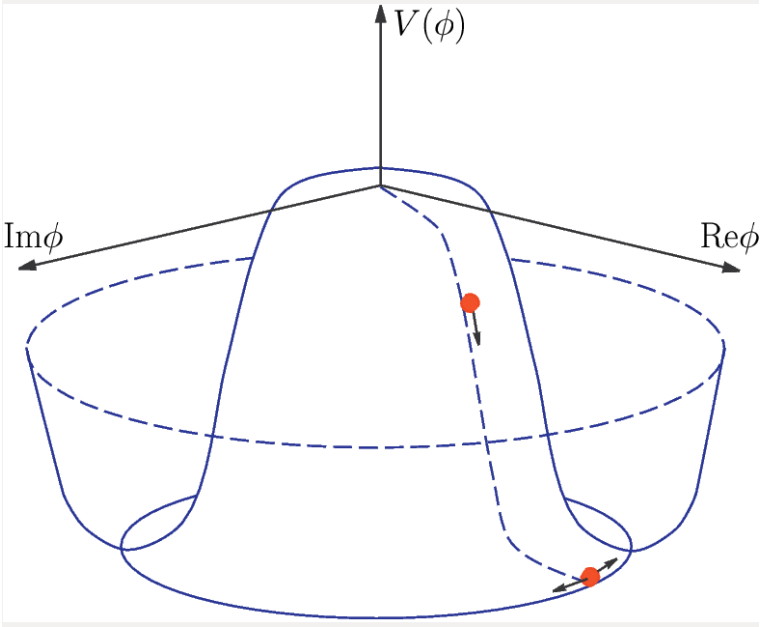

by the end of the rolling, perturbations of interest already have flat spectrum (and special form of non-Gaussianity)

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- we consider the second situation

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let us present a correlator in terms of the primordial Newtonian potential

$$\langle\Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\rangle =$$
$$= \left(\frac{\pi\mathcal{P}_\Phi}{2}\right)^{3/2} A(k_1,k_2,k_3)\delta\left(\sum_i \mathbf{k}_i\right)\cos(k_1\eta_*)\cos(k_2\eta_*)\cos(k_3\eta_*)$$

$A(k_1,k_2,k_3)$ is initial shape function

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here $\Delta_l(k\eta_0)$ are standard CMB transfer functions,

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let us denote: $\mathbf{y}_i \equiv \mathbf{k}_i\eta_0$, $z \equiv -\frac{\eta_*}{\eta_0}$, $A(y_1, y_2, y_3) \equiv \frac{A(k_1,k_2,k_3)}{\eta_0^6}$

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  we assume the ordering $l_1 \leq l_2 \leq l_3$ in what follows

$$\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = i^{l_1+l_2+l_3} \mathcal{P}_\Phi^{3/2} \int d\mathbf{y}_1 d\mathbf{y}_2 \Delta_{l_1}(y_1) \Delta_{l_2}(y_2) \Delta_{l_3}(|\mathbf{y}_1+\mathbf{y}_2|) \times$$

$$\times Y^*_{l_1 m_1}(\theta_1, \phi_1) Y^*_{l_2 m_2}(\theta_2, \phi_2) Y^*_{l_3 m_3}(\theta_3, \phi_3) \times$$

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- second one is more general: we use approximation for spherical functions

$$Y_{lm}(\theta, \phi) = \frac{1}{\pi \sqrt{\sin\theta}} \cos\left[\left(l + \frac{1}{2}\right)\theta - \frac{\pi}{4} + \frac{\pi m}{2}\right] e^{im\phi} + \mathcal{O}\left(\frac{1}{l}\right)$$

In general case the answer is

$$\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = i^{l_1+l_2+l_3} \cdot \frac{\pi \mathcal{P}_\Phi^{3/2}}{4} \cdot B^{l_1 m_1}_{l_2 m_2; l_3 m_3} \times$$

$$\times \int dy_1 dy_2 \cdot y_1^2 \cdot y_2^2 \cdot \Delta_{l_1}(y_1) \Delta_{l_2}(y_2) \Delta_{l_3}(y_1+y_2) \cdot A(y_1, y_2, y_1+y_2) \times$$

$$\times \frac{2(y_1+y_2)}{z y_1 y_2} \cdot \sin\left\{\frac{1}{2} \cdot \frac{y_2^2 l_1^2 + y_1^2 l_2^2}{y_1 y_2 (y_1+y_2) z}\right\} \cdot \cos\left\{\frac{l_1 l_2}{(y_1+y_2) z}\right\} \times$$

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$$+ (l_1, m_1 \leftrightarrow l_3, m_3) + (l_2, m_2 \leftrightarrow l_3, m_3)$$

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let us proceed to the qualitative results

bispectrum is suppresed by a duration of intermediate stage: $\eta_0^2/|\eta_*|^2$ or $\eta_0/|\eta_*|$, depending on a regime

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the fact, that amplitude is approximately constant is quite non-trivial and important for discriminating the models we discussed in this paper from inflationary scenarios, some of which predict the oscillatory behaviour of the bispectrum analogous to ours, but strongly peaked in flattened triangle limit.

THANK YOU FOR YOUR ATTENTION!