Chiral effects and physics of chiral media

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In external EM field there is non-conservation of chirality - axial anomaly of QFT:

$$\partial_\mu J_5^\mu = \frac{1}{8\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} = \frac{1}{2\pi^2} E \cdot B$$

This microscopic effect has macroscopic manifestation in chiral medium - chiral effects, which are transport phenomena closely tied with axial anomaly:

$$\vec{J}_5(x) = \frac{\mu}{2\pi^2}\vec{B} + \left(\frac{\mu^2 + \mu_5^2}{2\pi^2} + \frac{T^2}{6}\right)\vec{\Omega}$$

$$\vec{J}(x) = \frac{\mu_5}{2\pi^2}\vec{B} + \frac{\mu\mu_5}{\pi^2}\vec{\Omega}$$

Chiral effects were firstly obtained about 80s in papers of A. Vilenkin in linear response theory[1]:

$$J_\mu(x) = \int dx' \Pi^R_{\mu\nu} A^\nu(x')$$

$$\sigma_B = \lim_{k\to 0} \sum_{ij} \frac{i}{2k_j} \epsilon_{ijl} \Pi_{ij}|_{\omega=0} = \frac{\mu_5}{2\pi^2},$$

and as sum of LL contributions with the same answer

$$\vec{J}_{R(L)} = \sum_{np_y p_z} \langle \psi_n | \gamma^i | \psi_n \rangle f(\epsilon_{np_z} - \mu_{R(L)}) = \pm \frac{\mu_{R(L)}}{4\pi^2} B$$

$$\vec{J} = \vec{J}_R + \vec{J}_L = \frac{\mu_R - \mu_L}{4\pi^2} B = \frac{\mu_5}{2\pi^2} B$$

[1] see e.g. Phys. Rev. D 22, 3080

The similar question appeared in holography [2]. Here the starting point is the five dimensional Einstein-Maxwell action:

$$S = -\frac{1}{16\pi G_5}\int\left(\sqrt{-g}\left(R + 12 - F^2\right) - \frac{4\kappa}{3}\epsilon^{MNOPQ}A_M F_{NO} F_{PQ}\right) d^5x$$

It was shown that the theory at the boundary could be described by relativistic hydrodynamics:

$$\partial_\mu T^{\mu\nu} = 0\ ,\ \ T^{\mu\nu} = wu^\mu u^\nu + Pg^{\mu\nu} + \tau^{(1)\mu\nu}$$
$$\partial_\mu J^\mu = 0\quad ,\quad J^\mu = nu^\mu + \nu^{(1)\mu},$$

where $\nu^{(1)}$ and $\tau^{(1)\mu\nu}$ are corrections of the first order in spatial gradients.

[2] JHEP 0901 (2009) 055

One of the most intriguing points here is that $\nu^{(1)}$ contains new term which was forbidden in classical hydro[3]:

$$\nu^{(1)\mu} = ... + \xi\omega^{\mu}$$

where $\xi = \frac{3q^2\kappa}{16\pi G_5 m}$ and $\omega^{\mu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_{\nu}\partial_{\alpha}u_{\beta}$. To resolve this inconsistency one should consider ideal hydrodynamics modified by chiral anomaly[4]:

$$\partial_{\mu}T^{\mu\nu} = 0\ ,\ \partial_{\mu}J^{\mu} = CE\cdot B$$

and from entropy current conservation one finds that

$$\nu^{(1)\alpha} = C\mu B^{\alpha} + C\mu^2\omega^{\alpha}$$

[3] Landau course vol. VI

[4] Phys.Rev.Lett. 103 (2009) 191601

To find connection between these effects and anomalies one could consider effective field theory[5]:

$$S_{eff} = \int dx \left(i\bar{\psi}\gamma^{\rho} D_{\rho}\psi + \mu u_{\mu}\bar{\psi}\gamma^{\mu}\psi + \mu_5 u_{\mu}\bar{\psi}\gamma^{\mu}\gamma_5\psi \right) + S_{int}.$$

After calculating of anomaly and substituting hydro averaged currents we get

$$\partial_{\mu}\left(n_5 u^{\mu} + \frac{\mu^2+\mu_5^2}{2\pi^2}\omega^{\mu} + \frac{\mu}{2\pi^2}B^{\mu} \right) = -\frac{1}{4\pi^2}\epsilon_{\mu\nu\alpha\beta}\partial^{\mu}A^{\nu}\partial^{\alpha}A^{\beta}$$

$$\partial_{\mu}\left(n u^{\mu} + \frac{\mu\mu_5}{\pi^2}\omega^{\mu} + \frac{\mu_5}{2\pi^2}B^{\mu} \right) = 0$$

with the same coefficients as above in front of each term.

[5] Phys.Rev. D83 (2011) 105025

Let's compare CME with usual electric conductivity:

$$\vec{J} = \sigma_B \vec{B} \quad , \quad \vec{J} = \sigma_E \vec{E}$$

and for time time reversal quantities we have

$$\vec{J}^T = -\vec{J} \quad , \quad \vec{E}^T = +\vec{E} \quad , \quad \vec{B}^T = -\vec{B}$$

it means that effectively

$$\sigma_B^T = \sigma_B \quad , \quad \sigma_E^T = -\sigma_E$$

while it is obvious that σ_E must be positive.

So CME is of dissipation-free nature as it is in the case of Hall current which is also time reversal $J \sim E \times B$.

Radiative corrections:

$$\delta J_5 \; = \; -\frac{\alpha_{el} eB\mu}{2\pi^3}\Big(\ln\frac{2\mu}{m_f} + \ln\frac{m_\gamma^2}{m_f^2} + \frac{4}{3}\Big) \; ,$$

Back reaction of a medium results in the:

$$\mathbf{curl} B = \sigma_B B \quad \Rightarrow \quad (\Delta + \sigma^2) B = 0$$

where is a pole on real axe which signals the instability[6]. Moreover partial sum of diagram series results in modification of CME:

$$\vec{J}_{CME} = \lim_{k\to 0} \frac{\sigma_B k^2}{k^2 - \sigma_B^2} \vec{B} = 0$$

This result has a partner in holography with spatially modulated field-current distribution[7]

[6] Phys.Rev.Lett. 111 (2013) 052002

[7] Phys.Rev.Lett. 106 (2011) 061601

Taking the anomaly into account the axial charge is not conserved

$$\partial_t Q_5^{(0)} = -\frac{1}{2\pi^2} \int \vec{E} \cdot \vec{B} d^3x$$

but it could be generalized to

$$Q_5 = Q_5^{(0)} + \frac{e^2}{4\pi^2} \int_x \vec{A} \cdot \vec{B}$$

and the instability gives rise to the chirality transfer from micro to macro level while redefined charge is conserved

$$Q_5^{(0)} \to \frac{e^2}{4\pi^2} \int_x \vec{A} \cdot \vec{B}$$

Could we get some additional information from macroscopic physics? The answer is yes! It was shown that there is an universal modiffication of the theory in a dense medium

$$eA_\mu \to eA_\mu + \mu u_\mu$$

and it results in additional anomaly-like terms in the generalized charge for hydro:

$$\partial_t \left(\int_x n_5 u^0 + \frac{\mu}{2\pi^2} \int_x u \cdot B + \frac{\mu^2}{4\pi^2} \int_x \epsilon_{ijk} u_i \partial_j u_k \right) = -\frac{1}{2\pi^2} \int_x \vec{E} \cdot \vec{B}$$

and one can compare it with the answer from the pure field theory

$$\partial_t Q_5^{(0)} = -\frac{1}{2\pi^2} \int_x \vec{E} \cdot \vec{B}$$

Now we have a hydro expectation for the conserving charge. New terms are manifestation of a macroscopic ordered motion of microscopic chiral constituents. Charge conservation imposes additional constraints as a requirement of classical conservation:

$$\eta \to 0 \quad , \quad \sigma_E \to \infty$$

and in this limit electric field is screened

$$\partial_t Q_5 = 0$$

However chiral effects are still there through a modiffication of the Hamiltonian:

$$H = H_0 + \mu_5 Q_5 \quad , \quad J = \mu_5 \frac{\delta Q_5}{\delta A}$$

- We have seen that conservation of the axial charge implies that classically chiral media are perfect liquids, with no dissipation.
- It is interesting to consider physics of CE in quantum states (e.g. some kinds of superfluidity).
- Holographic fluids are good place to check these constraints and instabilities since of nearly ideal behaviour.
- There is a close connection between CME and α-dynamo so it is interesting to check consequences for the turbulence stage.

Thanks!