

On the effect of surface pion condensate on the spectrum of neutron stars

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- **Introduction.**
- **Effective model.**
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Introduction

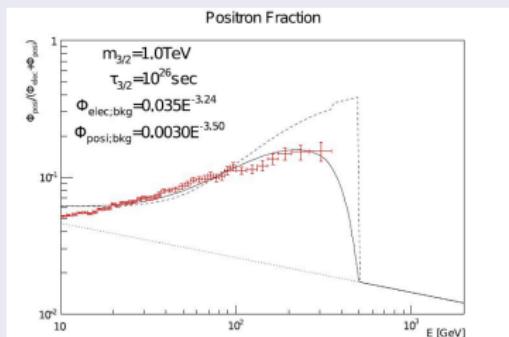
Dark Energy

Fermi-LAT, PAMELA, AMS-2:
anomalous excess of e^+e^- .

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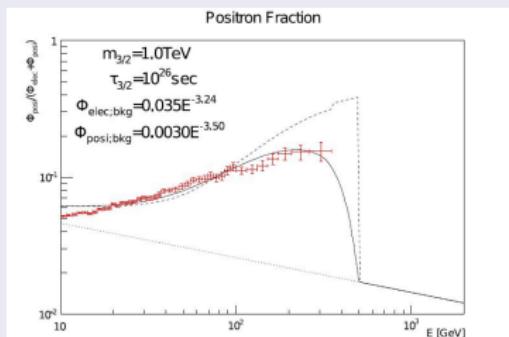


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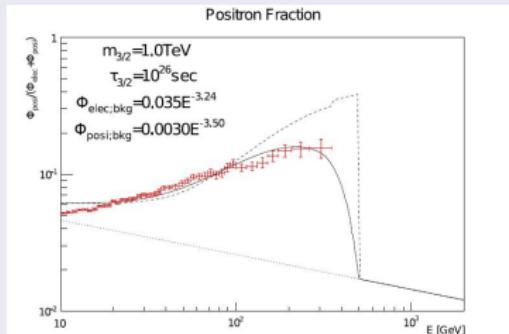
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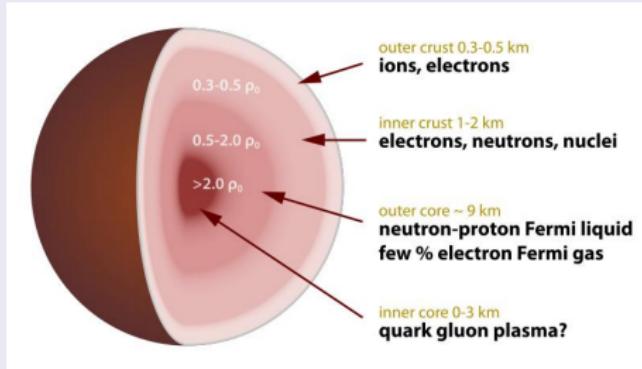
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Heavy ion physics

NA60, PHENIX: abnormal yield of lepton pairs (e, μ)

Neutron stars

Structure of a neutron star



Formation standing wave of σ and π^0 fields: axial wave condensation inside neutron stars.

(Koichi Takahashi, 2002)

Density of this condensate decreases along the radius and disappears outside a core of the star.

- In such assumption there will be a layer inside the star with broken parity in this region, which modify a spectrum of the star.

Model

- We discuss the X-ray region of neutron star radiation.
- Neutron star with condensed pions around the outer core
- We consider the small area near the point, where photon is leaving the parity-breaking medium

Effective Lagrangian (Andrianov, S.K., Soldati arXiv:1109.3440)

$$\mathcal{L} = -\frac{1}{4} F^{\alpha\beta}(x) F_{\alpha\beta}(x) - \frac{1}{4} F^{\mu\nu}(x) \tilde{F}_{\mu\nu}(x) a_{c\ell}(x) \quad (1)$$

A_μ and $a_{c\ell}$: the vector and effective background pseudoscalar fields,

$$a_{c\ell}(x) = \zeta x_1 \theta(-x_1) \quad (2)$$

The corresponding field equations are,

$$\square A^\nu + m^2 A^\nu + \zeta \varepsilon^{1\nu\sigma\rho} \theta(-x_1) \partial_\sigma A_\rho = 0 \quad (3)$$

Construction of the chiral polarization vectors $a_{c\ell}(x) = \zeta_\lambda x^\lambda \theta(-\zeta \cdot x)$

$$S_\lambda^\nu = \delta_\lambda^\nu D + k^\nu k_\lambda \zeta^2 + \zeta^\nu \zeta_\lambda k^2 - \zeta \cdot k (\zeta_\lambda k^\nu + \zeta^\nu k_\lambda); \quad D \equiv (\zeta \cdot k)^2 - \zeta^2 k^2$$

Transversal polarizations are,

$$\pi_\pm^{\mu\nu} \equiv \frac{S^{\mu\nu}}{2D} \pm \frac{i}{2} \varepsilon^{\mu\nu\alpha\beta} \zeta_\alpha k_\beta D^{-\frac{1}{2}}; \quad \varepsilon_\pm^\mu(k) = \pi_\pm^{\mu\lambda} \epsilon_\lambda^{(0)}$$

Scalar and longitudinal polarizations,

$$\varepsilon_S^\mu(k) \equiv \frac{k^\mu}{\sqrt{k^2}}, \quad \varepsilon_L^\mu(k) \equiv (D k^2)^{-\frac{1}{2}} (k^2 \zeta^\mu - k^\mu \zeta \cdot k)$$

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Spatial CS vector. $\zeta_\mu = (0, -\zeta_x, 0, 0)$: dispersion laws

$$\begin{cases} k_{1L} = k_{10} = \sqrt{\omega^2 - m^2 - k_\perp^2} \\ k_{1\pm} = \sqrt{\omega^2 - m^2 - k_\perp^2 \mp \zeta_x \sqrt{\omega^2 - k_\perp^2}} \end{cases}$$

$$\hat{k} = (\omega, k_2, k_3), \hat{x} = (x_0, x_2, x_3) : \hat{k} \cdot \hat{x} = -\omega x_0 + k_2 x_2 + k_3 x_3.$$

Proca-Stückelberg solution

$$A_{\text{PS}}^\mu(x) = \int d\hat{k} \theta(\omega^2 - k_\perp^2 - m^2) \sum_{r=1}^3 \left[\mathbf{a}_{\hat{k}, r} u_{\hat{k}, r}^\mu(x) + \mathbf{a}_{\hat{k}, r}^\dagger u_{\hat{k}, r}^{\mu*}(x) \right]$$

$$u_{\hat{k}, r}^\nu(x) = [(2\pi)^3 2k_{10}]^{-1/2} e_r^\nu(\hat{k}) \exp\{i k_{10} x_1 + i \hat{k} \cdot \hat{x}\} \quad (r = 1, 2, 3)$$

Chern-Simons solution

$$A_{\text{CS}}^\nu(x) = \int d\hat{k} \sum_{A=\pm, L} \theta(k_{1A}^2(\omega, k_\perp)) \left[c_{\hat{k}, A} v_{\hat{k} A}^\nu(x) + c_{\hat{k}, A}^\dagger v_{\hat{k} A}^{\nu*}(x) \right]$$

$$v_{\hat{k} A}^\nu(x) = [(2\pi)^3 2k_{1A}]^{-\frac{1}{2}} \varepsilon_A^\nu(k) \exp\{ik_{1A} x_1 + i \hat{k} \cdot \hat{x}\} \quad (A = L, \pm)$$

$$[A_{\text{PS}}^\mu(x) - A_{\text{CS}}^\mu(x)]|_{\zeta \cdot x = 0} = 0$$

Bogolubov Transformations

$$v_{\hat{k},A}^{\nu}(\hat{x}) = \sum_{s=1}^3 \left[\alpha_{sA}(\hat{k}) u_{\hat{k},s}^{\nu}(\hat{x}) - \beta_{sA}(\hat{k}) u_{\hat{k},s}^{\nu*}(\hat{x}) \right]$$

relations between the creation-destruction operators are,

$$\mathbf{a}_{\hat{k},r} = \sum_{A=\pm,L} \left[\alpha_{rA}(\hat{k}) c_{\hat{k},A} - \beta_{rA}^*(\hat{k}) c_{\hat{k},A}^\dagger \right]$$

$$c_{\hat{k},A} = \sum_{r=1}^3 \left[\alpha_{Ar}^*(\hat{k}) \mathbf{a}_{\hat{k},r} + \beta_{Ar}^*(\hat{k}) \mathbf{a}_{\hat{k},r}^\dagger \right]$$

There are two different Fock vacua,

$$\mathbf{a}_{\hat{k},r} |0\rangle = 0 \quad c_{\hat{k},A} | \Omega \rangle = 0$$

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$$\langle 0 | \mathbf{a}_{\hat{p},s} c_{\hat{k},A}^\dagger | 0 \rangle = \delta(\hat{k} - \hat{p}) \alpha_{As}(\hat{k})$$

The latter quantity can be interpreted as the relative probability amplitude that particle is transmitted from the left face to the right face.

Vacuum as a squeezed state

$$|0\rangle_{\hat{k}} = \sum_{p,m,l=0}^{\infty} f_{pml} \frac{(c_{\hat{k},+}^\dagger)^p (c_{\hat{k},-}^\dagger)^m (c_{\hat{k},L}^\dagger)^l}{\sqrt{p!m!l!}} |\Omega\rangle_{\hat{k}}$$

To find f_{pml} we use the equality $a_{\hat{k},r}|0\rangle_{\hat{k}} = 0$.

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To find f_{pml} we use the equality $\mathbf{a}_{\hat{k},r} |0\rangle_{\hat{k}} = 0$.

$$|0\rangle_{\hat{k}} = \exp \left[\frac{\beta_{r+}^*(\hat{k})}{2\alpha_{r+}(\hat{k})} (c_{\hat{k},+}^\dagger)^2 + \frac{\beta_{r-}^*(\hat{k})}{2\alpha_{r-}(\hat{k})} (c_{\hat{k},-}^\dagger)^2 + \frac{\beta_{rL}^*(\hat{k})}{2\alpha_{rL}(\hat{k})} (c_{\hat{k},L}^\dagger)^2 \right] |\Omega\rangle_{\hat{k}}$$

$$|\Omega\rangle_{\hat{k}} = \exp \left[\frac{-\beta_{A1}^*(\hat{k})}{2\alpha_{A1}^*(\hat{k})} (\mathbf{a}_{\hat{k},1}^\dagger)^2 + \frac{-\beta_{A2}^*(\hat{k})}{2\alpha_{A2}^*(\hat{k})} (\mathbf{a}_{\hat{k},2}^\dagger)^2 + \frac{-\beta_{A3}^*(\hat{k})}{2\alpha_{A3}^*(\hat{k})} (\mathbf{a}_{\hat{k},3}^\dagger)^2 \right] |0\rangle_{\hat{k}}$$

Vacuum as a squeezed state

In the correct normalization, $\langle 0|0 \rangle = 1$, $\langle \Omega|\Omega \rangle = 1$.
Going to the continuum limit for \hat{k} ,

$$|0\rangle = \exp \left[\int \left(\sum_{A=\pm,L} \frac{\beta_{rA}^*(\hat{k})}{2\alpha_{rA}(\hat{k})} (c_{\hat{k},A}^\dagger)^2 \theta(k_{1A}^2(\hat{k})) \right) d\hat{k} \right] |\Omega\rangle$$

$$|\Omega\rangle = \exp \left[\int \theta(\omega^2 - m^2 - k_\perp^2) \left(\sum_{r=1,2,3} \frac{-\beta_{Ar}^*(\hat{k})}{2\alpha_{Ar}^*(\hat{k})} (\mathbf{a}_{\hat{k},r}^\dagger)^2 \right) d\hat{k} \right] |0\rangle$$

Classical solutions

$$\zeta_\mu = (0, -\zeta, 0, 0)$$

$$\square A^\nu + m^2 A^\nu + \zeta \varepsilon^{1\nu\sigma\rho} \theta(-x_1) \partial_\sigma A_\rho = 0$$

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A_1 may be found in the whole space,

$$A_1 = \int \frac{d\hat{k}}{(2\pi)^3} (\tilde{u}_{1\rightarrow}(\omega, k_2, k_3) e^{ik_{10}x_1} + \tilde{u}_{1\leftarrow}(\omega, k_2, k_3) e^{-ik_{10}x_1}) e^{i\hat{k}\hat{x}}$$

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Solution for A_ν ($\nu = 0, 2, 3$)

$$A_\nu = \int \frac{d\hat{k}}{(2\pi)^3} \tilde{A}_\nu e^{i\hat{k}\hat{x}}$$

$$\tilde{A}_\nu = \begin{cases} \tilde{u}_{\nu\rightarrow}(\omega, k_2, k_3) e^{ik_{10}x_1} + \tilde{u}_{\nu\leftarrow}(\omega, k_2, k_3) e^{-ik_{10}x_1}, & x_1 > 0 \\ \sum_A [\tilde{v}_{\nu A\rightarrow}(\omega, k_2, k_3) e^{ik_{1A}x_1} + \tilde{v}_{\nu A\leftarrow}(\omega, k_2, k_3) e^{-ik_{1A}x_1}], & x_1 < 0 \end{cases}$$

Matching conditions

$$\tilde{u}_{\nu \rightarrow}^{(A)} = \frac{1}{2} (\tilde{v}_{\nu A \rightarrow} \left(\frac{k_{1A} + k_{10}}{k_{10}} \right) - \tilde{v}_{\nu A \leftarrow} \left(\frac{k_{1A} - k_{10}}{k_{10}} \right))$$

$$\tilde{u}_{\nu \leftarrow}^{(A)} = \frac{1}{2} (-\tilde{v}_{\nu A \rightarrow} \left(\frac{k_{1A} - k_{10}}{k_{10}} \right) + \tilde{v}_{\nu A \leftarrow} \left(\frac{k_{1A} + k_{10}}{k_{10}} \right))$$

Escaping from the parity-breaking medium

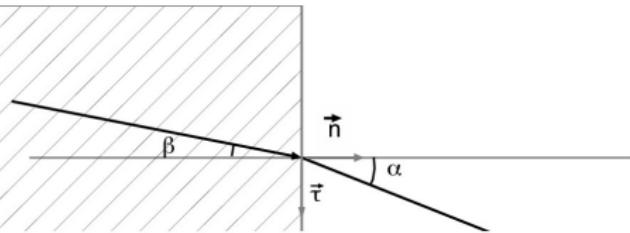
Using the relations obtained before, it is possible to find, which part is reflected

$$\tilde{v}_{\nu \pm \leftarrow} = \frac{k_{1\pm} - k_{10}}{k_{1\pm} + k_{10}} \tilde{v}_{\nu \pm \rightarrow}$$

and which pass through the boundary,

$$\tilde{u}_{\nu \rightarrow}^{(\pm)} = \frac{2k_{1\pm}}{k_{10} + k_{1\pm}} \tilde{v}_{\nu \pm \rightarrow}$$

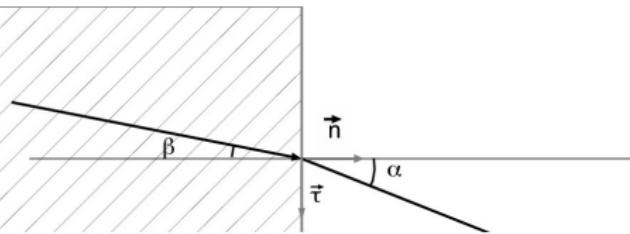
Neutron star. Photon escaping



- We are interested only in one direction of outgoing photons (angle α)
- $\vec{k} = k_n \vec{n} + k_{\perp} \vec{r}$
- $\frac{k_n}{k_{\perp}} = \cot(\alpha); \frac{k_n^{CS}}{k_{\perp}} = \cot(\beta)$

$$k_{n\pm}^{CS} = \sqrt{\omega^2 - k_{\perp}^2 \mp \zeta \sqrt{\omega^2 - k_{\perp}^2}}; k_n = \sqrt{\omega^2 - k_{\perp}^2} \quad \Rightarrow k_{n\pm}^{CS} = \sqrt{k_n^2 \mp \zeta k_n}$$

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We consider the photons on the mass shell, so in vacuum $\omega = |\vec{k}|$, and one can use $k_n = \omega \cos \alpha$. Now it is easy to write our transmission coefficient as a function of angle,

$$k_{tr}^{\pm} = \frac{2k_{n\pm}^{CS}}{k_{n\pm}^{CS} + k_n} = \frac{2\sqrt{\omega \cos \alpha \mp \zeta}}{\sqrt{\omega \cos \alpha \mp \zeta} + \sqrt{\omega \cos \alpha}} \quad (4)$$

Neutron star. Photon escaping

Since we can only see a neutron star from a very large distance, it is natural to consider only parallel rays, coming from this object.

$$N_-(\omega, \zeta) \sim \int_0^{\frac{\pi}{2}} 2\pi R^2 d\alpha \sin \alpha \frac{2\sqrt{\omega \cos \alpha + \zeta}}{\sqrt{\omega \cos \alpha + \zeta} + \sqrt{\omega \cos \alpha}} \quad (5)$$

$$N_+(\omega, \zeta) \sim \int_0^{\arccos \frac{\zeta}{\omega}} 2\pi R^2 d\alpha \sin \alpha \frac{2\sqrt{\omega \cos \alpha - \zeta}}{\sqrt{\omega \cos \alpha - \zeta} + \sqrt{\omega \cos \alpha}} \quad (6)$$

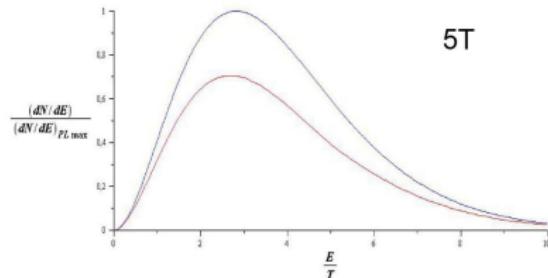
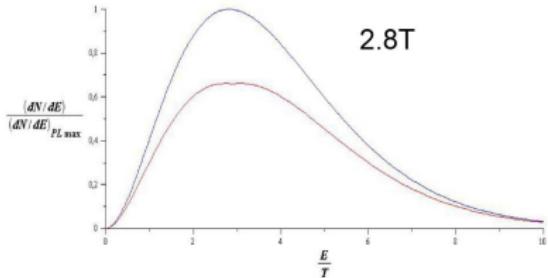
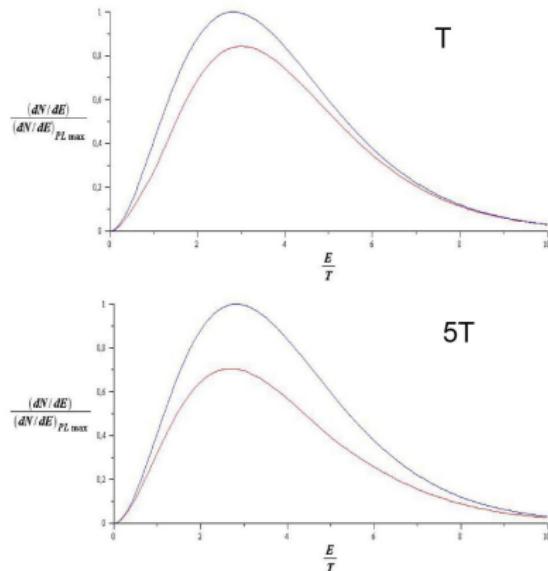
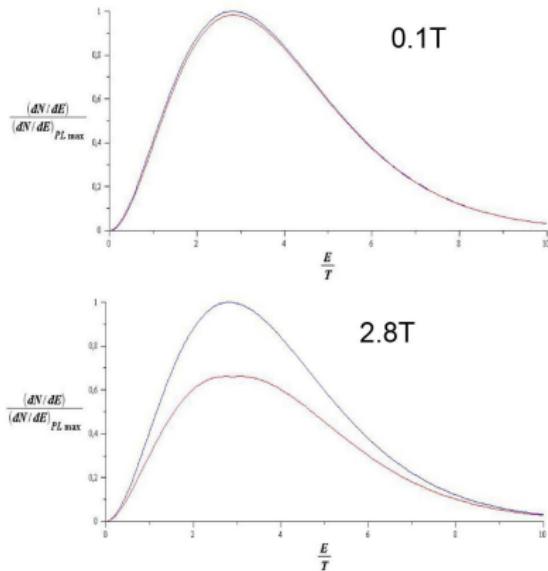
The second expression starts to contribute only when $\frac{\zeta}{\omega} < 1$.

$$k_{n+}^{CS} = \sqrt{\omega \cos \alpha (\omega \cos \alpha - \zeta)} \quad \cos \alpha > \frac{\zeta}{\omega}$$

In case of positive polarization at some value of angle α it is kinematically forbidden for photon to leave the parity odd medium.

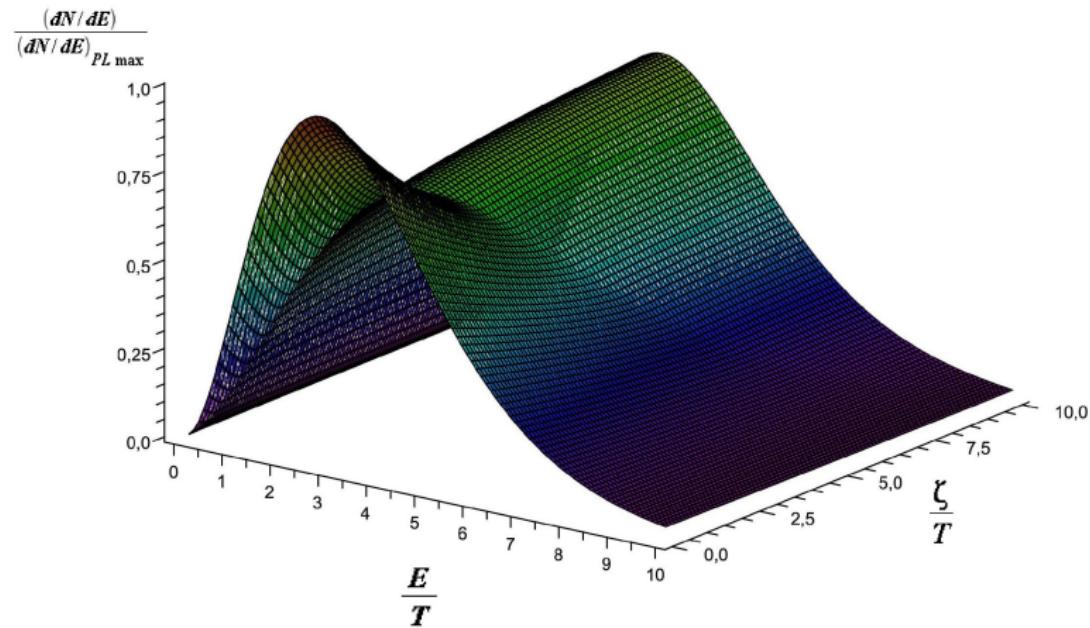
Spectrum

We present here the influence of a described pion condensate on the thermal radiation with effective temperature T and spectral radiance $B(\omega, T) \sim \frac{\omega^3}{e^{\frac{\omega}{T}} - 1}$.



Spectrum

We may also plot a three-dimensional picture, where ζ will be a changing parameter. After some value ($\zeta \approx 2.8 T$) with increasing ζ an effect decreases.



Now let's recall the graph, where $\zeta = 2.8 T$, in this plot we saw a plateau which is actually a very interesting region. We show this region separately,

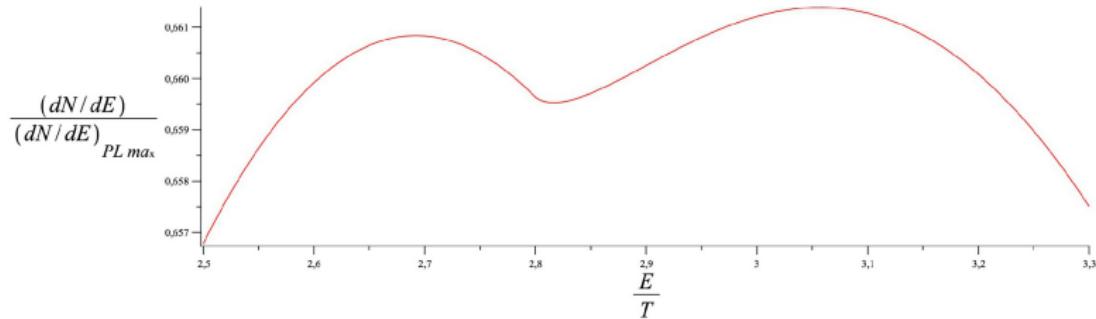


Figure: $\zeta = 2.8 T$

There are two maximums of the spectrum. They arise due to the second transversal polarization which begin to contribute at the value of $\zeta = 2.8 T$, when ζ is near the maximum of the spectrum.

Experiments

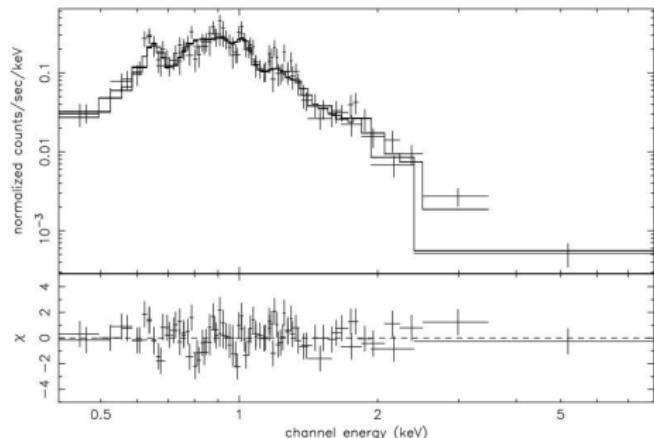


Fig. 9. Energy spectrum of the rim emission underneath RX J0822–4300 as observed in April 2001 with the EPIC-MOS1/2 detector and simultaneously fitted to an absorbed non-equilibrium ionization collisional plasma model (upper panel) and contribution to the χ^2 fit statistic (lower panel).

Bartlett et al. (arXiv:1309.2658)

Typical effective temperature of a neutron star is $T \sim \text{keV}$, so the boundary effects arises for $\zeta \sim O(\text{keV})$.

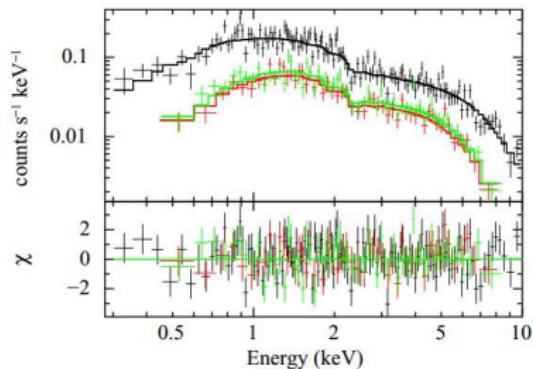


Figure 5. The 0.2–10.0 keV EPIC-pn (black), EPIC-MOS1 (red), EPIC-MOS2 (green) spectra of Swift J045106.8-694803. Top panel displays the background subtracted spectrum with best fit $\text{phabs}*\text{vphabs}(\text{powerlaw}+\text{bbbody})$ model, bottom panel shows the residuals.

Hui et al. (arXiv:0508655)

Conclusion

- Results:

- * The expression that relates two different physical vacua was found. It was shown that each of this vacua can be presented as a squeezed state in terms of another one;
- * We have discussed the simplest model, which give an idea how may the spectrum of neutron star change;
- * We have shown, that even in this model there is a significant (observable) effect.

- X-ray neutron star physics, including *Chandra* and *XMM-Newton*.

However, an accuracy of modern experiments does not allow us to say confidently, is there any anomalies in spectrum or the strange regions are just fluctuations and instruments' errors.

Of course, discussing model is not precise, one should take into account lots of processes taking place in neutron stars to make a theoretical prediction of the spectrum of these objects. Moreover, one should pay attention to the pion decay into photons, that will also change final spectrum.



