INTERSECTION POINTS OF MAGNETIC, YANG-MILLS AND GRAVITATIONAL FIELDS

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Talk based on papers by

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Plan of the Talk

1. When QED meets QCD.
2. Magnetic Collapse scenario for hadrons.
3. Two keystones of the formalism.
4. The effective Hamiltonian.
5. Treating confinement and gluon exchange. Eliminating color Coulomb catastrophe.
6. Results for ρ-meson spectrum.
7. Neutron in strong magnetic field.
8. Magnetic focusing of hyperfine interaction in hydrogen.
9. Antihydrogen in crossed magnetic and gravitational fields.

1. When QED meets QCD

$eB \sim \Lambda^2_{QCD}$ becomes a reality in peripheral heavy ion collisions at RHIC and LHC.

MF Hierarchy (in G, GeV$^2 = 5.12 \cdot 10^{19} G$)

Medical MPI scan	10^4
Hand-held magnet	100
Earth'	0.6
LHC Magnets	$8 \cdot 10^4$
Strongest steady Lab	10^6
Explosion Lab	$3 \cdot 10^7$
Surface of magnetars	$10^{14} - 10^{15}$
RHIC and LHC	$10^{18} - 10^{19} \sim \Lambda^2_{QCD}$
Early Universe	$10^{24} - 10^{40}$!

Physical Scale

Atomic:

$$B_a = m_e^2 e^3 = 2.35 \cdot 10^9 G$$

$$a_H = \frac{eB_a}{2m_e} = Ry$$

At $B > B_a$ hydrogen atom becomes an elongated spheroid

Schwinger:

$$eB_c = m_e^2 = 4.4 \cdot 10^{13} G$$

Landau levels distance

$$\Delta E = \varepsilon_1 - \varepsilon_{LLL} = m_e$$

ρ-critical:

$$(eB_\rho)^{-1/2} \simeq a_\rho \simeq 0.6 \text{ fm}$$

$$eB_\rho \simeq 10^{19} G$$

The problem has to be investigated at the quark level.

2. Magnetic Collapse scenario for hadrons.

A new phase transition? Vacuum superconductivity.

The LLL Landau orbit of $\rho^{\pm}$ becomes unstable when

$$m_\rho^2 + eB(1 - g_\rho) < 0.$$

$\rho^{\pm}$ become massless and condense $\grave{a}$ la Ambjorn-Olesen, Neilsen-Olesen-Savvidi.

ρ – meson:

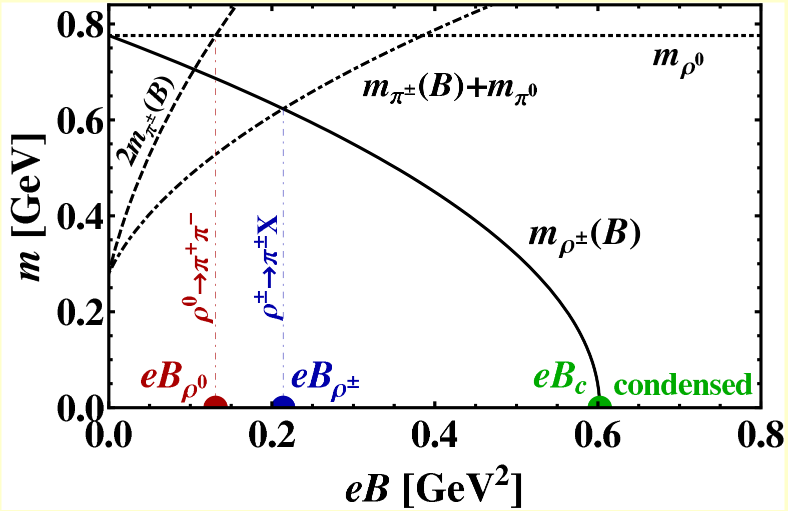
S.Schramm, B.Muller, A.Schramm 1992 (cautions against ignoring quark structure).

M.Chernodub 2010-1013.

Condensation of ρ mesons (naive)

The $\rho^{\pm}$ mesons become massless and condense at the critical value of the external magnetic field

$$B_c = \frac{m_\rho^2}{e} \approx 10^{16} \;\; Tesla$$



Kinematical impossibility of dominant decay modes

The pion becomes heavier while the rho meson becomes lighter

-The decay $\rho^{\pm} \to \pi^{\pm}\pi^{0}$ stops at certain value of the magnetic field $m_{\rho^\pm}(B_{\rho^\pm}) = m_{\pi^\pm}(B_{\rho^\pm}) + m_{\pi^0}$

-A similar statement is true for $\rho^0 \to \pi^+\pi^-$

Two keystones of the formalism.

1. Separation of the center of mass motion. In Quantum Mechanics for a system with total $Q = 0$ the problem was solved by W.E.Lamb, L.P.Gorkov and I.E.Dzyaloshinskii, B.Simon,...

Relativistic system: AKOS:

(i) neutral system, (ii) charged with $e_1 = e_2$, $m_1 = m_2$, (iii) 3-body neutral, $e - 1 = e_2$, $e_1 + e_2 + e_3 = 0$ (neutron)

$$\mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r}$$

$$\hat{\mathbf{F}} = \mathbf{P} + \frac{e}{2}(\mathbf{B} \times \boldsymbol{\eta}), \quad \boldsymbol{\eta} = \mathbf{r}_1 - \mathbf{r}_2$$

Factorization of the wave function

$$\hat{H} = \frac{1}{m}\left(-i\frac{\partial}{\partial \boldsymbol{\eta}} - \frac{e^2}{4}(\mathbf{B} \times \mathbf{R})\right)$$

$$\Psi(\mathbf{R}, \boldsymbol{\eta}) = \varphi(\boldsymbol{\eta}) \exp\left\{i\mathbf{P}\mathbf{R} - i\frac{e}{2}(\mathbf{B} \times \boldsymbol{\eta})\mathbf{R}\right\}$$

Two keystones of the formalism

2. Fock-Schwinger-Feynman representation for the $q\bar{q}$ Green's function

Fock-Schwinger

$$S = (m + \hat{D})^{-1} = (m - \hat{D}) \int_0^\infty ds e^{-s(m^2 - D^2)}$$

+ Feynman path integral.

$$G_{q_1 \bar{q}_2}(x, y) = \frac{T}{8\pi} \int_0^\infty \frac{d\omega_1}{\omega_1^{3/2}} \int_0^\infty \frac{d\omega_2}{\omega_2^{3/2}} \langle x | tr \hat{V} e^{-H_{q_1 \bar{q}_2} T} | y \rangle$$

ω_1, ω_2– "dynamical masses".

The effective Hamiltonian

$$\hat{H}\Psi = m(\omega_1, \omega_2)\Psi, \quad \frac{\partial M(\omega_1, \omega - 2)}{\partial \omega - i} = 0$$

$$\hat{H} = H_0 + H_\sigma + W$$

$$H_0 = \frac{1}{2\tilde{\omega}} \left(-\frac{\partial^2}{\partial \boldsymbol{\eta}^2} + \frac{e^2}{4} (\mathbf{B} \times \boldsymbol{\eta})^2 \right), \quad \tilde{\omega} = \frac{\omega_1 \omega_2}{\omega_1 + \omega_2}$$

$$H_\sigma = \frac{m_1^2 + \omega_1^2 - e\boldsymbol{\sigma}_1 \mathbf{B}}{2\omega_1} + \frac{m_2^2 + \omega_2^2 - e\boldsymbol{\sigma}_2 \mathbf{B}}{2\omega_2}$$

$$W = V_{conf} + V_{coul} + V_{SS} + \Delta M_{SE}$$

$$V_{conf} = \sigma\eta \to \tilde{V}_{conf} = \frac{\sigma}{2} \left(\frac{\eta^2}{\gamma} + \gamma \right)$$

Treating confinement and gluon exchange

$$(H_0 + \tilde{V}_{conf})\psi_{(\boldsymbol{\eta})} = M(\omega_1, \omega_2, \gamma)\Psi(\boldsymbol{\eta})$$

yields the wave function

$$\Psi(\boldsymbol{\eta}) = \frac{1}{\sqrt{\pi^{3/2} r_\perp^2 r_0}} \exp\left(-\frac{\eta_\perp^2}{2r_\perp^2} - \frac{\eta_2^2}{2r_0^2}\right)$$

At $B \to \infty$

$$r_\perp \propto \frac{1}{\sqrt{eB}}, \quad r_0 \propto \frac{1}{\sqrt{\sigma}}$$

$$M(\omega_1, \omega_2, \gamma) = \epsilon_{n_\perp, n_2} + \frac{m_1^2 + \omega_1^2 - e\mathbf{B}\sigma_{\mathbf{1}}}{2\omega_1} + \frac{m_2^2 + \omega_2^2 + e\mathbf{B}\sigma_{\mathbf{2}}}{2\omega_2}$$

4 states $M^{++}, M^{--}, M^{+-}, M^{-+}$

Treating confinement and gluon exchange

$$\triangle M_{OGE} \equiv \int V_{OGE}(q)\tilde{\psi}^2(q)\frac{d^3q}{(2d)^3} \propto -\sqrt{\sigma}\ln\ln\frac{eB}{\sigma}$$

Diverges for $eB \gg \sigma, \;\; r_0 \gg r_\perp$.

As in the case of hydrogen (Vysotsky et al.)

"color Coulomb catastrophe" is eliminated by quark loops with LLL level contribution.

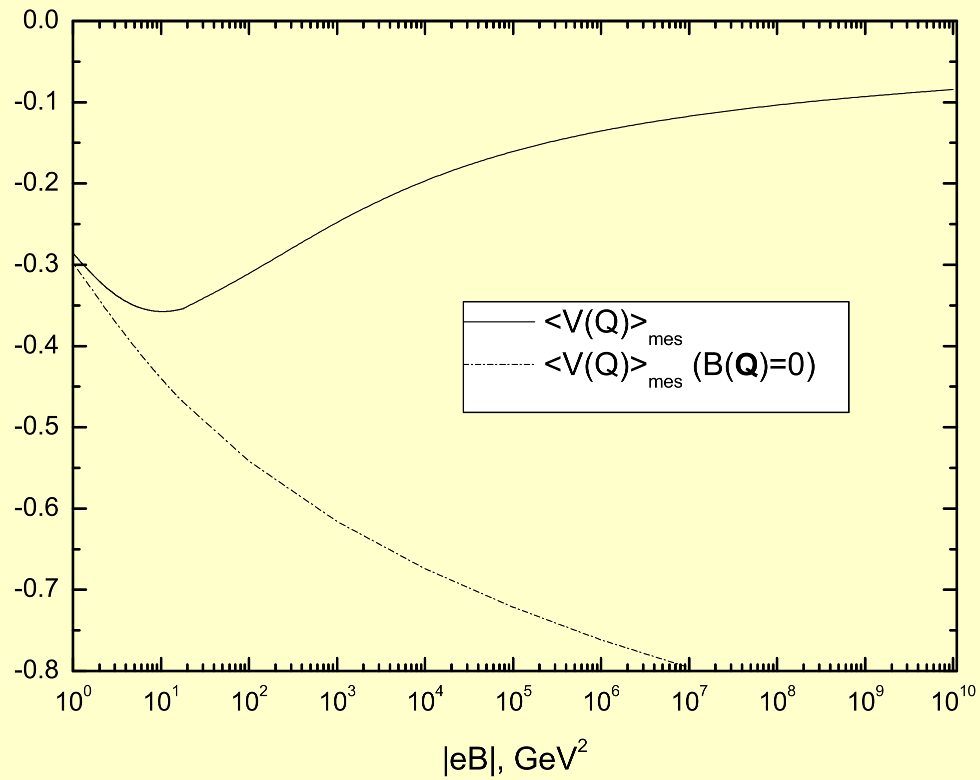


Figure 1: One gluon exchange correction $\triangle M_{OGE}$ to the meson mass in GeV as a function of magnetic field with (solid line) and without (broken line) account of quark loops contributions.

Results for ρ-meson spectrum

Spin-spin interaction

$$V_{SS} = \frac{8\pi\alpha_s}{9\omega_1\omega_2}\delta^{(3)}(\mathbf{r}\sigma_{\mathbf{1}}\sigma_{\mathbf{2}})$$

(i) Four new states instead of $M^{++}, M^{--}, M^{+-}, M^{-+}$

$$\rho^0(s_z = 0) = \cos\theta|\uparrow\downarrow\rangle + \sin\theta|\downarrow\uparrow\rangle$$

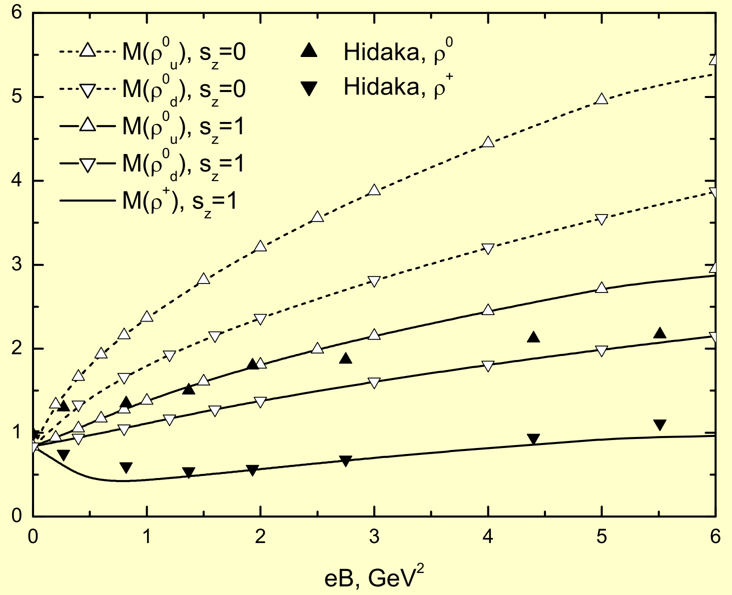
$$\pi^0(s_z = 0) = -\sin\theta|\uparrow\downarrow\rangle + \cos\theta|\downarrow\uparrow\rangle$$

$$\rho^0(s_z = +1) = |\uparrow\uparrow\rangle \qquad \rho^0(s_z = -1) = |\downarrow\downarrow\rangle$$

(ii) Smearing of $\delta(\mathbf{r})$

$$\delta^{(3)}(\mathbf{r}) \to \tilde{\delta}^{(3)}(\mathbf{r}) = \left(\frac{1}{\lambda\sqrt{\pi}}\right)^3 e^{-\mathbf{r}^2/\lambda^2},\ \lambda \sim 1\ GeV^{-1}$$

Results for ρ-meson spectrum



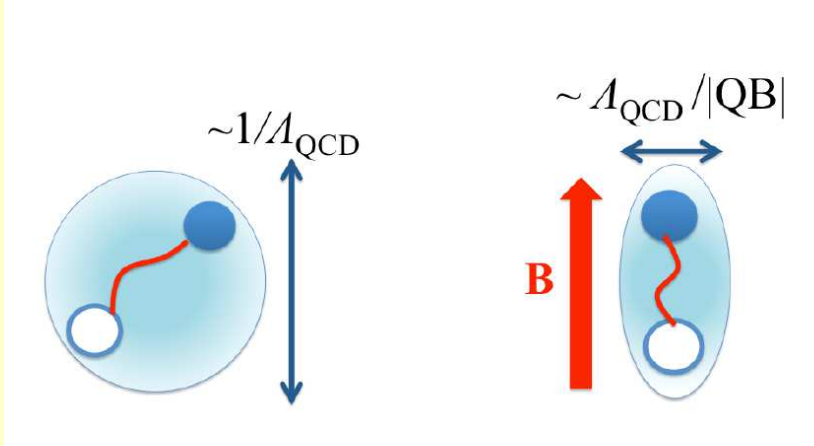
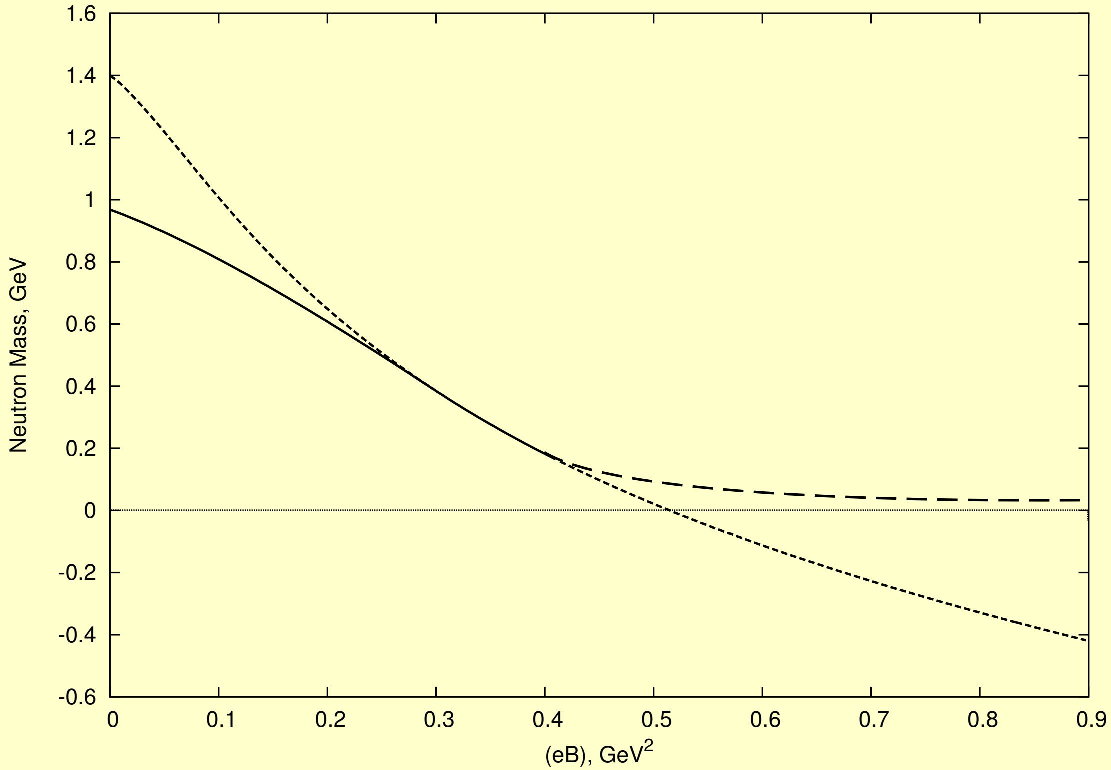


Figure 2: The masses of the systems in GeV as a functions of eB. See the text for explanations.

Neutron in Strong Magnetic Field

3q system in MF, pseudomomentum + Green's function path integral.

Striking result: $M_n(eB \sim 0.25 GeV^2 \sim 10^{19} G) \simeq \frac{1}{2} M_n^{(0)}$.



Magnetic focusing of hyperfine interaction in hydrogen

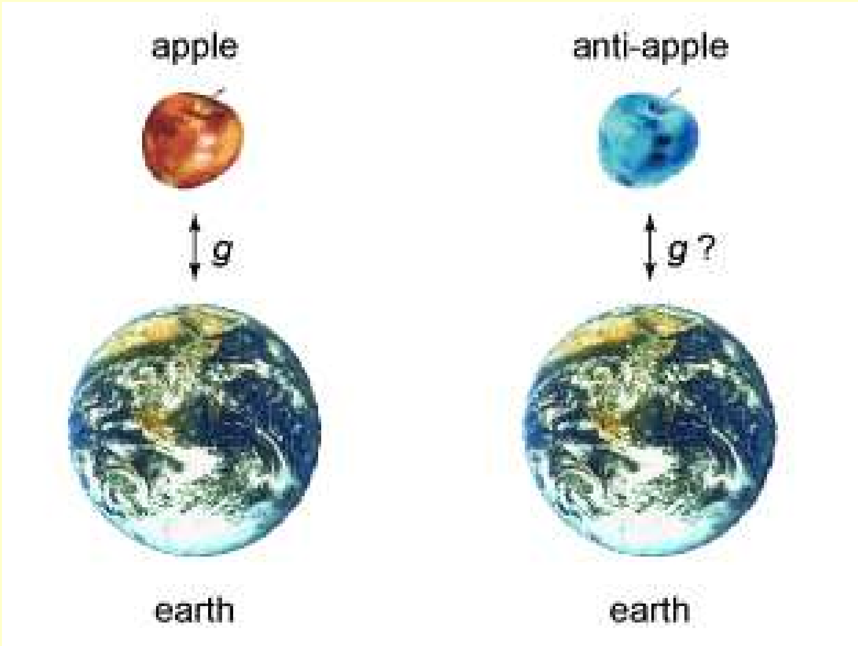
A new correction to the famous 21 cm line in magnetic field. In MF the hydrogen atom squeezes, $|\psi(0)|^2$ grows, the hf splitting acquires extra shift on top of Zeeman-Paschen-Back. In addition, MF-dependent tensor force appears.

$$H \gg 1 \quad \delta\nu \simeq \alpha^6 \left(\frac{m}{m_p}\right) (H \ln^2 H) \simeq 10^{-6} (H \ln^2 H) \; MHz$$

$$H \ll \alpha^2 \frac{m}{m_p} \delta\nu \simeq \delta E_{hfs} \left(1 - \frac{r_\perp^2}{r_z^2}\right)$$

Antihydrogen in crossed magnetic and gravitational fields

Motivation: (a) $\bar{H}$ traps at CERN keep hundreds of atoms for thousands of seconds (b) GBAR Project (V.Nesvizhevsky)



Take $\bar{H}$ as anti-apple. Impose a weak MF $\vec{B} \perp \vec{g}$. Will $\bar{H}$ fall, levitate, or what? Will gravitational levels survive?

Antihydrogen in crossed magnetic and gravitational fields

Tentative answer:

$\bar{H}$ will exhibit a diffusion-like behavior, obey Langevin or Fokker-Plank equations (Schmelcher and Cederbaum, Dumin, AKL in progress).

The interplay of the fast inner motion and the slow center-of-mass one.

Fast (electron) degrees of freedom serve as a stochastic force for the slow center-of-mass motion.

Will the gravitational states be washed out?

Conclusions

1) The new formalism has been developed for realistic quark systems in magnetic field.

2) The spectra of mesons in magnetic field have been obtained.

3) The results are in agreement with lattice data.

4) The neutron mass is fast decreasing with the magnetic field.

5) A new correction to hyperfine splitting in hydrogen has been derived.

6) Magnetic field may stimulate diffusion-like movement of $\bar{H}$ in gravitational field.