Scaling violation in logarithmic dimensions in massless scalar quantum field theories.

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Renormalization group equation:

$$\left(-p\frac{\partial}{\partial p}+\beta(g)\frac{\partial}{\partial g}+2\gamma(g)-2\right)D(p,g)=0$$

p is a monentum, g is a renormalized coupling constant (or its function), $\beta(g)$ is the beta function, $\gamma(g)$ is the anomalous dimension of field, D is a propagator.
Dimensionless variables:

$$s\equiv\frac{p}{\mu},\Phi\equiv\mu^2 D,$$

where μ is a parameter of renormalization with the dimension of mass.

$$\left(-s\frac{\partial}{\partial s}+\beta(g)\frac{\partial}{\partial g}+2\gamma(g)-2\right)\Phi(s,g)=0$$

A solution of the renormalization group equation:

$$\Phi(s,g) = \frac{1}{s^2}\Phi(1,\overline{g}(s,g))\exp\left(2\int\limits_{g}^{\overline{g}(s,g)}\frac{\gamma(x)}{\beta(x)}dx\right),$$

where $\overline{g}(s,g)$ is the invariant charge which is defined implicitly by the equation:

$$\ln s = \int\limits_{g}^{\overline{g}}\frac{dx}{\beta(x)}.$$

To find $\Phi(1,g)$ we have to solve the Dyson equation:

$$D^{-1}(p,g) = \Delta^{-1}(p) - \Sigma(p,g),$$

where $\Delta(p)$ is the bar propagator, $\Sigma(p,g)$ is the self-energy operator.

Within the minimal subtractions (MS) scheme it holds:

$$\Delta(p) = \frac{1}{p^2}$$

We introduce another dimensionless variable:

$$\Xi \equiv \mu^{-2}\Sigma$$

$$\Phi^{-1}(s,g) = s^2 - \Xi(s,g)$$

$$\Phi(1,g) = \frac{1}{1-\Xi(1,g)}.$$

$$D(p,g) = \frac{1}{p^2}\frac{1}{1-\Xi(1,\overline{g}(s,g))}\exp\left(2\int\limits_{g}^{\overline{g}(s,g)}\frac{\gamma(x)}{\beta(x)}dx\right)$$

To find $\Xi(1,g)$, we have to calculate some of the Feynman diagrams.

It is convinient to introduce the functions $\rho(g)$ and $V(g)$, such as:

$$\rho'(g) = \frac{1}{\beta(g)}$$

$$V'(g) = \frac{\gamma(g)}{\beta(g)}$$

Then

$$\ln s = \rho(\overline{g}) - \rho(g)$$

Or

$$\rho(\overline{g}) = \ln(e^{\rho(g)} s) \equiv \ln s_1$$

$$s_1 \equiv e^{\rho(g)} s$$

$$D(p, g) = e^{-2V(g)} \frac{1}{p^2} \frac{1}{1 - \Xi(1, \overline{g}(s_1))} e^{2V(\overline{g}(s_1))}$$

The asymptotic behavior.

$$\ln s = \int_{g}^{\overline{g}} \frac{dx}{\beta(x)}.$$

$$\overline{g}(1, g) = g$$

$\overline{g}(s, g) \to g_*$, when $\ln s \to \pm\infty$.
g_* is a zero of the beta function: $\beta(g_*) = 0$.
In the logarithmic dimension it holds: $\beta(g) = b_2 g^2 + ...$, and $g_* = 0$. That is, $\overline{g} \to 0$.
A type of asymptotic:
In the main approximation we have:

$$\ln s = -\frac{1}{b_2 \overline{g}} + ...$$

If $b_2 g > 0$, then IR.
If $b_2 g < 0$, then UV.

Corrections to the main approximation.

$$\beta(g) = b_2 g^2 + b_3 g^3 + b_4 g^4 + \ldots$$

$$\overline{g}(s,g) = -\frac{1}{b_2 \ln s}\left[1 + \frac{b_3}{b_2^2}\frac{\ln|\ln s|}{\ln s} - \rho(g)\frac{1}{\ln s} + \frac{b_3^2}{b_2^4}\frac{(\ln|\ln s|)^2}{(\ln s)^2} - \right.$$
$$- \left(\frac{b_3^2}{b_2^4} + \frac{2b_3}{b_2^2}\rho(g)\right)\frac{\ln|\ln s|}{(\ln s)^2} +$$
$$\left. + \left(\frac{b_2 b_4 - b_3^2}{b_2^4} + \frac{b_3}{b_2^2}\rho(g) + \rho(g)^2\right)\frac{1}{(\ln s)^2} + \ldots\right],$$

$$\overline{g}(s_1) = -\frac{1}{b_2 \ln s_1}\left[1 + \frac{b_3}{b_2^2}\frac{\ln|\ln s_1|}{\ln s_1} + \frac{b_3^2}{b_2^4}\frac{(\ln|\ln s_1|)^2}{(\ln s_1)^2} - \frac{b_3^2}{b_2^4}\frac{\ln|\ln s_1|}{(\ln s_1)^2} + \right.$$
$$\left. + \frac{b_2 b_4 - b_3^2}{b_2^4}\frac{1}{(\ln s_1)^2} + \ldots\right],$$

where $s_1 = e^{\rho(g)} s$,

$\rho(g)$:

$$\begin{cases} \rho'(g) = \frac{1}{\beta(g)} \\ \lim\limits_{g \to 0} \left(\rho(g) + \frac{1}{b_2 g} + \frac{b_3}{b_2^2} \ln |b_2 g| \right) = 0 \end{cases}$$

$$\rho(g) = -\frac{1}{b_2 g} - \frac{b_3}{b_2^2} \ln |b_2 g| + \frac{b_3^2 - b_2 b_4}{b_2^3} g + \dots$$

The asymptotic behavior of propagator.

$$D(p,g) = e^{-2V(g)} \frac{1}{p^2} \frac{1}{1 - \Xi(1, \overline{g}(s_1))} e^{2V(\overline{g}(s_1))}.$$

Suppose, we know the following approximation for the functions $\beta(g)$, $\gamma(g)$ and $\Xi(1,g)$:

$$\beta(g) = b_2 g^2 + b_3 g^3 + b_4 g^4 + \ldots$$
$$\gamma(g) = c_1 g + c_2 g^2 + c_3 g^3 + \ldots$$
$$\Xi(1,g) = a_1 g + a_2 g^2 + \ldots$$

The function $V(g)$ is uniquely determined by 2 conditions:

$$\begin{cases} V'(g) = \frac{\gamma(g)}{\beta(g)} \\ \lim\limits_{g\to 0} \left(V(g) - \frac{c_1}{b_2} \ln|b_2 g| \right) = 0 \end{cases}$$

$$V(g) = \frac{c_1}{b_2} \ln|b_2 g| + \frac{b_2 c_2 - b_3 c_1}{b_2^2} g +$$
$$+ \frac{b_3^2 c_1 - b_2 b_4 c_1 - b_2 b_3 c_2 + b_2^2 c_3}{2 b_2^3} g^2 + \dots$$

The expression for the propagator in terms of the invariant charge:

$$D(p,g) = e^{-2V(g)} \frac{1}{p^2} |b_2 \overline{g}|^{2c_1/b_2} \left[1 + \frac{-2b_3 c_1 + 2b_2 c_2 + b_2^2 a_1}{b_2^2} \overline{g} + \right.$$
$$+ \frac{1}{b_2^4} (b_2 b_3^2 c_1 - b_2^2 b_4 c_1 + 2 b_3^2 c_1^2 - b_2^2 b_3 c_2 - 4 b_2 b_3 c_1 c_2 + 2 b_2^2 c_2^2 +$$
$$\left. + b_2^3 c_3 - 2 b_2^2 b_3 c_1 a_1 + 2 b_2^3 c_2 a_1 + b_2^4 a_2 + b_2^4 a_1^2) \overline{g}^2 + \dots \right]$$

The propagator in terms of the monentum:

$$D(p,g) = e^{-2V(g)}\frac{1}{p^2}|\ln s_1|^{-2c_1/b_2}\left[1 + \frac{2b_3c_1}{b_2^3}\frac{\ln|\ln s_1|}{\ln s_1} + \right.$$

$$+ \frac{2b_3c_1 - b_2(2c_2 + b_2a_1)}{b_2^3}\frac{1}{\ln s_1} +$$

$$+ \frac{b_3^2c_1(b_2 + 2c_1)}{b_2^6}\frac{(\ln|\ln s_1|)^2}{(\ln s_1)^2} +$$

$$+ \frac{b_3(4b_3c_1^2 - b_2(b_2 + 2c_1)(2c_2 + b_2a_1))}{b_2^6}\frac{\ln|\ln s_1|}{(\ln s_1)^2} +$$

$$+ \frac{1}{b_2^6}[2b_3^2c_1^2 - b_2b_3c_1(b_3 + 4c_2) + b_2^3(c_3 + 2c_2a_1) +$$

$$\left. + b_2^2(b_4c_1 + 2c_2^2 - b_3(c_2 + 2c_1a_1)) + b_2^4a_2 + b_2^4a_1^2]\,\frac{1}{(\ln s_1)^2} + \ldots\right]$$

We can try to simplify more the expression by the transformation $s_2 = e^A s_1$.

$$D(p,g) = e^{-2V(g)} \frac{1}{p^2} |\ln s_2|^{-2c_1/b_2} \Bigg[1 + \frac{2b_3 c_1}{b_2^3} \frac{\ln|\ln s_2|}{\ln s_2} +$$

$$+ \left(\frac{2b_3 c_1 - b_2(2c_2 + b_2 a_1)}{b_2^3} - \frac{2c_1}{b_2} A \right) \frac{1}{\ln s_2} +$$

$$+ \frac{b_3^2 c_1 (b_2 + 2c_1)}{b_2^6} \frac{(\ln|\ln s_2|)^2}{(\ln s_2)^2} +$$

$$\left(\frac{b_3(4b_3 c_1^2 - b_2(b_2 + 2c_1)(2c_2 + b_2 a_1))}{b_2^6} - \frac{2b_2 b_3 c_1 + 4b_3 c_1^2}{b_2^4} A \right) \frac{\ln|\ln s_2|}{(\ln s_2)^2} \cdot$$

$$+ \left(\frac{1}{b_2^6} [2b_3^2 c_1^2 - b_2 b_3 c_1 (b_3 + 4c_2) + b_2^3 (c_3 + 2c_2 a_1) + \right.$$

$$+ b_2^2 (b_4 c_1 + 2c_2^2 - b_3(c_2 + 2c_1 a_1)) + b_2^4 a_2 + b_2^4 a_1^2] +$$

$$\left. \frac{a_1 b_2^3 + 2a_1 b_2^2 c_1 - 4b_3 c_1^2 + 2b_2^2 c_2 + 4b_2 c_1 c_2}{b_2^4} A + \frac{b_2 c_1 + 2c_1^2}{b_2^2} A^2 \right) \frac{1}{(\ln s_2)^2}$$

If $c_1 \neq 0$, then we take $A = \left(\frac{2b_3c_1 - b_2(2c_2 + b_2a_1)}{b_2^3}\right)\frac{b_2}{2c_1}$.
And we receive:

$$D(p,g) = e^{-2V(g)}\frac{1}{p^2}|\ln s_2|^{-2c_1/b_2}\left[1 + \frac{2b_3c_1}{b_2^3}\frac{\ln|\ln s_2|}{\ln s_2} + \right.$$
$$+ \frac{b_3^2c_1(b_2+2c_1)}{b_2^6}\frac{(\ln|\ln s_2|)^2}{(\ln s_2)^2} - \frac{2b_3^2c_1}{b_2^5}\frac{\ln|\ln s_2|}{(\ln s_2)^2} -$$
$$- \frac{1}{4b_2^4c_1}[a_1^2b_2^2(b_2 - 2c_1) + 4b_2^2(a_1c_2 - a_2c_1) - 4b_4c_1^2 +$$
$$\left. + 4b_3c_1c_2 + 4b_2(c_2^2 - c_1c_3)]\frac{1}{(\ln s_2)^2} + \ldots\right]$$

If $c_1 = 0$, then we have:

$$D(p,g) = e^{-2V(g)}\frac{1}{p^2}\left[1 - \frac{2c_2 + b_2 a_1}{b_2^2}\frac{1}{\ln s_2} - \right.$$
$$- \left(\frac{b_3(2c_2 + b_2 a_1)}{b_2^4}\right)\frac{\ln|\ln s_2|}{(\ln s_2)^2} +$$
$$+ \left(\frac{a_1^2 b_2^2 + a_2 b_2^2 + 2a_1 b_2 c_2 - b_3 c_2 + 2c_2^2 + b_2 c_3}{b_2^4} +\right.$$
$$\left.\left. + \ \frac{a_1 b_2 + 2c_2}{b_2^2}A\right)\frac{1}{(\ln s_2)^2} + \ldots\right]$$

and we choose $A = -\frac{a_1^2 b_2^2 + a_2 b_2^2 + 2a_1 b_2 c_2 - b_3 c_2 + 2c_2^2 + b_2 c_3}{b_2^2(a_1 b_2 + 2c_2)}$.

$$D(p,g) = e^{-2V(g)}\frac{1}{p^2}\left[1 - \frac{2c_2 + b_2 a_1}{b_2^2}\frac{1}{\ln s_2} - \right.$$
$$\left. - \left(\frac{b_3(2c_2 + b_2 a_1)}{b_2^4}\right)\frac{\ln|\ln s_2|}{(\ln s_2)^2} + \ldots\right]$$

The ϕ^3 theory in the Euclidian space.

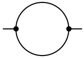
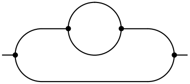
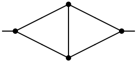
$$L = \frac{1}{2}(\partial\phi)^2 + \frac{\lambda}{3!}\phi^3$$

ϕ is a scalar field, λ is a coupling constant.
$d = 6$ (the logarithmic dimension for this theory).

$$\beta(g) = -\frac{3}{2}g^2 + -\frac{125}{72}g^3 - \left(\frac{33085}{10368} + \frac{5\zeta(3)}{4}\right)g^4 + ...$$
$$\gamma(g) = \frac{1}{12}g + \frac{13}{432}g^2 + \left(\frac{5195}{62208} - \frac{\zeta(3)}{24}\right)g^3 + ...,$$

where $\zeta(z)$ is the Riemann's zeta function, $g = \frac{\lambda^2}{64\pi^3}$.
$b_2 < 0$. If λ is real then $g > 0$, $b_2 g < 0$ and we get the ultraviolet asymptotic. But one usually takes λ to be imaginary that it holds $g < 0$ and we obtain the infrared asymptotic.

To find the desired accuracy for the propagator, we have to compute three Feynman diagrams:

$\Sigma = \frac{1}{2}$  $+\frac{1}{2}$  $+\frac{1}{2}$  $+ \ldots$

Line is the bar propagator $\frac{1}{p^2}$, verticle is the coupling constant λ. All these diagrams diverge in the logarithmic dimension ($d = 6$), and we make the dimensional regularization ($d = 6 - 2\varepsilon$) then the R-operation. The result is the fillowing:

$$\Sigma(p, g) = \Big[-\frac{8 + 3(\tau - 2\ln s)}{36} g - \\ - \frac{1789 + 1116(\tau - 2\ln s) + 180(\tau - 2\ln s)^2}{5184} g^2 + \ldots \Big] p^2,$$

and

$$\Xi(1, g) = -\frac{8 + 3\tau}{36} g - \frac{1789 + 1116\tau + 180\tau^2}{5184} g^2 + \ldots,$$

where $g = \frac{\lambda^2}{64\pi^3}$, $\tau = \ln 4\pi - \gamma_E$, and γ_E is the Euler's constant.

The infrared asymptotic of the propagator:

$$\Phi(p,g) = e^{-2V(g)}\frac{1}{p^2}|\ln s_2|^{1/9}\left[1 + \frac{125}{1458}\frac{\ln|\ln s_2|}{\ln s_2} - \right.$$
$$- \frac{15625}{531441}\frac{(\ln|\ln s_2|)^2}{(\ln s_2)^2} + \frac{15625}{236196}\frac{\ln|\ln s_2|}{(\ln s_2)^2} -$$
$$\left. - \left(\frac{11291 - 30132\tau + 1296\zeta(3)}{157464}\right)\frac{1}{(\ln s_2)^2} + \ldots\right]$$

where

$$s_2 = \exp\left(-\frac{65}{81} - \frac{\tau}{2}\right)\cdot s_1 = \exp\left(\rho(g) - \frac{65}{81} - \frac{\tau}{2}\right)\cdot s,$$

$$\rho(g) = \frac{2}{3g} + \frac{125}{162}\ln\left(\frac{3}{2}|g|\right) + \left(\frac{36755}{69984} + \frac{5\zeta(3)}{9}\right)g + \ldots,$$
$$V(g) = -\frac{1}{18}\ln\left(\frac{3}{2}|g|\right) + \frac{43}{972}g + \left(\frac{2375}{419904} + \frac{\zeta(3)}{27}\right)g^2 + \ldots$$

The O(N)-symmetric ϕ^4 theory in the Euclidian space.

$$L = \frac{1}{2}(\partial\phi)^2 + \frac{\lambda}{4!}(\phi^2)^2$$

ϕ is an N-componental field, λ is a coupling constant, $\lambda > 0$.
The logarithmic dimension for this theory is $d = 4$.
The beta function and the anomalous dimension of field have been computed up to 5 loops:

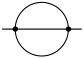
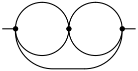
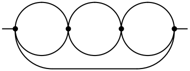
$$
\begin{aligned}
\beta(g) = {} & \frac{N+8}{3}g^2 - \frac{3N+14}{3}g^3 + \\
& + \frac{33N^2 + 922N + 2960 + 96\zeta(3)(5N+22)}{216}g^4 - \\
& - \frac{1}{3888}[-5N^3 + 6320N^2 + 80456N + 196648 + \\
& + 96\zeta(3)(63N^2 + 764N + 2332) - 288\zeta(4)(5N^2 + 62N + 176) + \\
& + 1920\zeta(5)(2N^2 + 55N + 186)]g^5 + \\
& + \frac{1}{62208}[13N^4 + 12578N^3 + 808496N^2 + 6646336N + 13177344 + \\
& + 16\zeta(3)(-9N^4 + 1248N^3 + 67640N^2 + 552280N + 1314336) - \\
& - 288\zeta(4)(63N^3 + 1388N^2 + 9532N + 21120) + \\
& + 256\zeta(5)(305N^3 + 7466N^2 + 66986N + 165084) - \\
& - 9600\zeta(6)(N+8)(2N^2 + 55N + 186) + \\
& + 112896\zeta(7)(14N^2 + 189N + 526) + \\
& + 768\zeta(3)^2(-6N^3 - 59N^2 + 446N + 3264)]g^6 + \ldots,
\end{aligned}
$$

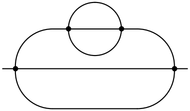
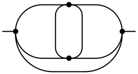
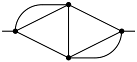
$$
\begin{aligned}
\gamma(g) = {} & \frac{N+2}{36}g^2 - \frac{(N+2)(N+8)}{432}g^3 + \\
& + \frac{5(N+2)(-N^2+18N+100)}{5184}g^4 - \\
& - \frac{N+2}{186624}[39N^3 + 296N^2 + 22752N + 77056 - \\
& - 48\zeta(3)(N^3 - 6N^2 + 64N + 184) + 1152\zeta(4)(5N+22)]g^5 + \ldots,
\end{aligned}
$$

where $g = \frac{\lambda}{16\pi^2} > 0$.

We have $b_2 > 0$ and $g > 0$, we therefore get the infrared asymptotic.

We need the self-energy operator ut to 4 loops:

$\Sigma = \frac{1}{6}$  $+\frac{1}{4}$  $+\frac{1}{8}$  $+$

$+\frac{1}{12}$  $+\frac{1}{4}$  $+\frac{1}{4}$  $+\ldots$

The result is the following:

$$
\begin{aligned}
\Xi(1,g) = &-\frac{N+2}{144}(13+4\tau)g^2- \\
&-\frac{(N+2)(N+8)}{2592}(167+84\tau+12\tau^2)g^3- \\
&-\frac{N+2}{41472}[1851N^2+41467N+174518+16(N^2-14N-68)\zeta(3)+ \\
&+24(54N^2+1081N+4466)\tau+ \\
&+16(21N^2+373N+1514)\tau^2+32(N+8)^2\tau^3]g^4+...,
\end{aligned}
$$

where $\tau = \ln 4\pi - \gamma_E$, $g = \frac{\lambda}{16\pi^2}$

The infrared asymptotic of the propagator.

$$
\begin{aligned}
D(p,g) = {} & e^{-2V(g)}\frac{1}{p^2}\Bigg[1-\frac{N+2}{2(N+8)^2}\frac{1}{\ln s_2}+\frac{3(N+2)(3N+14)}{2(N+8)^4}\frac{\ln|\ln s_2|}{(\ln s_2)^2}- \\
& -\frac{9(N+2)(3N+14)^2}{2(N+8)^6}\frac{(\ln|\ln s_2|)^2}{(\ln s_2)^3}+\frac{9(N+2)(3N+14)^2}{2(N+8)^6}\frac{\ln|\ln s_2|}{(\ln s_2)^3}+ \\
& +\frac{(N+2)}{48(N+8)^6}[319N^4+9942N^3+116469N^2+607364N+1204452+ \\
& +168(N+8)^4\tau+24(N+8)^4\tau^2-384\zeta(3)(N+8)(5N+22)]\frac{1}{(\ln s_2)^3}+ \\
& +\frac{27(N+2)(3N+14)^3}{2(N+8)^8}\frac{(\ln|\ln s_1|)^3}{(\ln s_1)^4}-\frac{135(N+2)(3N+14)^3}{4(N+8)^8}\frac{(\ln|\ln s_2|)^2}{(\ln s_2)^4}- \\
& -\frac{3(N+2)(3N+14)}{32(N+8)^8}[319N^4+9942N^3+115173N^2+595268N+ \\
& 1176228+168(N+8)^4\tau+ \\
& +24(N+8)^4\tau^2-384\zeta(3)(N+8)(5N+22)]\frac{\ln|\ln s_2|}{(\ln s_2)^4}- \\
& -\frac{N+2}{384(N+8)^8}[7068N^6+322295N^5+6183232N^4+63882945N^3+ \\
& 374808430N^2+1182947372N+1567304328+ \\
& +96\zeta(3)(N+8)(25N^3+1096N^2+9052N+21984)+ \\
& 1920\zeta(5)(N+8)^2(2N^2+55N+186)+ \\
& +12(N+8)^4(461N^2+6606N+25948)\tau+ \\
& +24(N+8)^4(63N^2+953N+3778)\tau^2+144(N+8)^6\tau^3]\,\frac{1}{(\ln s_2)^4}+\dots\Bigg]
\end{aligned}
$$

The ϕ^6 theory.

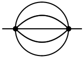
$$L = \frac{1}{2}(\partial\phi)^2 + \frac{\lambda}{6!}\phi^6$$

ϕ is a scalar field, λ is a coupling constant, $\lambda > 0$.
The logarithmic dimension is $d = 3$.

$$\beta(g) = \frac{20}{3}g^2 - \left(\frac{1124}{15} + \frac{15\pi^2}{2}\right)g^3 + ...,$$
$$\gamma(g) = \frac{1}{90}g^2 - \frac{2}{81}g^3 + ...,$$

Where $g = \frac{\lambda}{64\pi^2} > 0$.
Using the renormalization group equation we get the IR-asymptotic.

We need only 1 diagram for Σ:

$$\Sigma = \frac{1}{120}$$  $+ \ldots$

We obtain:

$$\Xi(1, g) = -\frac{13 + 3\tau}{270} g^2 + \ldots$$

where $\tau = \ln 4\pi - \gamma_E$.
The result for the propagator:

$$D(p, g) = e^{-2V(g)} \frac{1}{p^2} \left(1 - \frac{1}{2000} \frac{1}{\ln s_2} + \frac{3(2248 + 225\pi^2)}{8000000} \frac{\ln |\ln s_2|}{(\ln s_2)^2} + \ldots \right)$$

Conclusion.

Using the renormalization group equation, we have calculated the infrared asymptotic of the propagator for the theories ϕ^3, ϕ^4 and ϕ^6. The equation includes a beta function and an anomalous dimension of field. An analysis has shown that these data are not enough to find the asymptotic behavior of the propagator. We need also to know a self-energy operator as a function of a coupling constant with a fixed value of momentum. To find this function it required to sum Feynman diagrams.
For the ϕ^3 theory the propagatir in the main approximation is power with logarithm (scaling is violated), and in the theories ϕ^4 and ϕ^6 in the main approximation scaling is not violated. Corrections in all these cases are expressed in terms of the logarithm and the logarithm logarithm of the momentum. There are universal terms, which are not changed with the scale transformation $p \to \alpha p$, and non-universal, which are changed.
Triviality. The ϕ^4 theory with arbitrary N is not trivial (corrections include logarithms), but in the limit $N \to \infty$ it becomes trivial. It is also interesting that the triviality appears with $N = -2$.

Thank you for your attention!!!