Precision tests of QED

Alexander Penin

University of Alberta & TTP Karlsruhe

QUARKS 2014

Suzdal, Russia, June 02-08, 2014

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with positronium bound state

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  - *theory*
  - *experiment*
  - *"puzzles"*
- Hyperfine splitting to $\mathcal{O}(\alpha^7 m_e)$:
  - *one-photon annihilation contribution*

Based on

M. Baker, P. Marquard, A. Penin, J. Piclum and M. Steinhauser,

Phys. Rev. Lett. **112**, *120407 (2014)*

High precision tests of QED

- Main idea
  - *spectroscopy of a bound electron*

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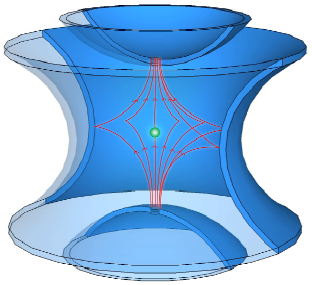
- Geonium “atom”:



QED in a nutshell

$$\overline{\Psi}\left(i\gamma^{\mu}D_{\mu}-m_{e}\right)\Psi-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

How accurate is QED?

- Fine structure constant ❶
  - *Rydberg constant from hydrogen/deuterium spectrum*
    $$R_\infty = \frac{\alpha^2 m_e c}{4\pi\hbar}$$
  - *electron/rubidium mass ratio from cyclotron frequency*
    $$\frac{m_e}{m_{Rb}} = \frac{\omega_{Rb}}{\omega_e}$$
  - *rubidium mass/Planck constant ratio from recoil*
    $$v_{rec} = \frac{\hbar k}{m_{Rb}}$$

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- Fine structure constant ❷
  - *electron anomalous magnetic moment* ($\bar{\mu} = \frac{ge}{2m_e c}\bar{s}$)
    $$\frac{g}{2} = 1 + \frac{\alpha}{2\pi} + \ldots$$
  - *geonium spectrum*
    $$\frac{g}{2} = \frac{\omega_s}{\omega_c}$$

How accurate is QED?

- Most precise prediction/measurement
  - *Fine structure constant (hydrogen spectrum, Rb recoil)*

    $$\alpha^{-1} = 137.035999 05(9)$$

    R. Bouchendira, P Clade, S. Guellati-Khelifa, F. Nez, and F. Biraben (2011)
  - *Fine structure constant (Electron $g-2$, geonium spectrum)*

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    T. Aoyama, M. Hayakawa, T. Kinoshita, M. Nio (2012)
- Biggest problem so far
  - *$\sim 7\sigma$ mismatch of proton charge radius (hydrogen vs muon hydrogen Lamb shift)*

Positronium bound state

- Basic facts
  - *pure QED system*
  - *"ortho" and "para" spin states*
  - *decays:* *p-Ps*$\longrightarrow n\gamma$, *o-Ps*$\longrightarrow (2n+1)\gamma$
  - *best observables: width* Γ_o*, hyperfine splitting* $\Delta\nu = E_o - E_p$
  - *QED prediction:* $\mathcal{O}(\alpha^3 \ln(\alpha))$

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  - *QED prediction:* $\mathcal{O}(\alpha^3 \ln(\alpha))$
- Why interesting?
  - *o-Ps mixes with an off-shell photon at Born level*
  - ➥ *sensitive to exotic new physics*

New physics signals?

- Large extra dimensions

S.Gninenko, N.Krasnikov, A. Rubbia (2003)

S. Dubovsky, V. Rubakov, , P. Tinyakov (2000)

  - *modified gravitational potential* $V(r) = -G\frac{m_1 m_2}{r}\left(1 + \frac{1}{k^2 r^2}\right)$
  - *effect on decay width* $\delta\Gamma_o \sim \frac{1}{\alpha^2}\frac{m_e}{k}\Gamma_o$

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- Mixing of “normal” photon with “dark” or “mirror” photon

Glashow (1986)

  - *kinetic mixing* $\epsilon F^{\mu\nu} F_{\mu\nu}$
  - *effect on HFS* $\delta\Delta\nu \sim \epsilon\Delta\nu$

Positronium bound state

- "Puzzles"
  - $\sim 5\sigma$ *mismatch of QED and experiment on* Γ_o
  - $\sim 2.5\sigma$ *mismatch of QED and experiment on* $\Delta\nu$

Orthopositronium life time measurements

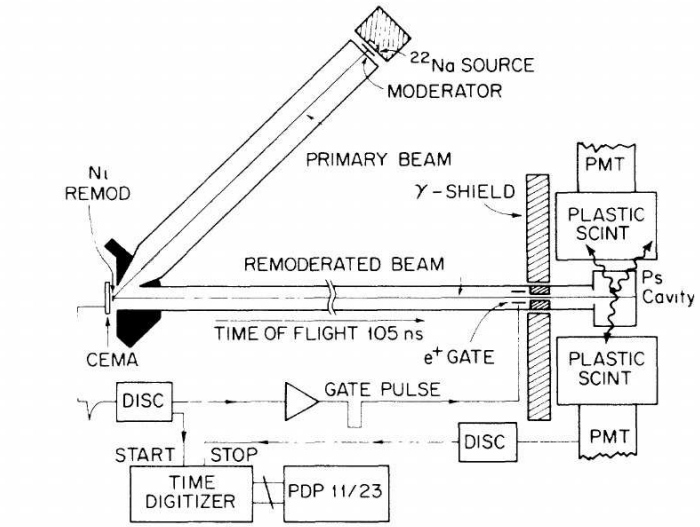
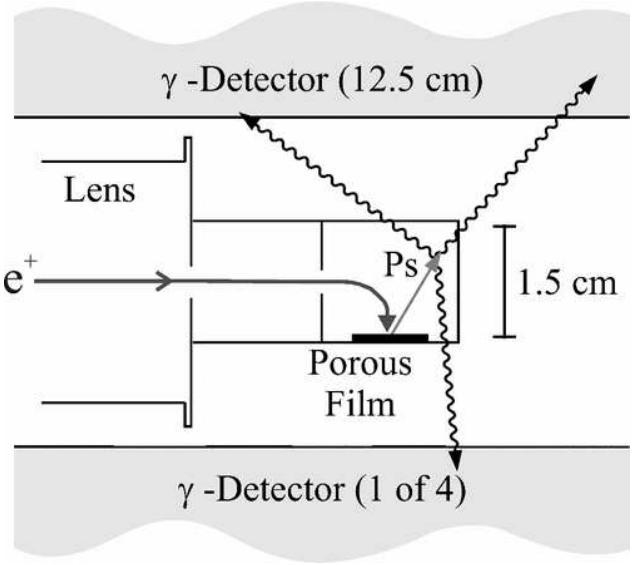


FIG. 1. Time-tagged and gated slow positron beam used to measure the orthopositronium decay rate.

Ann Arbor experiment 1990



Ann Arbor experiment 2003

Positronium bound state

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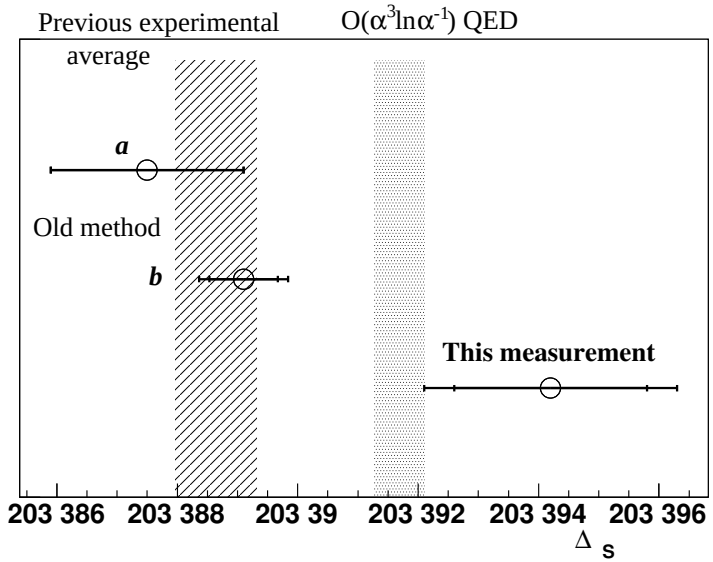
RESOLVED

Positronium HFS



Experiment

$\Delta\nu^{\rm exp} = 203.387\,5(16)$ GHz — A. P. Mills, Jr., *et al.* Phys. Rev. Lett. **34**, 246 (1975)

$\Delta\nu^{\rm exp} = 203.389\,10(74)$ GHz — M. W. Ritter, *et al.* Phys. Rev. A **30**, 1331 (1984).

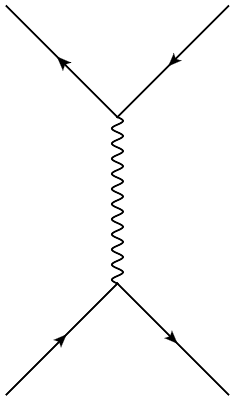
$\Delta\nu^{\rm exp} = 203.394\,2(16)_{\rm stat.}(13)_{\rm syst.}$ GHz — A. Ishida, *et al.* arXiv:1310.6923 [hep-ex].

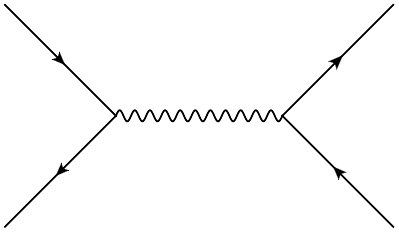
Theory

$\Delta\nu^{\rm th} = 203.391\,69(41)$ GHz — B. A. Kniehl and A. A. Penin, Phys. Rev. Lett. **85**, 5094 (2000).

Basic theory

- Born/Breit spin-dependent interaction





$$\delta_{hfs}\mathcal{H} = \left(\left[\frac{4}{3}\right]_{sct} + [1]_{ann} \right) \frac{\pi\alpha}{m_e^2} \delta(\boldsymbol{r}) \boldsymbol{S}^2 \,,$$

- Leading order HFS

$$\Delta\nu^{LO} = \left(\left[\frac{1}{3}\right]_{sct} + \left[\frac{1}{4}\right]_{ann} \right) \alpha^4 m_e$$

QED corrections

J. Pirenne, Arch. Sci. Phys. Nat. **29**, 265 (1947).
V. B. Berestetski and L. D. Landau, Zh. Eksp. Teor. Fiz. (USSR) **19**, 673 (1949).
R. A. Ferrell, Phys. Rev. **84**, 858 (1951).
R. Karplus and A. Klein, Phys. Rev. **87**, 848 (1952).
S. J. Brodsky and G. W. Erickson, Phys. Rev. **148**, 26 (1966).
R. Barbieri, J. A. Mignaco and E. Remiddi, Nuovo Cim. A **11**, 824 (1972).
W. E. Caswell and G. P. Lepage, Phys. Rev. A **20**, 36 (1979).
J. R. Sapirstein, E. A. Terray and D. R. Yennie, Phys. Rev. D **29**, 2290 (1984).
G. S. Adkins, M. H. T. Bui and D. Zhu, Phys. Rev. A **37**, 4071 (1988).
G. S. Adkins, Y. M. Aksu and M. H. T. Bui, Phys. Rev. A **47**, 2640 (1993).
A. H. Hoang, P. Labelle and S. M. Zebarjad, Phys. Rev. Lett. **79**, 3387 (1997)
K. Pachucki, Phys. Rev. Lett. **79**, 4120 (1997)
K. Pachucki and S. G. Karshenboim, Phys. Rev. Lett. **80**, 2101 (1998).
A. Czarnecki, K. Melnikov and A. Yelkhovsky, Phys. Rev. Lett. **82**, 311 (1999)
G. S. Adkins and J. Sapirstein, Phys. Rev. A **58**, 3552 (1998)
S. G. Karshenboim, Zh. Eksp. Teor. Fiz. **103**, 1105 (1993)
R. J. Hill, Phys. Rev. Lett. **86**, 3280 (2001).
K. Melnikov and A. Yelkhovsky, Phys. Rev. Lett. **86**, 1498 (2001).
B. A. Kniehl and A. A. Penin, Phys. Rev. Lett. **85**, 5094 (2000).

QED corrections

$$\begin{aligned}\Delta\nu^{\rm th} &= \Delta\nu^{LO}\left\{1-\frac{\alpha}{\pi}\left(\frac{32}{21}+\frac{6}{7}\ln 2\right)-\right.\\ &+ \left(\frac{\alpha}{\pi}\right)^2\left[-\frac{5}{14}\pi^2\ln\alpha+\frac{1367}{378}-\frac{5197}{2016}\pi^2+\left(\frac{6}{7}+\frac{221}{84}\pi^2\right)\ln 2-\frac{159}{56}\zeta(3)\right]\\ &+ \left.\left(\frac{\alpha}{\pi}\right)^3\left[-\frac{3}{2}\pi^2\ln^2\alpha+\left(-\frac{62}{15}+\frac{68}{7}\ln 2\right)\pi^2\ln\alpha+D\right]\right\},\end{aligned}$$

- Anatomy of $\mathcal{O}(\alpha^2)$ nonlogarithmic term
  - *47% scattering contribution*
  - *32% one-photon annihilation contribution*
- This work
  - *one-photon annihilation contribution to* D

Formula of success

pNRQED + Dim.Reg. = $\mathcal{O}(\alpha^7 m_e)$

Nonrelativistic effective theory

- Multiscale problem:
  - *hard* m_e
  - *soft* vm_e
  - *ultrasoft* $v^2 m_e$
- Coulombic bound state
  ➥ Schrödinger equation
- How to derive Schrödinger equation from QED?
  ➥ pNRQED

QED $\rightarrow$ NQED $\rightarrow$ pNRQED (Caswell, Lepage; Pineda, Soto)

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$$\overline{\Psi}\left(i\gamma^{\mu}D_{\mu} - m_e\right)\Psi$$

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$$\overline{\Psi}\left(i\gamma^\mu D_\mu - m_e\right)\Psi$$

$\downarrow$ hard modes integrated out

$$\psi^\dagger\left(iD_0 + \frac{\boldsymbol{D}^2}{2m_e}\right)\psi + \frac{1}{8m_e^3}\psi^\dagger\boldsymbol{D}^4\psi - \frac{c_F\, e}{2m_e}\psi^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{B}\psi + \ldots$$

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$\downarrow$ soft modes integrated out

$$\psi^\dagger\left(i\partial_0 + \frac{\boldsymbol{\partial}^2}{2m_e} - e\boldsymbol{r}\cdot\boldsymbol{E}\right)\psi + \ldots$$

Loops in the Effective Theory

- How to separate the regions of virtual momenta?

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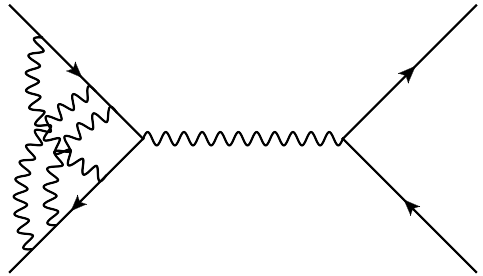
➥ **Regions are separated in dimensional regularization!**

- Effective theory in dimensional regularization

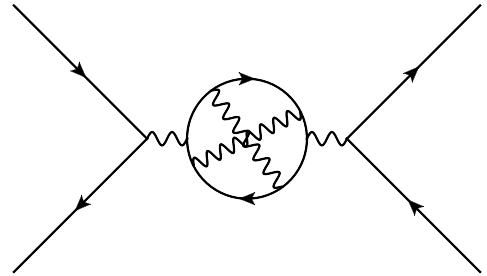
  (Pineda, Soto; Czarnecki, Melnikov, Yelkhovsky; Beneke, Signer, Smirnov; Kniehl, Penin, Smirnov, Steinhauser)

  - *no new scales*
  - *gauge, lorenz invariance*
  - *"build-in" matching*

Structure of the corrections to HFS



irreducible



reducible

- Master formula

$$\Delta_{ann}^{1-\gamma}\nu = \frac{\alpha^4 m_e}{4}\frac{R_o}{1+P_o}.$$

Irreducible contribution

- Vacuum polarization at the bound state pole

$$\lim_{E\to E_o} \Pi(q^2) = \frac{\alpha}{4\pi}\frac{R_o}{E/E_o - 1 - i\varepsilon},$$

- Effective theory decomposition

$$R_o = \left(c_v - \frac{E_o}{m_e}\frac{d_v}{6} + \ldots\right)^2 \left(1 + \frac{E_o}{2m_e}\right)^{-2} \frac{|\psi_o(0)|^2}{|\psi^C(0)|^2}$$

- Positronium wave function

$$\left(-\frac{\boldsymbol{\partial}^2}{m_e} - \frac{\alpha}{|\boldsymbol{r}|} + \delta\mathcal{H} - E\right)\psi_o(\boldsymbol{r}) = 0$$

  - *subject to ultrasoft corrections*

Irreducible contribution

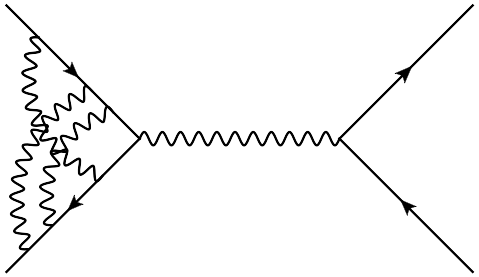
- Similar to $\mathcal{O}(\alpha_s^3)$ corrections to $\Gamma(\Upsilon(1S) \to e^+e^-)$

  M. Beneke, Y. Kiyo, P. Marquard, A. Penin, J. Piclum, D. Seidel and M. Steinhauser, Phys. Rev. Lett. **112**, 151801 (2014)

Irreducible contribution

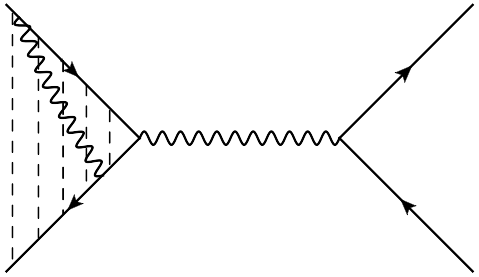
- Bottlenecks:
  - *three-loop hard vertex correction*

    P. Marquard, J. Piclum, D. Seidel and M. Steinhauser, Phys. Rev. D **89**, 034027 (2014)



  - *ultrasoft corrections*

    M. Beneke, Y. Kiyo and A. A. Penin, Phys. Lett. B **653**, 53 (2007)

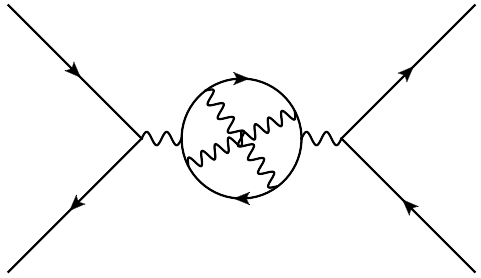


Reducible corrections

- Regular part of vacuum polarization

$$P_o = \lim_{E \to E_o} \left(e^2 \Pi(q^2) - \frac{\alpha^2 R_o}{E/E_o - 1 - i\varepsilon} \right).$$

- Bottlenecks
  - *Three-loop hard vacuum polarization*



Results

- Irreducible contribution $R_o = 1 + \sum_{n=1} \left(\frac{\alpha}{\pi}\right)^n r^{(n)}$

$$
\begin{aligned}
r^{(3)} &= -\frac{383}{18} + \left[-\frac{3}{2}\ln^2\alpha + \left(-\frac{7}{90} + 8\ln 2\right)\ln\alpha \right. \\
&- \left. \frac{1019}{180} - 4\ln 2 + \delta_o^{us}\right]\pi^2 + 2\zeta(3) - \frac{109}{864}\pi^4 + 2c_{v\,0}^{(3)}.
\end{aligned}
$$

- Reducible contribution $P_o = \sum_{n=1} \left(\frac{\alpha}{\pi}\right)^n p^{(n)}$

$$
p^{(3)} = (2\ln\alpha - 3)\,\pi^2 + p_{h\,0}^{(3)},
$$

- Cross-check of $\mathcal{O}(m_e\alpha^6)$ one-photon annihilation

A. H. Hoang, P. Labelle and S. M. Zebarjad, Phys. Rev. Lett. **79**, 3387 (1997)

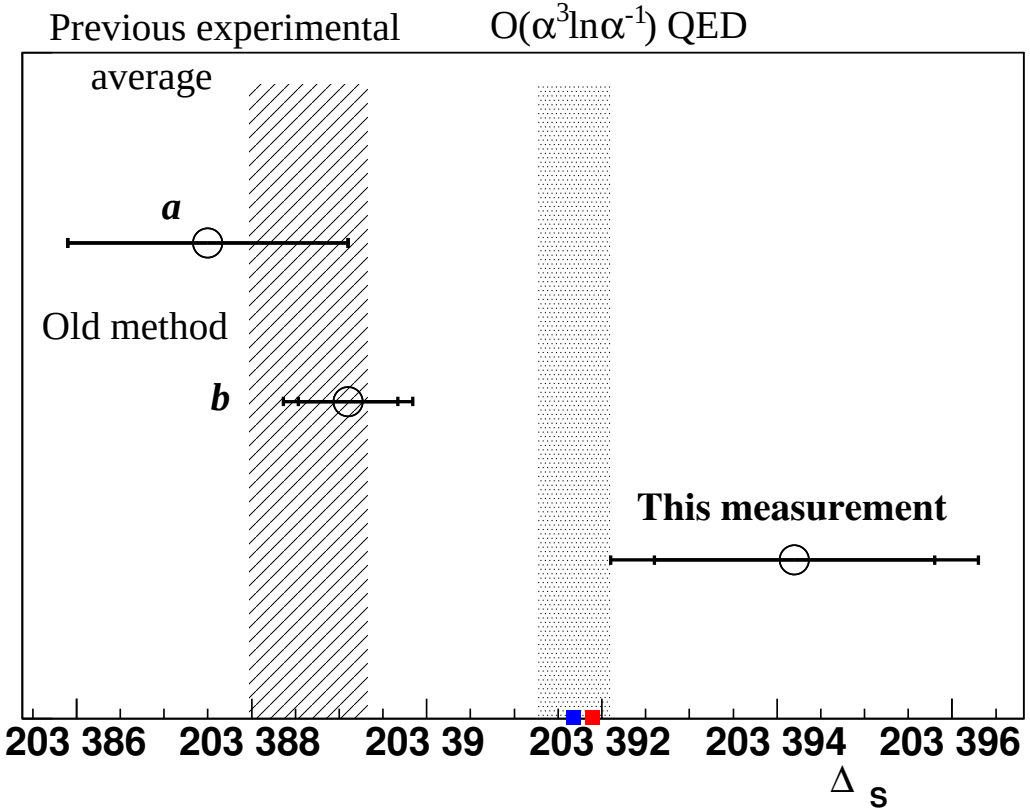
Final result

$$D_{ann}^{1-\gamma} = \frac{3}{7}\left[-\frac{49309}{1458}+\left(\frac{16573}{3240}-\frac{65}{9}\ln 2+\delta_o^{us}\right)\pi^2-\frac{221}{18}\zeta(3)\right.$$
$$\left.-\frac{109}{864}\pi^4+2c_{v\,0}^{(3)}-p_{h\,0}^{(3)}\right] = 84.8 \pm 0.5$$

- Structure of the corrections $\mathcal{O}(m_e\alpha^7)$
  - *Bethe logarithm* δ_o^{us} *gives* $D_{ann}^{1-\gamma} \approx 80$
  - *scattering contribution estimate* $D_{sct} \approx \frac{4\pi^2}{7}\delta_o^{us} \approx 106$
  - *relativistic contributions (electron* $g-2$*, electron loops):* $D \sim 1$

G. Adkins, R. Fell; M. Eides, V. Shelyuto

Final result



$$\Delta_{ann}^{1-\gamma}\nu = 217 \pm 1 \text{ kHz}$$

Summary

- Hyperfine splitting in positronium to $\mathcal{O}(m_e\alpha^7)$
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- QED is doing rather well so far
- Full $\mathcal{O}(m_e\alpha^7)$ result and more accurate measurements are crucial to give QED a hard time
- *Positronium could be an alternative gate to a BSM physics in the era of the total SM success at the LHC*