Localization on a thick brane in the presence of defect

Oleg O. Novikov

in collaboration with A.A.Andrianov and V.A.Andrianov

V.A.Fock Department of Theoretical Physics
Saint-Petersburg State University

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Introduction

Brane world models - universe is a 4D (mem)brane in a higher dimensional spacetime (bulk). Extra dimensions may be large or even infinite

Popular scenarios of brane world: ADD, RSI, RSII
Ignited by discovery of extended objects in string theory

However much earlier - 1980s - V.Rubakov, M.Shaposhnikov; K.Akama
"Thick branes" - the brane is a domain wall produced by QFT in the bulk

Some fields develop nonzero expectation values forming topologically nontrivial vacuum configuration, e.g. kink

$$\Phi \simeq M \tanh \beta y,$$

With appropriate interaction light states may be localized near the jump

Localization of various fields

- Scalar fields can form localized vacuum configurations
  Light fluctuations can form localized states
- Fermions - Yukawa interaction $\bar{\psi}\Phi\psi$
  Localized chiral fermions as zero-modes
- Gauge fields - problems appear
  Localized zero-mode - problems with charge universality
  Making it delocalized can lead to unremovable infrared divergencies
  (discovered by M.Smolyakov, see also work by D.Kirpichnikov)
- Let us include gravity in the model $\Rightarrow$ 5D diffeomorfism invariance
  In invariant sector mixing of the scalars and graviscalars happen
  This may change drastically the spectrum of light scalar states
  (e.g. M.Giovannini, 2001, 2003; A.Andrianov, L.Vecchi, 2007)

Formulation of the model

Consider 5D spacetime

$$X^A = (x^\mu, y), x^\mu = (x^0, x^1, x^2, x^3), \eta^{AB} = \mathrm{diag}(+,-,-,-,-)$$

with two real scalar fields Φ и H and a defect modelled by **rigid** thin brane

$$\begin{aligned} S[g,\Phi,H] &= \int d^5X\sqrt{|g|}\left(-\frac{1}{2}M_*^3R + \mathcal{L}_{mat}(g,\Phi,H)\right) \\ &\quad -3M_*^3\lambda_b\int\limits_{y=0} d^4X\sqrt{\left|{}^{(4)}g\right|} + S_{GH}, \\ \mathcal{L}_{mat} &= Z\left(\frac{1}{2}(\partial_A\Phi\partial^A\Phi + \partial_A H\partial^A H) - V(\Phi,H)\right), \end{aligned}$$

where S_{GH} – Gibbons-Hawking-York compensating term.

Minimal model

We consider as example the following minimal model,

$$
\begin{aligned}
S[g,\Phi,H] &= \int d^5X\sqrt{|g|}\left(-\frac{1}{2}M_*^3R+\mathcal{L}_{mat}(g,\Phi,H)\right)\\
&\quad -3M_*^3\lambda_b\int\limits_{y=0} d^4X\sqrt{\left|{}^{(4)}g\right|}+S_{GH},\\
\mathcal{L}_{mat} &= \frac{3\kappa M_*^3}{2M^2}\Big(\partial_A\Phi\partial^A\Phi+\partial_AH\partial^AH+2M^2\Phi^2+2\Delta_HH^2\\
&\quad -(\Phi^2+H^2)^2-M^4\Big)+M_*^3\Lambda_c,
\end{aligned}
$$

where $\kappa\sim M^3/M_*^3\ll 1$ is a small parameter characterizing the strength of gravity. From now on $M^2>\Delta_H$.

Background solutions

$\Phi = \Phi(y), H = H(y)$ - we limit ourselves to the solution conserving 4D Lorentz invariance
We select the following ansatz for background metrics.

$$g_{AB}dx^A dx^B = e^{-2\rho(y)}dx^\mu dx^\nu - dy^2$$

Equation of motion for classical background solutions taking into account matching at $y = 0$

$$\rho'' = \frac{Z}{3M_*^3}(\Phi'^2 + H'^2) + 2\lambda_b\delta(y),$$
$$\frac{2Z}{3M_*^3}V(\Phi, H) = \rho'' - 4(\rho')^2 - 2\lambda_b\delta(y),$$
$$\Phi'' - 4\rho'\Phi' = \frac{\partial V}{\partial \Phi}, \quad H'' - 4\rho' H' = \frac{\partial V}{\partial H}.$$

Only three are independent (one equation fixes the 5D cosmological constant). Smooth limit $\kappa, \lambda_b \to 0$

The solutions of minimal model for small κ, λ_b

First phase

$$\Phi = M \tanh My + O(\kappa), H = 0$$

Second phase (critical point at $\mu = 0$)

$$\Phi = M \tanh \tau + O(\kappa), H = \frac{\mu}{\cosh \tau} + O(\kappa),$$

Metric factor in the leading order,

$$\rho = \frac{2\kappa}{3} \ln \cosh \tau + \frac{\kappa}{6} \tanh^2 \tau + O(\kappa^2)$$

$$\tau \equiv M\beta y, \quad \beta = \sqrt{1 - \frac{\mu^2}{M^2}} + O(\kappa), \quad \Delta_H = \frac{M^2 + \mu^2}{2} + O(\kappa)$$

If we suppose to use H as mass-generating field we are interested in the second phase

Perturbation theory for background fields

Treat κ, λ_b and μ/M as perturbation parameters.

$$\Phi(\tau) = M \sum_{l,m,n=0}^{\infty} \kappa^l \left(\frac{\lambda_b}{M}\right)^m \left(\frac{\mu}{M}\right)^{2n} \Phi_{l,m,n}(\tau), \qquad \Phi_{n,0,0} \equiv \Phi_n,$$

$$H(\tau) = M \sum_{l,m,n=0}^{\infty} \kappa^l \left(\frac{\lambda_b}{M}\right)^m \left(\frac{\mu}{M}\right)^{2n+1} H_{l,m,n}(\tau), \quad H_{n,0,0} \equiv H_n,$$

$$\rho(\tau) = \kappa \sum_{n,m=0}^{\infty} \kappa^n \left(\frac{\mu}{M}\right)^{2m} \rho_{n+1,m}(\tau), \qquad \rho_{n,0} \equiv \rho_n,$$

To simplify equations introduce $\tau = \beta y$ and consider parameters as series

$$\Delta_H = \Delta_{H,c}(\kappa) + \frac{1}{2}\mu^2, \Delta_{H,c}(\kappa) = \frac{1}{2}M^2 \sum_{m,n=0}^{\infty} \kappa^m \left(\frac{\lambda_b}{M}\right)^n \Delta_H^{m,n},$$

$$\frac{1}{\beta^2} = \frac{1}{M^2} \sum_{l,m,n=0}^{\infty} \kappa^l \left(\frac{\lambda_b}{M}\right)^m \left(\frac{\mu}{M}\right)^{2n} \left(\frac{1}{\beta^2}\right)_{l,m,n};$$

Some results of the perturbation theory

$$\Phi_{1,0,0} = -\frac{2}{9}\frac{\sinh\tau}{\cosh^3\tau},\ H_{1,0,0} = \frac{2}{27\cosh\tau}\left(C^H_{1,0,0} - 2\log\cosh\tau + 3\tanh^2\tau\right),$$

$$\Phi_{0,1,0}|_{y>0} = -\frac{1}{9}\frac{\tanh\tau}{\cosh^2\tau}\Big(9\cosh^2\tau + 6\cosh^4\tau - 6\sinh\tau\cosh\tau^3$$
$$-12\cosh\tau\sinh\tau\Big),$$
$$H_{0,1,0}|_{y>0} = \frac{1}{3\cosh\tau}\Big[5(\cosh^2\tau - \cosh\tau\sinh\tau) + \tau + 2\mathrm{Li}_2(-e^{-2\tau})$$
$$+C^H_{0,1,0} + 4(\ln\cosh\tau - \tau)\cosh\tau\sinh\tau + 4(\cosh^2\tau - \tau)\ln 2 + 4\tanh\tau\Big]$$

$$\left(\frac{1}{\beta^2}\right)_{1,0,0} = \frac{4}{3},\ \Delta_H^{1,0} = -\frac{44}{27},\ \Delta_H^{0,1} = -\frac{4}{3}(1+2\ln 2),\ \left(\frac{1}{\beta^2}\right)_{0,1,0} = 2,$$

Small fluctuations near classical solutions

Consider the fluctuations of metrics h_{AB} and scalar fields ϕ, χ,

$$g_{\mu\nu}(X) = e^{-2\rho}(\eta_{\mu\nu} + h_{\mu\nu}(X)), g_{5\nu} = e^{-2\rho} v_\mu(X), g_{55} = -1 + S(X)$$

$$\Phi(X) = \Phi(y) + \phi(X), H(X) = H(y) + \chi(X),$$

Let us separate longitudinal and transverse components,

$$h_{\mu\nu} = b_{\mu\nu} + \partial_\mu F_\nu + \partial_\nu F_\mu + \partial_\mu\partial_\nu E + \eta_{\mu\nu}\psi, \qquad v_\mu = v_\mu^\perp + \partial_\mu \eta,$$

$$\partial^\mu b_{\mu\nu} = 0, \qquad \partial_\mu F^\mu = 0, \qquad \partial^\mu v_\mu^\perp = 0$$

Then in the quadratic order different spin components decouple. We are interested in the scalar sector

Gauge invariance

Infinitesimal diffeomorfisms induce gauge symmetry

$$\eta \to \eta - \zeta_5 - C', \quad E \to E - 2C, \quad \psi \to \psi + 2\rho'\zeta_5, \quad S \to S - 2\zeta_5',$$

$$\phi \to \phi + \Phi'\zeta_5, \chi \to \chi + H'\zeta_5$$

To study invariant spectrum we use gauge invariant variables

$$\check{\eta} = E' - 2\eta + \frac{1}{\rho'}\psi, \quad \check{S} = S - \frac{1}{\rho'}\psi' + \frac{\rho''}{(\rho')^2}\psi$$
$$\check{\phi} = \phi + \frac{\Phi'}{2\rho'}\psi, \quad \check{\chi} = \chi + \frac{H'}{2\rho'}\psi$$

$\check{\eta}$ is a lagrangian multiplier for the constraint $\rho'\check{S} = \frac{2Z}{3M_*^3}(\Phi'\check{\phi} + H'\check{\chi})$
In the end there are only two independent variables $\check{\phi}$ and $\check{\chi}$

Fluctuation equations

One can obtain the following spectral equation,

$$(-\partial_y^2 + \partial^2 V + \hat{\mathcal{M}}_{NP} - 2\rho'' + 4(\rho')^2) \begin{pmatrix} \phi^{(m)} \\ \chi^{(m)} \end{pmatrix} = e^{2\rho} m^2 \begin{pmatrix} \phi^{(m)} \\ \chi^{(m)} \end{pmatrix}$$

$$\hat{\mathcal{M}}_{NP} = \frac{2Z}{3M_*^3} \left[\frac{\rho'' + 4(\rho')^2}{(\rho')^2} \begin{pmatrix} (\Phi')^2 & \Phi' H' \\ \Phi' H' & (H')^2 \end{pmatrix} - \frac{1}{\rho'} \partial_y \begin{pmatrix} (\Phi')^2 & \Phi' H' \\ \Phi' H' & (H')^2 \end{pmatrix} \right],$$

that should be completed by the matching conditions at $y = 0$,

$$\left[\partial_y \phi^{(m)}\right]_\pm = -\frac{2Z}{3M_*^3} \frac{\Phi'|_{y=0}}{\rho'|_{0+}} \left(\Phi' \phi^{(m)} + H' \chi^{(m)}\right)|_{y=0} - 4\rho'|_{0+} \phi^{(m)},$$

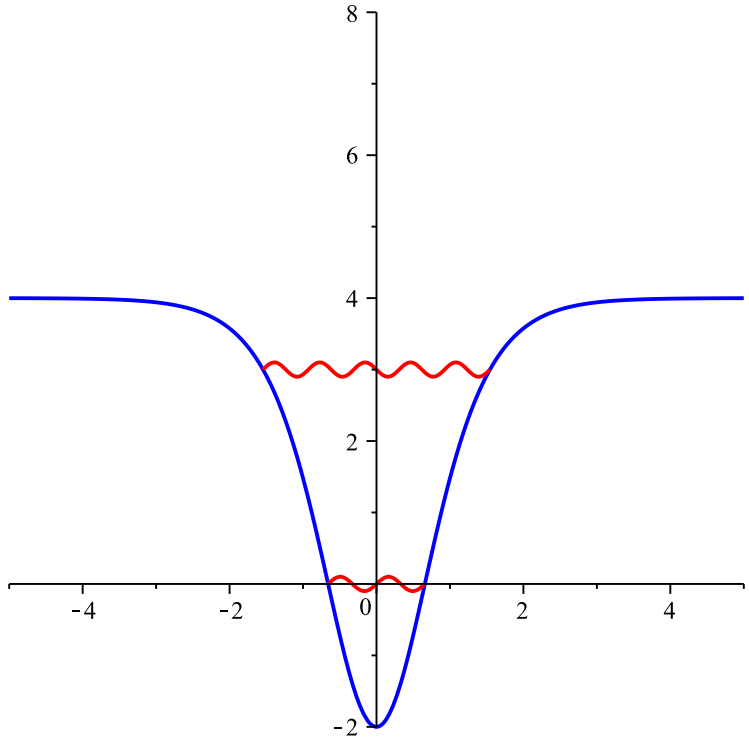
$$\left[\partial_y \chi^{(m)}\right]_\pm = -\frac{2Z}{3M_*^3} \frac{H'|_{y=0}}{\rho'|_{0+}} \left(\Phi' \phi^{(m)} + H' \chi^{(m)}\right)_{y=0} - 4\rho'|_{0+} \chi^{(m)}$$

Generally speaking the terms $\sim \frac{Z}{M_*^3} \frac{1}{\rho'}$ do not vanish in the limit $\kappa, \lambda_b \to 0$

ϕ-channel without gravity

If $\langle H \rangle = 0$ the channels ϕ and χ decouple
If there is no gravity and defect there are two localized states

$$\phi^{(0)} = \Phi' = \frac{1}{\cosh^2 \tau}, \qquad \phi^{\sqrt{3}M} = \frac{\tanh \tau}{\cosh \tau}$$



ϕ-channel with gravity

Using EoM for background solutions the ϕ-channel equation can be rewritten in the factorized form,

$$\left(-\partial_\tau + \frac{\rho''}{\rho'} - \frac{\Phi''}{\Phi'} + 2\rho'\right)\left(\partial_\tau + \frac{\rho''}{\rho'} - \frac{\Phi''}{\Phi'} + 2\rho'\right)\phi^{(m)} = \left(\frac{m^2}{M^2\beta^2}\right)e^{2\rho}\phi^{(m)}$$

This potential may have singularities if $\rho' = 0$
The matching condition

$$\left[\partial_\tau \phi^{(m)}\right]_\pm = -\left(2\kappa\frac{(\Phi'|_{\tau=0})^2}{\rho'|_{0+}} + 4\rho'|_{0+}\right)\phi^{(m)}$$

is equivalent to addition of δ-function to the potential

ϕ-channel solutions outside the brane

In the limit $\kappa \to 0$, $\lambda_b \simeq \kappa M \beta b$, $\rho \simeq \kappa \tilde{\rho}_1 + \kappa b \tau$

$$Q_b Q_b^\dagger \phi^{(m)} = \left(\frac{m^2}{M^2 \beta^2}\right) \phi^{(m)}, \quad Q_b = -\partial_\tau + \frac{\tilde{\rho}_1''}{\tilde{\rho}_1' + b} - \frac{\Phi''}{\Phi'}$$

The factorization connects this potential with exactly solvable superpotential not depending on b

$$Q_b^\dagger Q_b = \tilde{Q} \tilde{Q}^\dagger + 3, \quad \tilde{Q} = -\partial_\tau + \tanh \tau, \quad \tilde{Q}^\dagger \tilde{Q} = -\partial_\tau^2 + 1$$

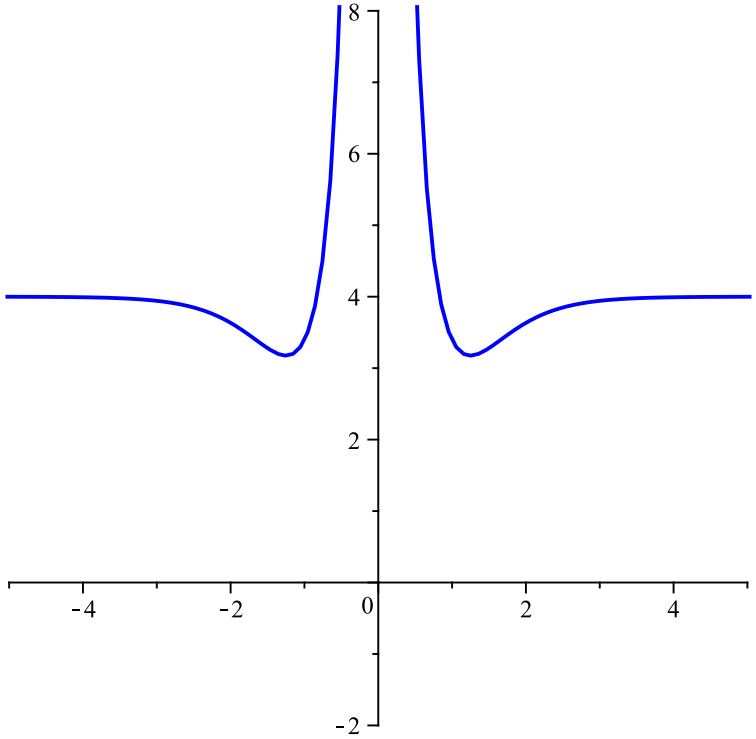
$$\phi^{(m)} = Q_b \tilde{Q} \sin k\tau, \quad \tilde{\phi}^{(m)} = Q_b \tilde{Q} \cos k\tau, \quad m^2 = (4 + k^2) M^2$$

$$\phi_b^{(\sqrt{3}M)} = Q_b \frac{1}{\cosh \tau}, \quad \tilde{\phi}_b^{(\sqrt{3}M)} = Q_b \left(\sinh \tau + \frac{x}{\cosh \tau}\right)$$

$$\phi^{(0)} = \frac{\Phi'}{\tilde{\rho}' + b}, \quad \tilde{\phi}^{(0)} = \phi^{(0)} \int^\tau d\tau' \frac{1}{(\phi^{(0)}(\tau'))^2}$$

ϕ-channel in the model with gravity, no defect

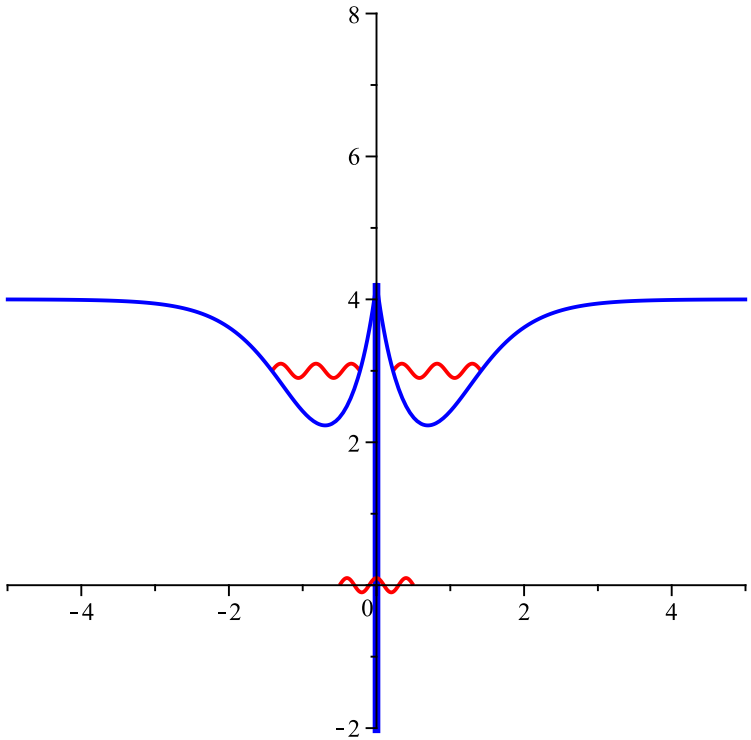
Without defect $\hat{\mathcal{M}}_{NP}$ produces a singular barrier $U \sim 2/\tau^2$ - no localized states (A.Andrianov, L.Vecchi,2007) Continuous spectrum from $m > 2M$



ϕ-channel: positive defect tension

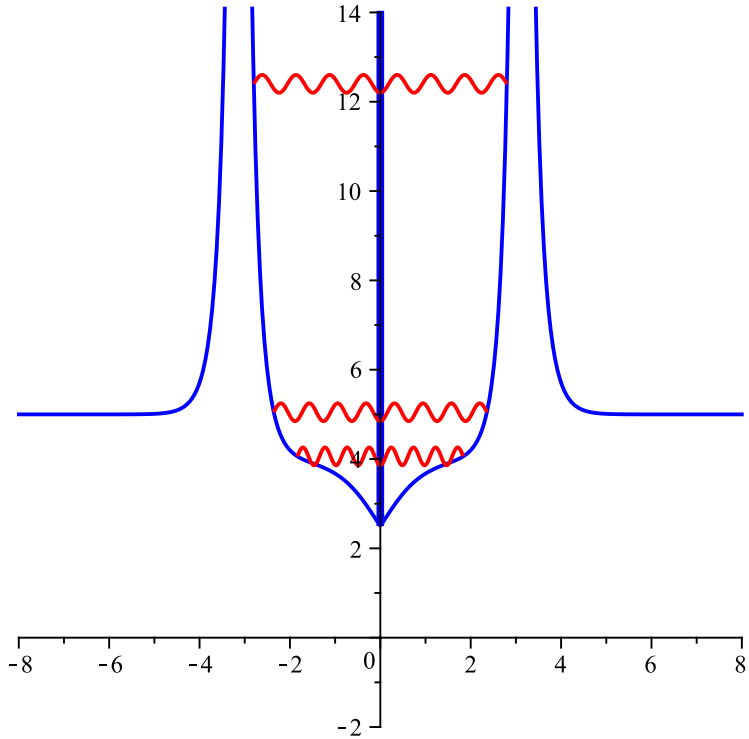
$\kappa \to 0$, $\lambda_b = \kappa M \beta b > 0$ - the singular barriers are cut of continuous spectrum from $m > 2M$

The heavy localized states and the pseudogoldstone mode are recovered (thanks to the δ-well for any b)



ϕ-channel: negative defect tension

$\kappa \to 0$, $\lambda_b = \kappa M \beta b < 0$, $|b| < \frac{2}{3}$ - to ensure AdS-geometry
Singular barriers at $\pm\tau_b$ form infinite walls of the potential well - the continuous spectrum at both sides of the well and the discrete one inside, decoupled from each other

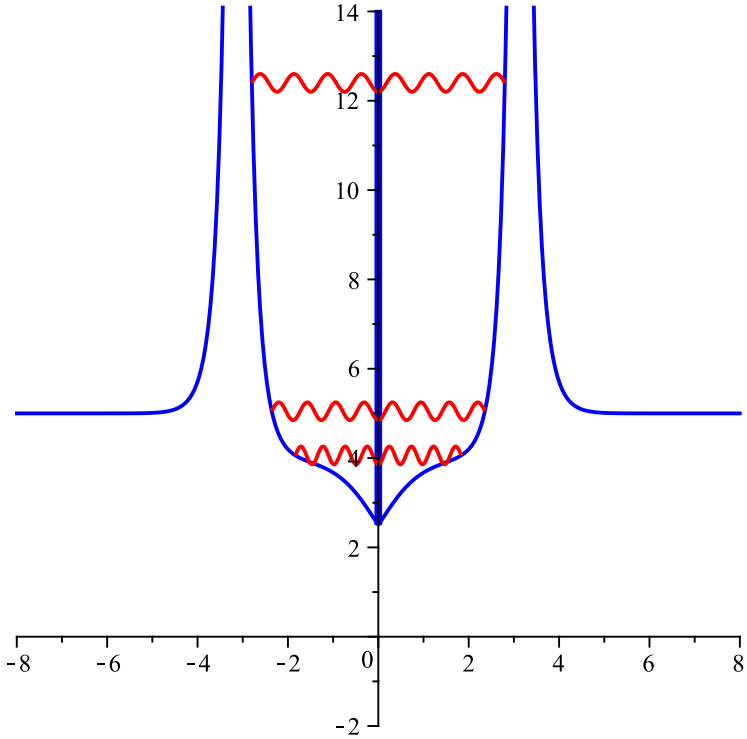


ϕ-channel: negative defect tension

This discrete spectrum consists of two series,

$$k_n = -\frac{\arctan\tanh\tau_b}{\tau_b} + \pi, \quad \tan k\tau_b = k\frac{(1+k^2)\tanh^3\tau_b - 3k^2\tanh\tau_b}{-3k^2 - 3(1+k^2)\tanh^2\tau_b + (1+k^2)\tanh^4\tau_b}$$

$$m^2 = (4+k^2)M^2$$



χ-channel for $\langle H \rangle = 0$

When $\langle H \rangle = 0$, the channels ϕ and χ decouple
From now on we assume $\lambda_b = \kappa M \beta b$
χ-channel in the limit $\kappa \to 0$ smoothly reproduce the model without gravity

$$\Big(-\partial_\tau + \tanh\tau\Big)\Big(\partial_\tau + \tanh\tau\Big)\chi = \Big(\frac{m^2}{M^2\beta^2}\Big)\chi$$

Similarly to the model without gravity this potential has one localized state

$$\chi_0 = \frac{1}{\cosh\tau}$$

In the phase with $\langle H \rangle = 0$ its squared mass is positive $\Delta_H > \Delta_H^{(c)}$,
otherwise this phase is unstable
At the critical point this is a zero-mode

Light scalar state mass $b \geqslant 0$

In the phase with $\langle H \rangle \neq 0$ this state can be obtained by the perturbation theory of κ and μ/M

$$m^2 = M^2 \sum_{n,k}^{\infty} \kappa^n \left(\frac{\mu}{M}\right)^k (m^2)_{n,k}$$

If $b \geqslant 0$ the leading order mass happen to be the same as in the model without gravity

$$(m^2)_{0,1} = 2$$

Without the defect NLO mass can be obtained analytically:

$$(m^2)_{0,2} = -128\sqrt{3}\operatorname{arctanh}\frac{\sqrt{3}}{3} + 146 + \frac{4}{3}\ln 2 \cdot (1 + \ln 2) - \frac{\pi^2}{9} \approx +0.4817$$

While in the model without gravity

$$(m^2)^{NG}_{0,2} = -\frac{130442}{121275} \approx -1.0756$$

Mass of the light scalar state $b < 0$

If the defect tension is negative $b < 0$ the leading order of mass depends on b

$$(m^2)_{0,1}\Big|_{b<0} = \frac{1}{3\tau_b(3\cosh^2\tau_b + 1) - \sinh\tau_b\cosh\tau_b(5\cosh^2\tau_b\cdot(\cosh 2\tau_b + 2) - 3)}\cdot$$
$$\cdot\Big[8\ln\cosh\tau_b\cdot\sinh 2\tau_b\cosh^2\tau_b\cdot(\sinh 2\tau_b + 2\tau_b(\cosh 2\tau_b + 2)) -$$
$$-\frac{1}{2}\sinh 2\tau_b\cdot(2\cosh\tau_b - \sinh\tau_b)\cdot(-3\sinh\tau_b + 3\sinh 3\tau_b + 8\cosh\tau_b + 4\cosh 3\tau_b) +$$
$$+\tau_b(7\sinh 2\tau_b - 2\sinh 4\tau_b - \sinh 6\tau_b + 9\cosh 2\tau_b + 15) + 4\tau_b^2(3\cosh 2\tau_b + 5) -$$
$$-32\left((\ln\cosh\tau_b)^2 + \tau_b^2\right)\cdot\cosh^6\tau_b\Big]$$

In the limit $b \to 0$ one obtains the same value $(m^2)_{0,1} = 2$

In the limit $b \to -\frac{2}{3}$, $(m^2)_{0,1} \longrightarrow \frac{14}{5} + \frac{16}{5}(\ln 2)^2 + \frac{16}{5}\ln 2 \approx 6.5555$

Conclusions

1. The mixing of the scalars and graviscalars leads to nontrivial changes in the invariant spectrum of the scalar states
2. In the minimal model without defect the translational Goldstone mode and heavy resonance disappear due to the singular barrier in the potential
3. Introduction of the defect in the form of a thin brane with small positive tension removes this singular barrier and restores pseudogoldstone mode and heavy resonance
4. If the curious case of the defect with negative tension singular barriers form the potential well with infinitely tall walls, inside which discrete spectrum of the localized states appears
5. These effects influence the mass of the light scalar state in the model with two scalar fields

Thank you for your attention!