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QCD Flux Tube Spectrum from Approximate Integrability

arXiv:1301.2325 and arXiv:1404.0037

w/

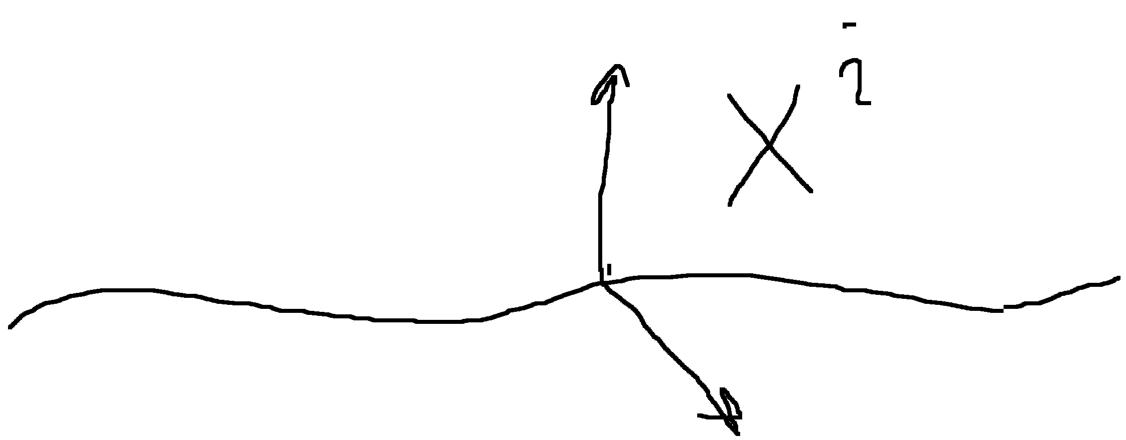
Sergei Dubovsky and Raphael Flauger

Quarks 2014

What are Flux Tubes:

- String-like objects, i.e. stretched between the quarks
- Crucial for gauge theory dynamics, responsible for confinement
- For an effective field theorist - D-2 Goldstone bosons of

$$ISO(1, D-1) \to ISO(1,1) \times SO(D-2)$$



Drawing by SD

What are Flux Tubes:

- Effective 2D theory with cutoff $\ell_s^{-1} \sim \Lambda_{QCD}$
- Action consists of geometrical invariants constructed from

$$h_{\alpha\beta} = \eta_{\alpha\beta} + \partial_\alpha X^i \partial_\beta X^j$$

$$S_{string} = -\int d^2\sigma \sqrt{-\det h_{\alpha\beta}} \left(\ell_s^{-2} + \frac{1}{\alpha_0} \left(K^i_{\alpha\beta}\right)^2 + \dots \right)$$

on-shell

- No one-loop counterterms on-shell, which makes low-energy predictions more universal.
- Perturbatively:

$$S_{string} = -\ell_s^{-2} \int d^2\sigma \frac{1}{2}(\partial_\alpha X^i)^2 - \frac{1}{8}(\partial_\alpha X^i)^4 + \frac{1}{4}(\partial_\alpha X^i \partial_\beta X^j)^2 + \dots$$

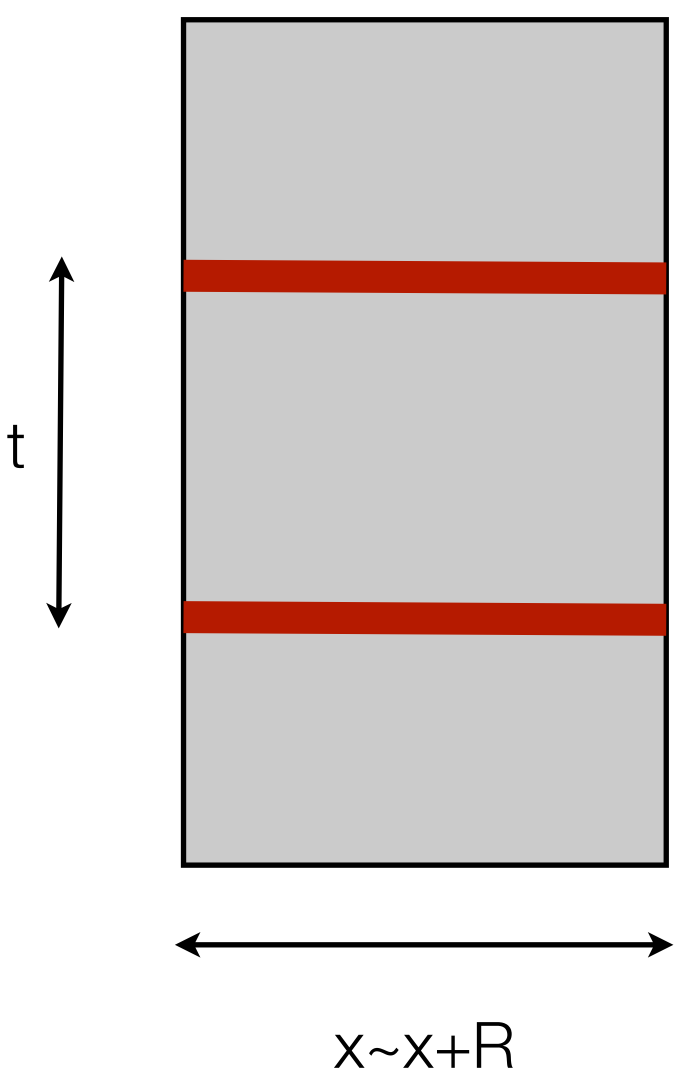
The spectrum

- We can calculate the spectrum of this theory on $S_1 \times R$
- From the bulk point of view it will be the spectrum of closed flux tubes with $w = 1$ along some compact direction.
- Naive approach (Aharony and Klinghoffer 2010): reduce to QM and do perturbation theory universal up to $(\ell_s/R)^5$
- Wrong approach - quantize in Light Cone, don't care about Lorentz anomaly

$$E_{LC}(N, \tilde{N}) = \sqrt{\frac{4\pi^2(N-\tilde{N})^2}{R^2} + \frac{R^2}{\ell_s^4} + \frac{4\pi}{\ell_s^2}\left(N + \tilde{N} - \frac{D-2}{12}\right)}$$

On the lattice: 4D SU(3) YM with one short direction

$$\int \mathcal{D}A \, e^{-S_{YM}} \mathcal{O}(0)\mathcal{O}^\dagger(t) = \langle | \mathcal{O}(0) | n \rangle \langle n | \mathcal{O}^\dagger(t) | \rangle \sim e^{-E_0 t}$$

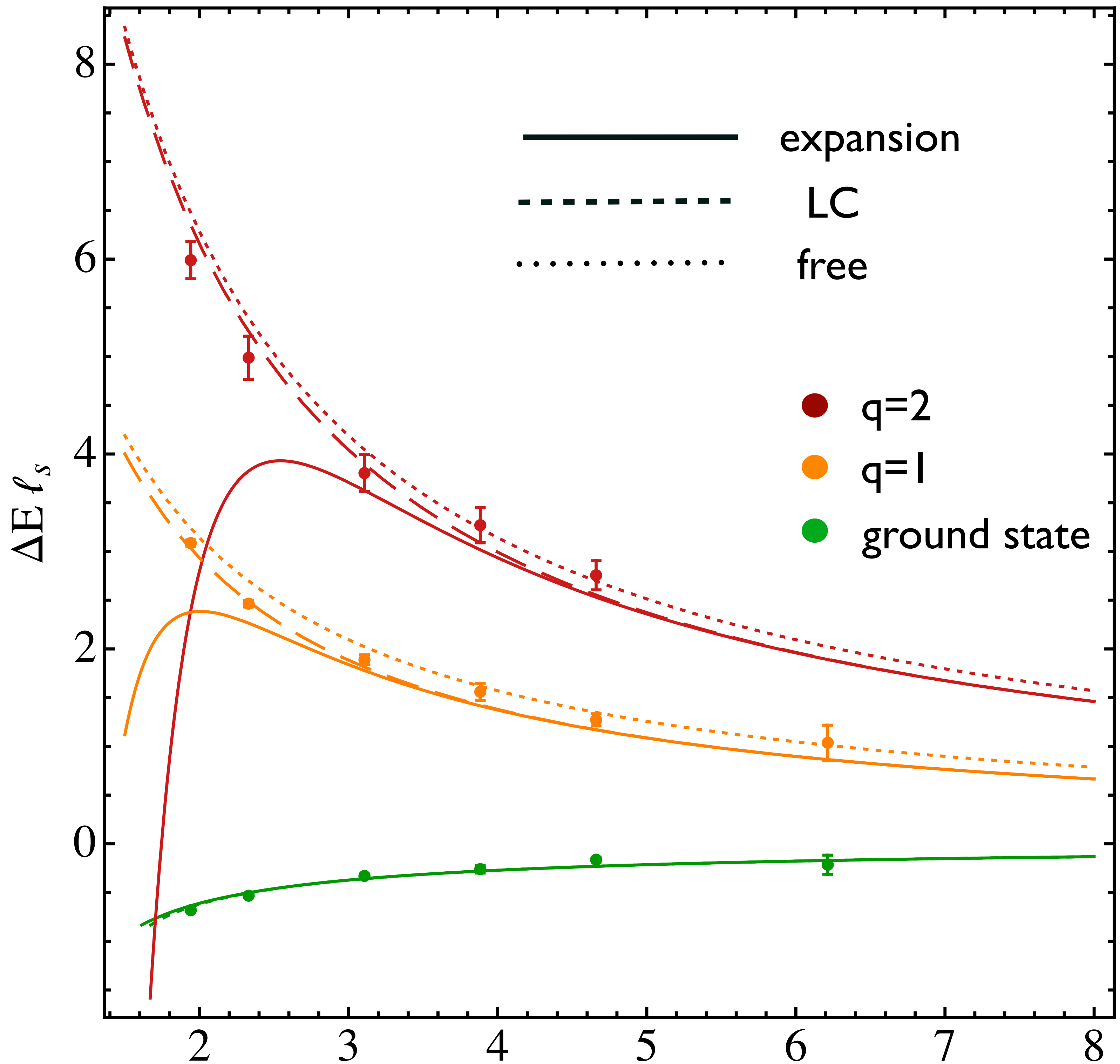


$$\mathcal{O} = P \exp \left\{ i \oint A \right\}$$ "Creates" a flux tube

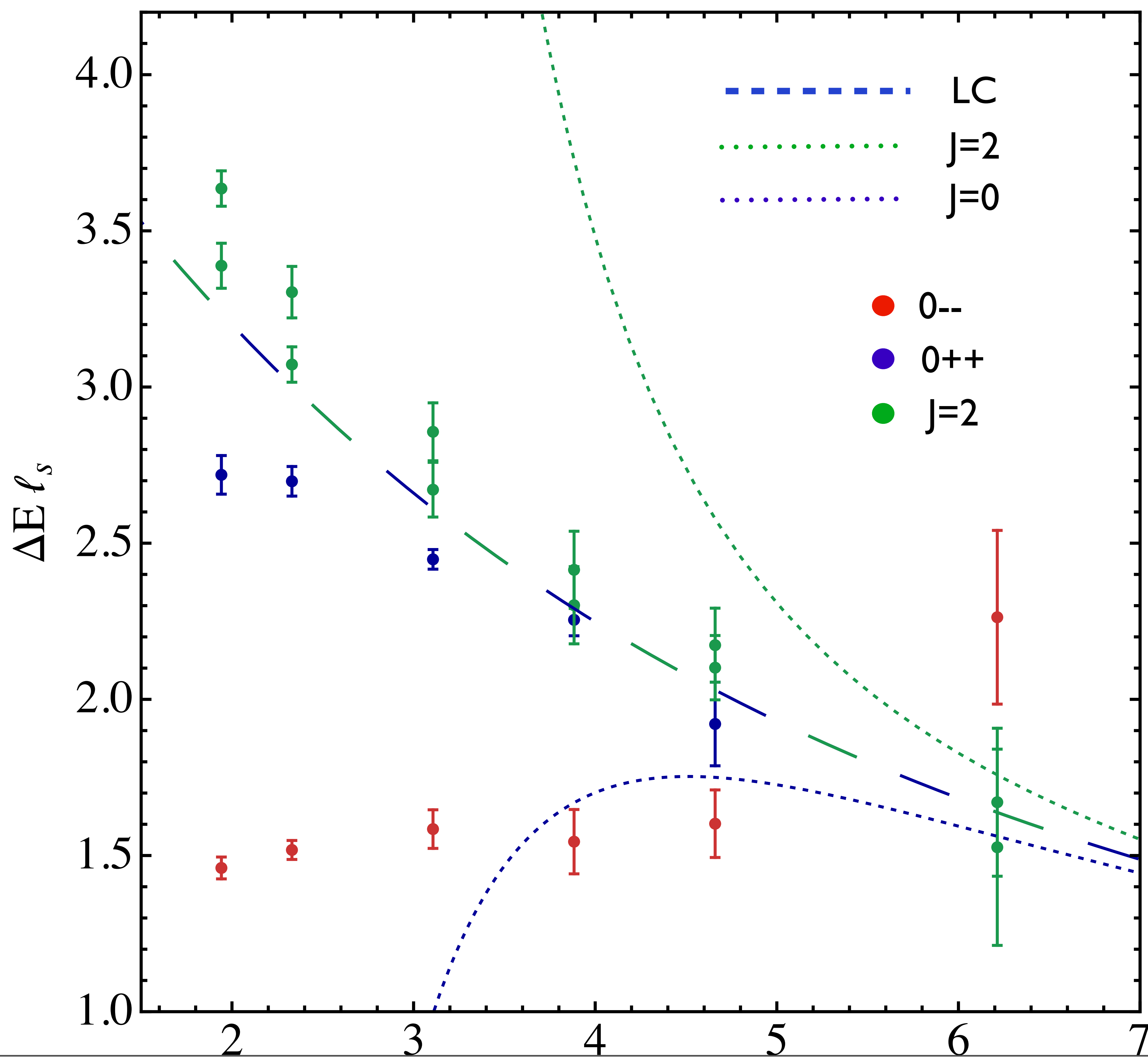
Then deform the paths to get excited states

Ground and one-particle states

Data from Athenodorou, Bringoltz, and Teper 2010



Two-particle states



Need some smarter way to calculate

- Let's see why perturbation theory fails on the example of LC theory:

$$\sqrt{\frac{R^2}{\ell_s^4} + \frac{4\pi}{\ell_s^2}\left(N + \tilde{N} - \frac{D-2}{12}\right)}$$

$4\pi(N + \tilde{N})$ is a large number

- In fact

$$E = \ell_s^{-1}\mathcal{E}(p_i\ell_s, \ell_s/R)$$

- Need to disentangle these two functional dependences

2-step procedure:

I. Calculate the (infinite-volume) S-matrix

II. Obtain spectrum from this S-matrix

Step I. we just do perturbation theory in $p\ell_s$

$$E = \ell_s^{-1}\mathcal{E}(p_i\ell_s, \ell_s/R)$$

From S-matrix to the spectrum?

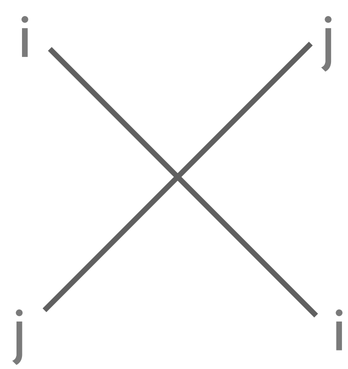
- Should be possible in general, but hard in practice
- Exact procedure - known for integrable 2D theories - Thermodynamic Bethe Anzats (Zamolodchikov 1990)
- Fortunately, our theory is *close to LC integrable theory* - their tree level lagrangians are the same
- Immediately explains why LC is close to the data
- Non-integrability shows up only at one-loop 6pt function

$$\sim (p\ell_s)^6$$

What is TBA:

First, Asymptotic Bethe Anzats:

Consider $\Psi(x_1,x_2) = \langle 0|X^i(x_1)X^j(x_2)|p_L^{(i)},p_R^{(j)}\rangle$



for large R (ignoring wrapping interactions)

$$x_1 > x_2 \quad \Psi(x_1,x_2) = e^{-ip_L x_1} e^{ip_R x_2}$$

$$x_1 < x_2 \quad \Psi(x_1,x_2) = e^{-ip_L x_1} e^{ip_R x_2} e^{2i\delta(p_L,p_R)}$$

Periodicity of the wave function then implies

$$p_{L,R}R + 2\delta(p_L,p_R) = 2\pi n_{L,R}$$

ABA can be inverted to get S-matrix from the spectrum

$$e^{2i\delta(s)} = e^{-i((\Delta E(N,N))-2(\Delta E(N,0)))R/2}$$

Light-Cone quantized NG string:

$$S_{2\to 2} = e^{is\ell_s^2/4}$$ + Factorizability

What is TBA:

Dorey, Tateo '96

$$p^{(i)}_{k\,L} R + \sum_{j,m} 2\delta(p^{(i)}_{k\,L}, p^{(j)}_{m\,R}) + \sum_{j=1}^{D-2} \int_0^\infty \frac{dp'}{2\pi} \frac{d\,2\delta(p^{(i)}_{k\,L}, p')}{dp'} \ln\left(1 - e^{-R\epsilon^j_R(p')}\right) = 2\pi n^{(i)}_{k\,L}$$

$$\epsilon^i_L(p) = p + \frac{1}{R} \sum_{j,k} 2\delta(p, \hat{p}^{(j)}_{k\,R}) + \frac{1}{2\pi R} \sum_{j=1}^{D-2} \int_0^\infty dp' \frac{d\,2\delta(p,p')}{dp'} \ln\left(1 - e^{-R\epsilon^j_R(p')}\right)$$

$$E(R) = R + \sum_{j,k} p^{(j)}_{k\,L} + \sum_{j=1}^{D-2} \int_0^\infty \frac{dp'}{2\pi} \ln\left(1 - e^{-R\epsilon^j_L(p')}\right)$$

+right-movers

Light-Cone quantized string

$$E_{LC}(N,\tilde{N}) = \ell_s^{-1}\sqrt{\frac{R^2}{\ell_s^2} + \frac{4\pi^2\ell_s^2(N-\tilde{N})^2}{R^2} + 4\pi\left(N+\tilde{N}-\frac{D-2}{6}\right)}$$

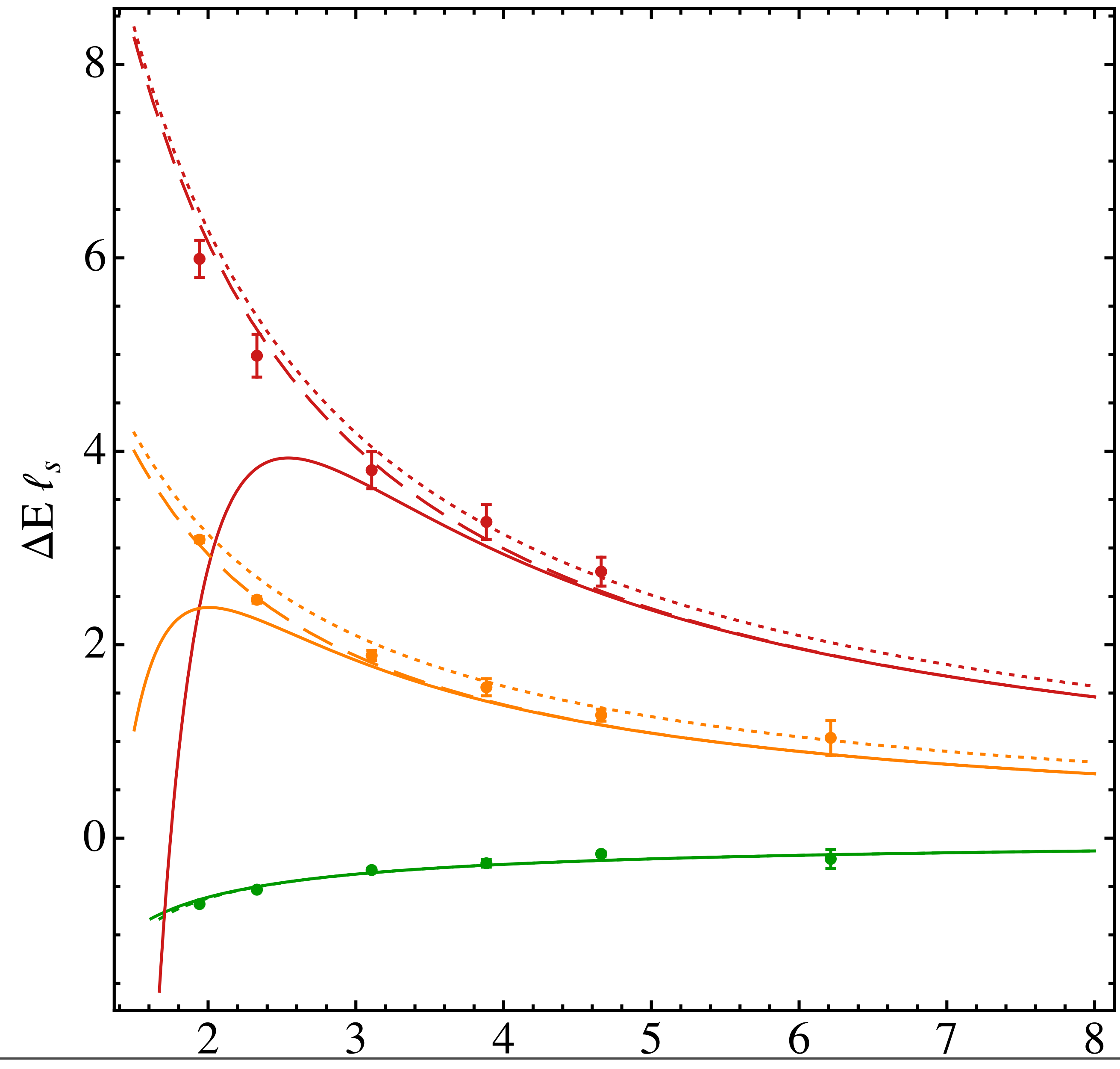
$\updownarrow$

$$S_{LC} = e^{2i\delta_{LC}}\delta_i^k\delta_j^l = e^{is\ell_s^2/4}\delta_i^k\delta_j^l$$

TBA for one-loop phase shift:

$$2\delta = 2\delta_{LC} + 2\delta_{1-loop} = p^2\ell_s^2 \pm \frac{22}{24\pi}p^4\ell_s^4$$

Ground and 1-particle states

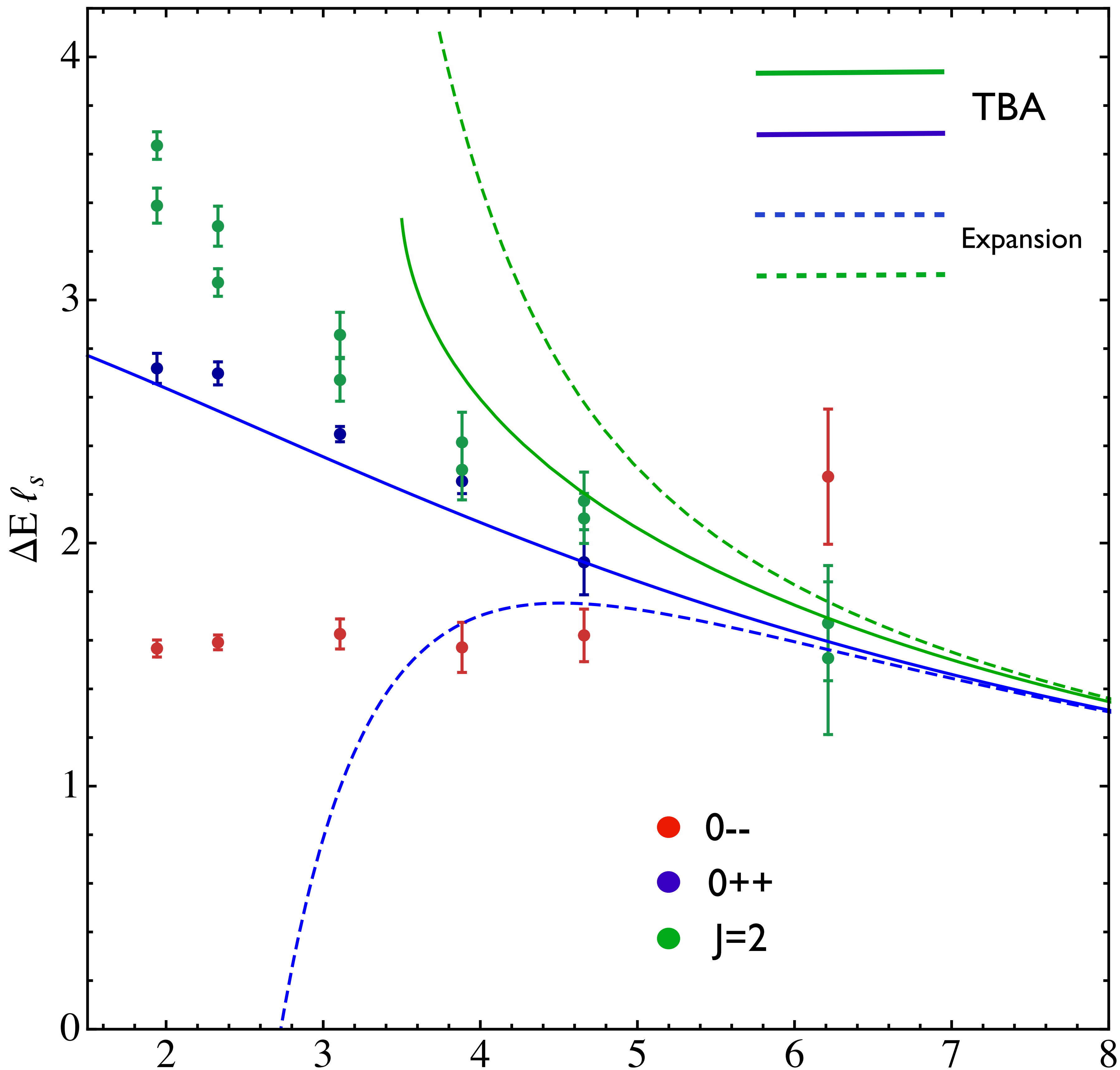


Dashed - TBA

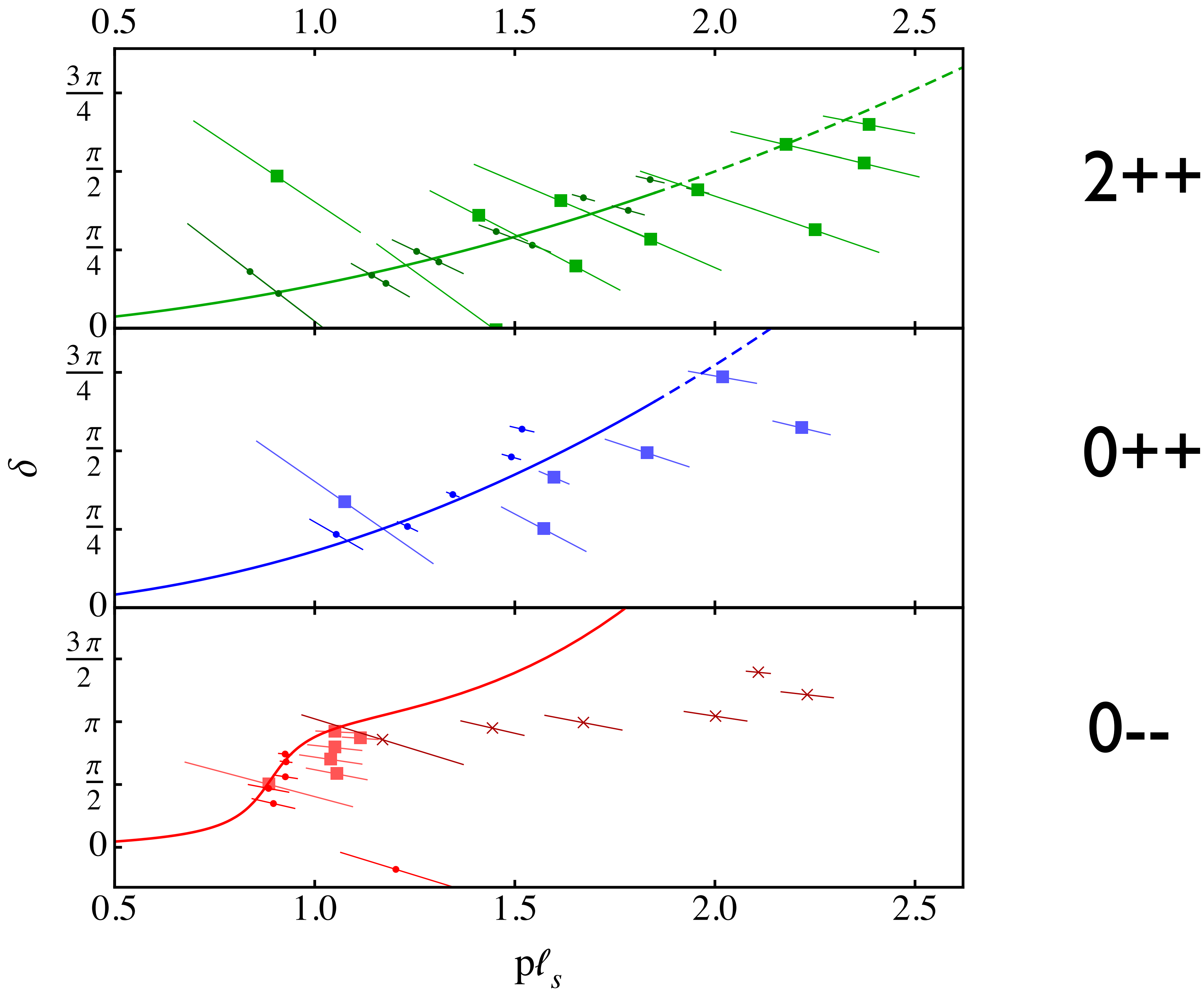
Solid - standard effective field theory expansion

Dotted - free theory (=ABA in this case)

2-particle states: Naive expansion and TBA



Phase shift from excited states



How do we include this massive state?

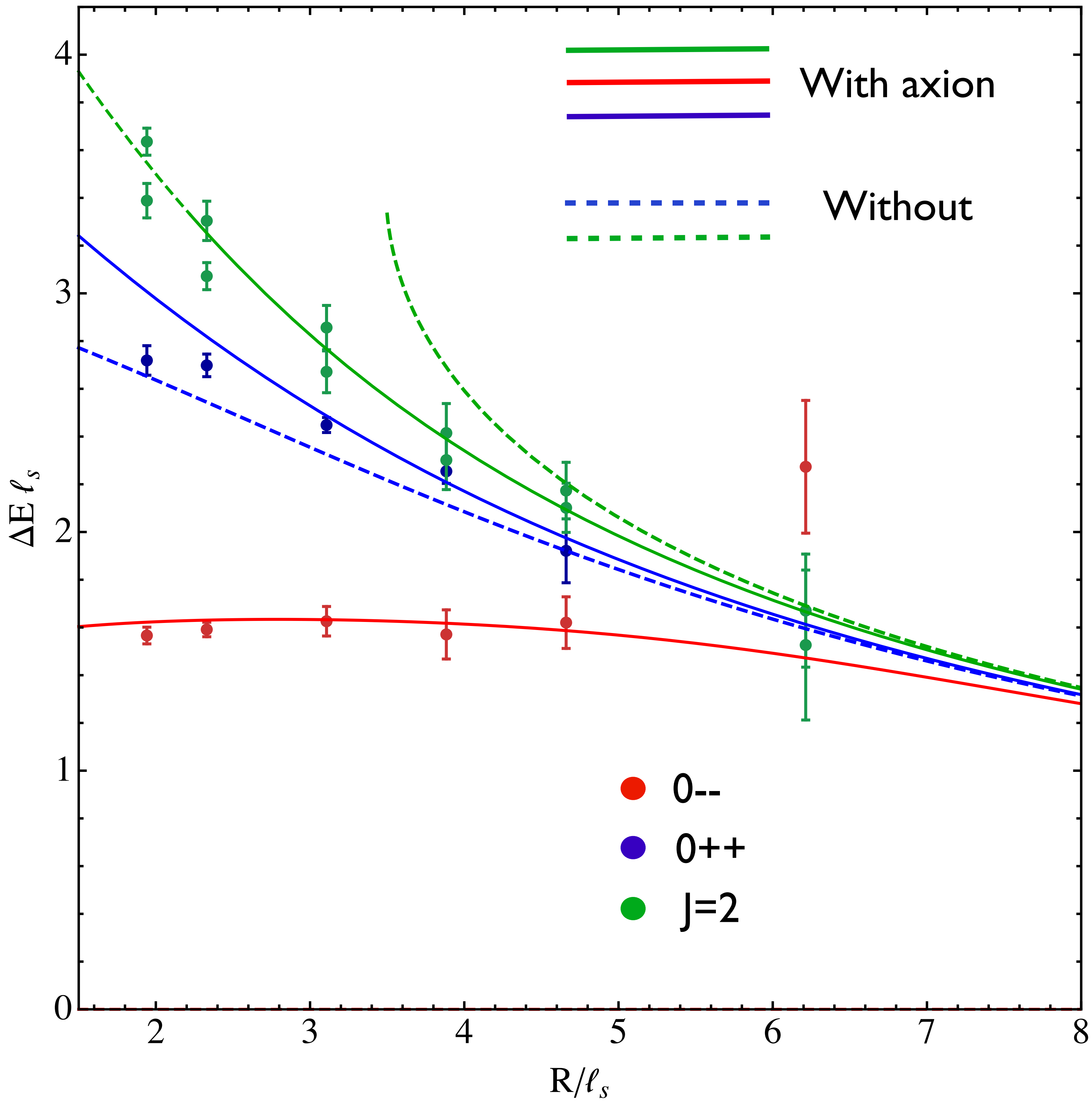
Contributes to scattering of Goldstone's and changes the phase shifts. In particular, it appears as a resonance in the antisymmetric channel. We can calculate contributions from

$$S = \int d^2\sigma \frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - \frac{1}{2}m^2\phi^2 + \frac{\alpha}{8\pi}\phi\epsilon^{\alpha\beta}\epsilon_{ij}K^i_{\alpha\rho}K^j{}_\beta{}^\rho$$

the resonant contribution to the phase shift is

$$\delta = \arctan\left(\frac{m\Gamma(s/m)^3}{m^2 - s}\right)$$

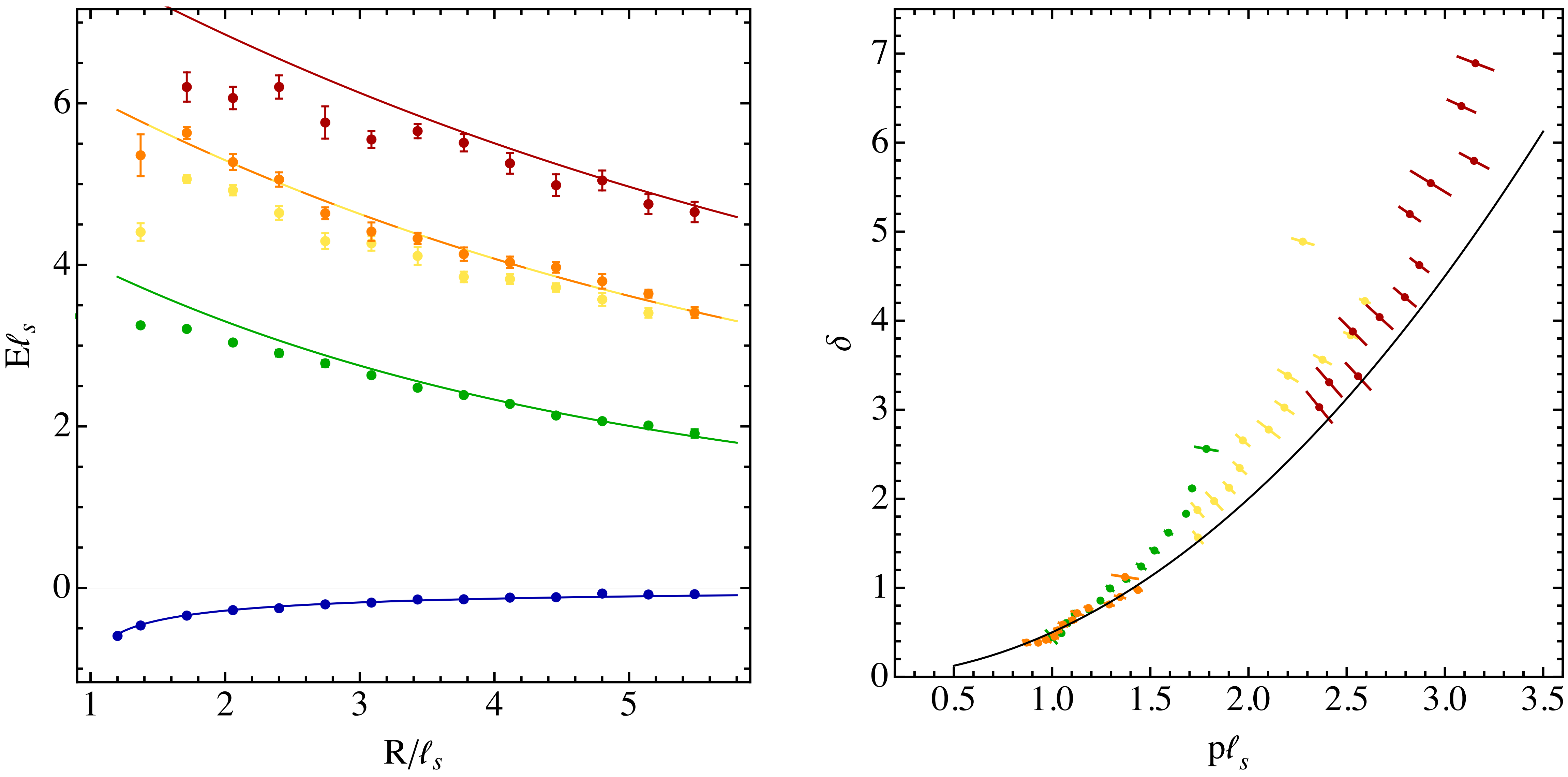
TBA with the resonance



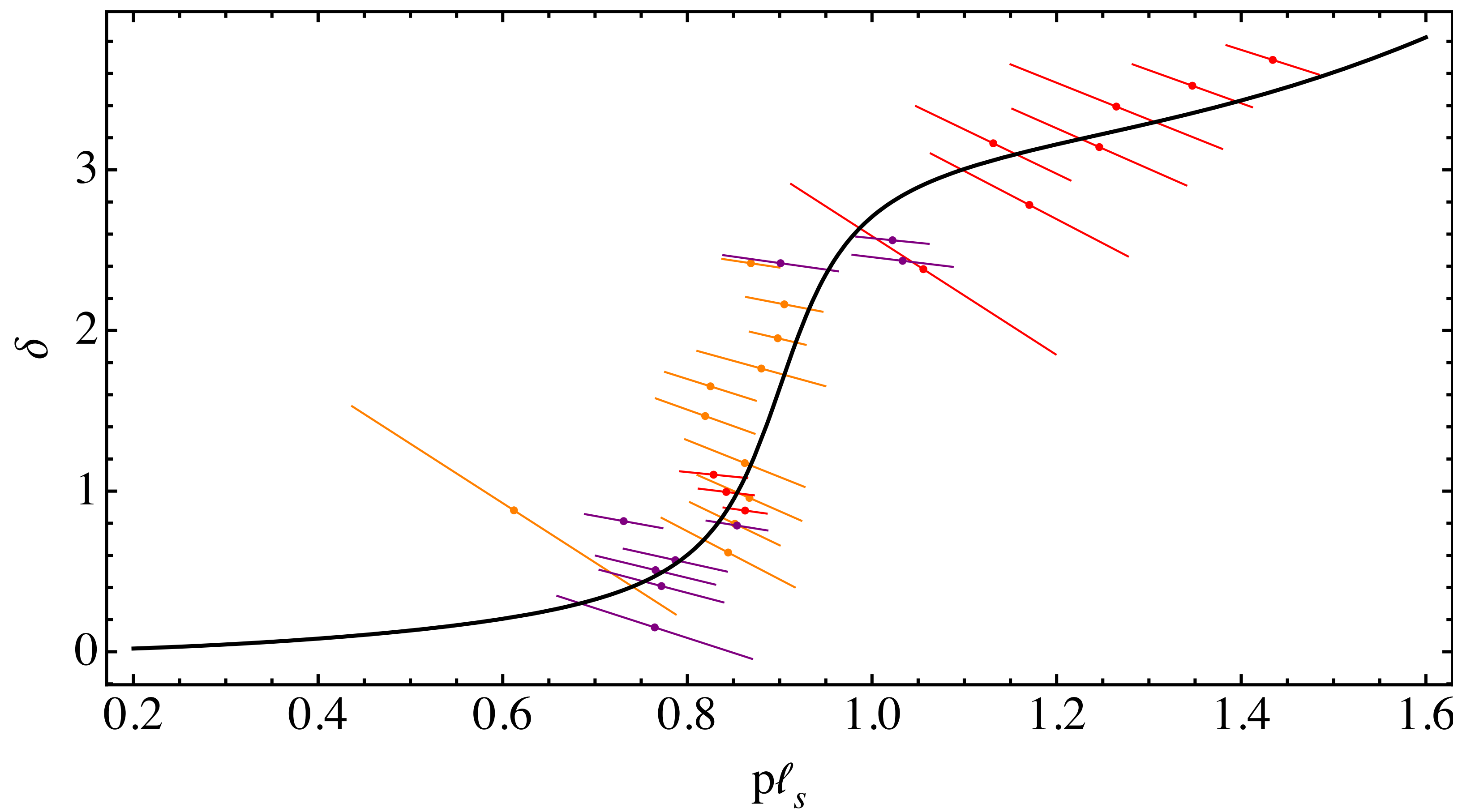
$$m = 1.85\ell_s^{-1}$$

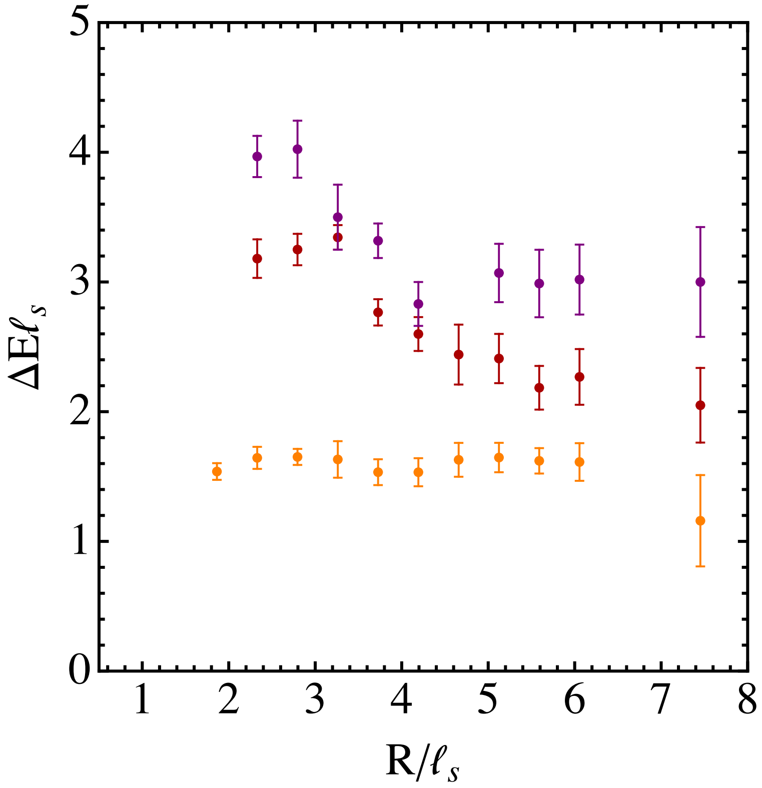
$$\Gamma = 0.4\ell_s^{-1}$$

3D SU(6), q=0 states



3A-string in D=3 SU(6), q=0 states





Worldsheet Axion

$$\epsilon^{ij}\epsilon^{\alpha\beta}K^i_{\alpha\gamma}K^{j\gamma}_{\beta} \sim F\tilde{F}\big|_{ws}$$

$$\int_\Sigma d\sigma d\tau \epsilon^{ij}\epsilon^{\alpha\beta}K^i_{\alpha\gamma}K^{j\gamma}_{\beta} \sim \text{Self-intersection number}$$

$$\epsilon^{\alpha\beta}K^i_{\alpha\gamma}K^{j\gamma}_{\beta} = \nabla_\alpha J^\alpha$$

$$\int d\sigma J^0 \sim \text{twisting}$$

To conclude

- Approximate integrability of world sheet theory allows to calculate flux tube spectrum beyond perturbation theory
- Analysis of lattice data identified the first massive excitation of QCD flux tube
- More lattice studies can reveal more particles
- Same methods can be applied to open strings and perhaps make contact with real experiment

Lorentz-invariantly quantized string

Integral equations reduce to algebraic:

$$p_L R + 2\delta(p_L, p_R) - \frac{\pi}{6Rc_R} p_L = 2\pi N_L$$

$$c_L = 1 + \frac{p_R}{R} - \frac{\pi}{6R^2 c_R}$$

Same for c_L p_R

$$E_{N_L, N_R} = p_L + p_R - \frac{\pi}{6Rc_L} - \frac{\pi}{6Rc_R}$$

Our prediction for the spectrum

$$L_\phi = -M^2\phi^2 - (\partial\phi)^2 + \alpha\phi\epsilon^{ij}\epsilon^{\alpha\beta}K^i_{\alpha\gamma}K^{j\gamma}_\beta$$