18th International Seminar on High Energy Physics QUARKS-2014

Suzdal, Russia, 2-8 June, 2014

Primordial black hole constraints on some models of dissipative inflation (a case of the axion inflation with gauge field production)

Peter Klimai, Edgar Bugaev

(pklimai@gmail.com)

Institute for Nuclear Research of RAS, Moscow

Plan of the talk

- Review of the axion inflation model
- Curvature perturbation power spectrum
- Probability distribution function (PDF) and non-Gaussianity
- Primordial black holes (PBH) constraints on the model
- Comparison with other works
- Conclusions

Based on

E.V. Bugaev & P.A. Klimai, arXiv:1312.7435 [astro-ph.CO]

The Model

It is widely assumed that early Universe went through an inflationary phase of accelerated expansion. To drive inflation, a vacuum-like equation of state is needed, which can be provided by one or more scalar fields.

One problem of inflationary paradigm is the need to have a rather flat inflaton potential, so that “slow-roll” parameters

$$\epsilon = \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 , \quad \eta = M_p^2 \frac{V''}{V} .$$

are small.

In large-field inflation models which are phenomenologically favorable due to the generation of observable amount of tensor modes, infinite number of higher-dimensional corrections are generally expected.

A simple solution to this problem is to assume that the inflaton is a pseudo-Nambu-Goldstone boson (PNGB). In this case the inflaton enjoys a shift symmetry $\varphi \to \varphi + \mathrm{const}$ which is broken at non-perturbative level due to instanton effects as well as explicitly. The slow-roll parameters in this case are under control due to smallness of symmetry breaking.

PNGBs arise rather often in models of particle physics. This happens every time when approximate global symmetry is spontaneously broken (e.g. QCD axion [Peccei & Quinn (1977)]). Axion-like particles are abundant in string theory as well [see review in Cicoli & Quevedo, CQG 28 (2011) 204001].

The first explicit example of axion inflation was the natural inflation model [Freese, Frieman, Olinto, PRL 65, 3233(1990)] in which the shift symmetry is broken down to a discrete subgroup, resulting in a periodic (due to instanton effects) potential

$$V(\varphi) = \Lambda^4 \left[1 - \cos\left(\frac{\varphi}{f}\right)\right]$$

(*f* is called the axion decay constant). This potential, unfortunately, is only able to drive slow-roll inflation for $f > M_P$.

Such large values of *f* are not allowed in UV-complete string theory [Banks, Dine, Fox, Gorbatov, JCAP 0306, 001 (2003); Arkani-Hamed, Motl, Nicolis, Vafa, JHEP 0706, 060 (2007).]

There are several groups of models in which the large field inflation is possible with sub-Planckian axion decay constants: "Racetrack inflation" [J.J. Blanco-Pillado et al, JHEP 0411, 063 (2004)], N-flation [S.Dimopoulos et al, JCAP 0808, 003 (2008)], assisted inflation [A.Liddle et al, PRD 58, 061301 (1998); E. Copeland et al, PRD 60,083506 (1999)], axion monodromy inflation [L. McAllister et al, PRD 82, 046003 (2010); R. Flauger et al, JCAP 1006, 009 (2010); N.Kaloper, PRL 102, 121301 (2009); N.Kaloper et al, JCAP 1103, 023 (2011); M. Berg et al, PRD 81, 103535 (2010)].

In particular, L. McAllister et al, PRD 82, 046003 (2010) showed that in IIB string theory the presence of suitable wrapped branes leads to the potential energy of the axion no longer being a periodic function of it. Rather, it grows linearly with the field:

$$V_{sr}(\varphi) \approx \mu^3 \varphi$$

The different realization of the monodromy idea (not based on string theory) is contained in [N.Kaloper et al, PRL 102, 121301 (2009); N.Kaloper et al, JCAP 1103, 023 (2011)]. In these works by introducing in the action the coupling of axion to a 4-form, the quadratic potential is obtained:

$$V_{sr}(\varphi) = \frac{m^2\varphi^2}{2}$$

The same potential has been introduced in the original work on chaotic inflation [A.Linde, Phys. Lett. B 129, 177 (1983)].

Generally,

$$V(\varphi) = V_{sr}(\varphi) + V_{inst}(\varphi)$$

In this work, we will assume that the second term is subdominant.

In order to have a reheating phase, inflaton must be coupled to something. In axion inflation models, there generally exists a coupling to U(1) gauge fields of the form (allowed by the shift symmetry) [Anber & Sorbo, JCAP 0610, 018 (2006); PRD 81, 043534 (2010)]

$$\mathcal{L}_{int} = -\frac{\alpha}{4f}\varphi F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\tilde{F}^{\mu\nu} = \eta^{\mu\nu\omega\theta} F_{\omega\theta}/(2\sqrt{-g})$$

Such a coupling leads to a production of gauge quanta during inflation.

Due to inverse decay of these quanta into inflaton perturbations, additional scalar perturbations are produced in this model.

The expected signatures are a rise of non-Gaussianity effects and violation of scale-invariance for perturbation amplitudes.

In particular, a rather essential primordial black hole (PBH) formation becomes possible [C.-M. Lin, K.-W.Ng, Phys. Lett. B 718, 1181 (2013); A.Linde, S.Mooij, E.Pajer, Phys.Rev. D87 103506 (2013)]. At the same time, the model is consistent with observations on CMB scales.

The equations of motion for the inflaton field are

$$\ddot{\varphi} + 3H\dot{\varphi} + V' = \frac{\alpha}{f}\langle \vec{E} \cdot \vec{B} \rangle$$

$$\vec{B} \equiv \frac{1}{a^2}\vec{\nabla} \times \vec{A}\,, \qquad \vec{E} \equiv -\frac{1}{a^2}\vec{A}'$$

$$3H^2 M_P^2 = \frac{1}{2}\dot{\varphi}^2 + V + \frac{1}{2}\langle \vec{E}^2 + \vec{B}^2 \rangle$$

For the gauge field, the following equation (after decomposing to Fourier modes) and solution are obtained [Anber & Sorbo, PRD 81, 043534 (2010)] :

$$\frac{d^2 A_\pm(\tau, k)}{d\tau^2} + \left[k^2 \pm 2\,k\,\frac{\xi}{\tau}\right] A_\pm(\tau, k) = 0$$

$$\xi \equiv \frac{\alpha\dot{\varphi}}{2fH}$$

$$A_+(\tau, k) \simeq \frac{1}{\sqrt{2\,k}}\left(\frac{k}{2\,\xi\,aH}\right)^{1/4} e^{\pi\,\xi - 2\sqrt{2\xi\,k/aH}}\,.$$

$$(8\xi)^{-1} \lesssim k/(aH) \lesssim 2\xi$$

$$\langle \vec{E} \cdot \vec{B} \rangle \approx -2.4 \times 10^{-4}\,\frac{H^4}{\xi^4} e^{2\pi\xi}$$

$$\frac{1}{2}\langle \vec{E}^2 + \vec{B}^2 \rangle \approx 1.4 \times 10^{-4}\,\frac{H^4}{\xi^3} e^{2\pi\xi}$$

The power spectrum normalization on the CMB scales $\mathcal{P}_\zeta(k_*) \approx 2.4 \times 10^{-9}.$

For $V_{sr}(\varphi) = \dfrac{m^2\varphi^2}{2}$:

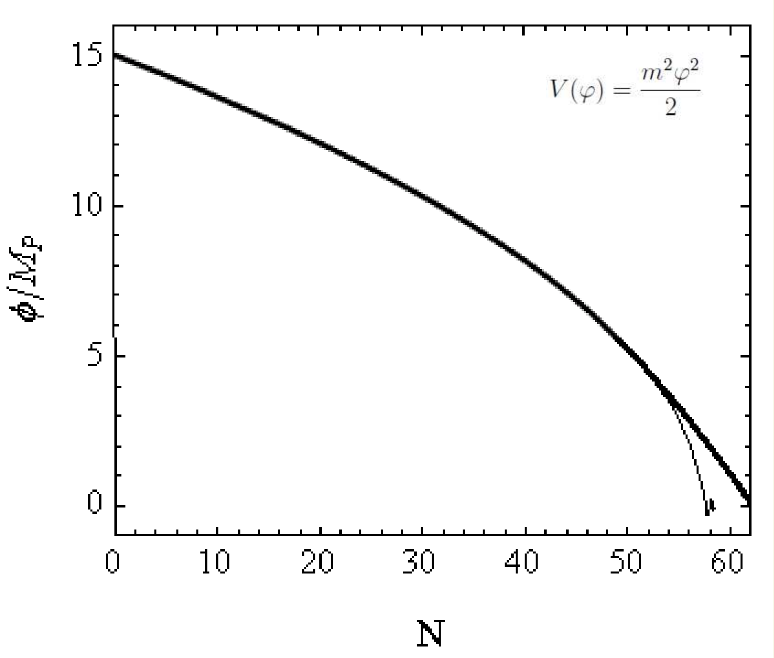
$m = 6.8 \times 10^{-6} M_P$

$\varphi_0 \approx 15 M_P$

$H_0 \approx 4.2 \times 10^{-5} M_P$

$r = 0.14$

BICEP2 result [Ade et al, arxiv:1403.3985] $r = 0.20^{+0.07}_{-0.05}$



For $V_{sr}(\varphi) \approx \mu^3\varphi$:

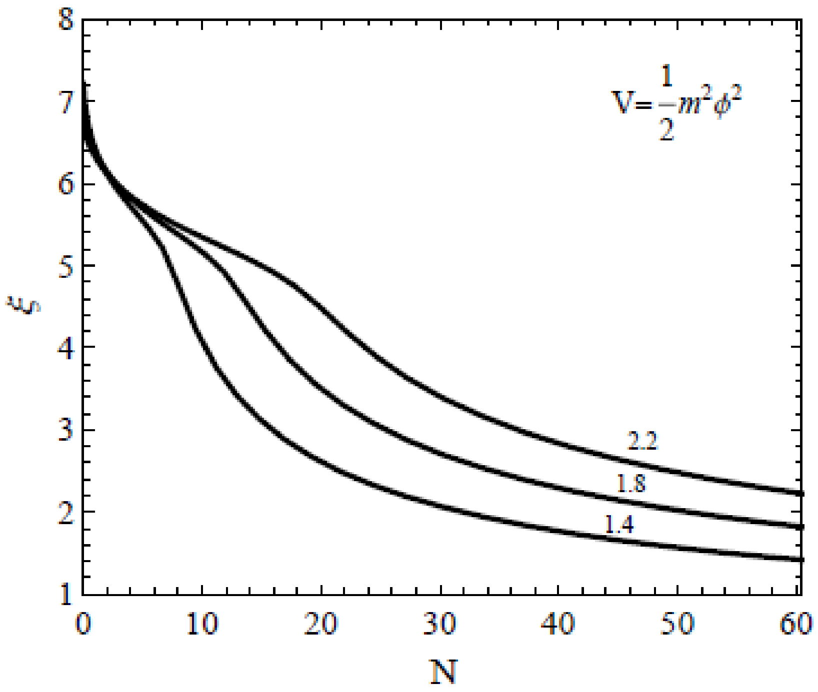
$\mu \approx 6.3 \times 10^{-4} M_P$

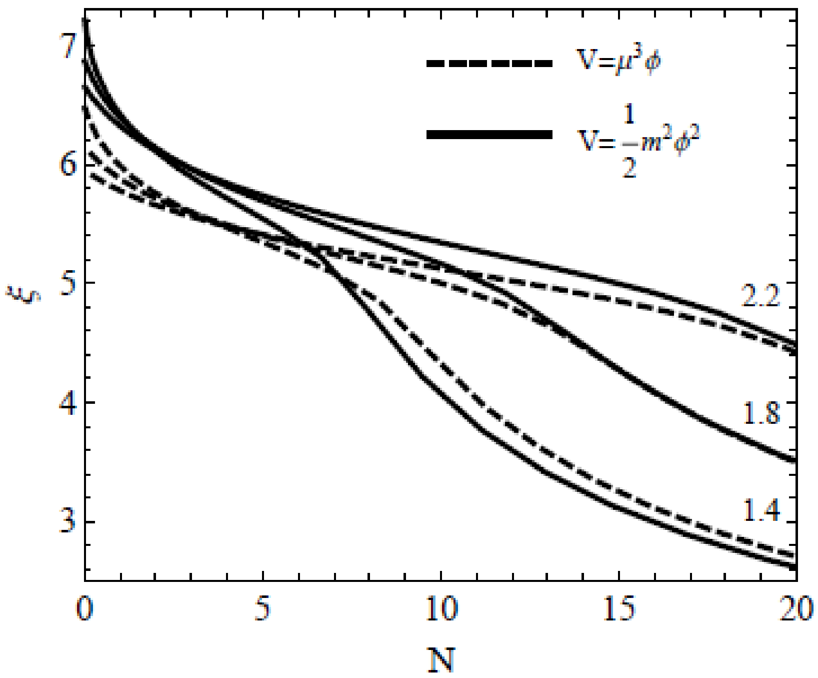
$\varphi_0 \approx 10.6 M_P,$

$H_0 \approx 2.9 \times 10^{-5} M_P$

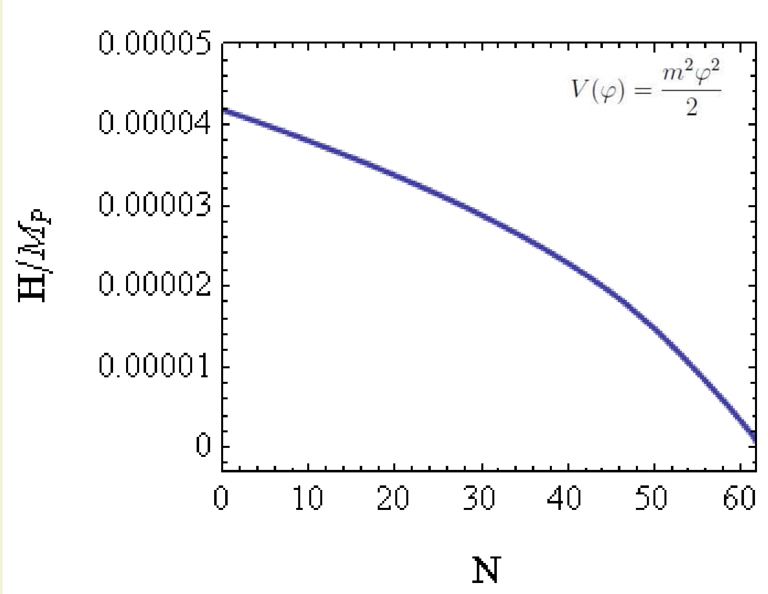
$r = 0.07$

Thick line is full calculation, thin line is calculation without including E,B terms in the background equations





The dependence of parameter ξ on *N* is shown.



Hubble parameter during inflation as a function of *N*.

Equation for field perturbations [Anber & Sorbo, PRD 81, 043534 (2010)]

$$\delta\ddot{\varphi} + 3\beta H\delta\dot{\varphi} - \frac{\nabla^2}{a^2}\delta\varphi + V''\delta\varphi = \frac{\alpha}{f}\left[\vec{E}\cdot\vec{B} - \langle\vec{E}\cdot\vec{B}\rangle\right]$$

$$\beta \equiv 1 - 2\pi\xi\frac{\alpha}{f}\frac{\langle\vec{E}\cdot\vec{B}\rangle}{3H\dot{\varphi}}$$

Note the quadratic dependence on A in r.h.s. (strong non-Gaussianity).

The approximate solution [A.Linde, S.Mooij, E.Pajer, Phys.Rev. D87 103506 (2013)] of this equation at horizon crossing (k=aH) is obtained by approximating d/dt=H (1st and 3rd terms in l.h.s. cancel each other) and neglecting 4th term:

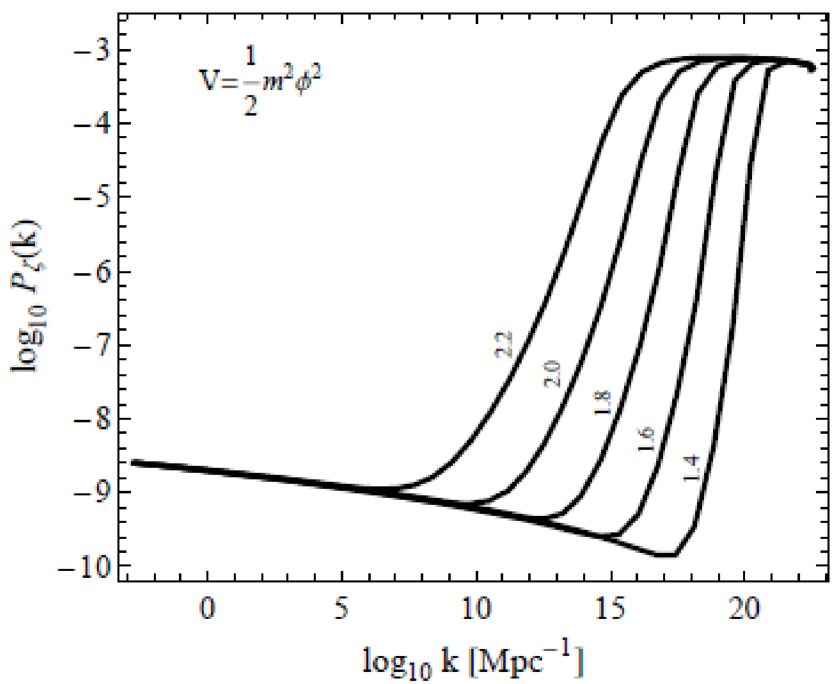
$$\delta\varphi \approx \frac{\alpha}{f}\frac{\left(\vec{E}\cdot\vec{B} - \langle\vec{E}\cdot\vec{B}\rangle\right)}{3\beta H^2}$$

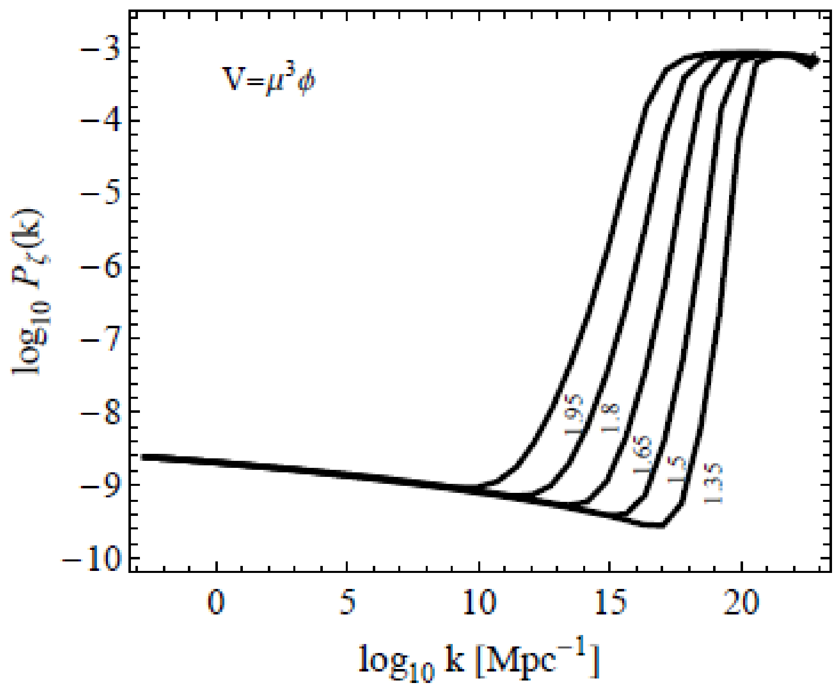
For the curvature perturbation

$$\langle\zeta(x)^2\rangle = \frac{H^2}{\dot{\varphi}^2}\langle\delta\varphi^2\rangle \approx \frac{\alpha^2}{f^2}\frac{\langle\vec{E}\cdot\vec{B}\rangle^2}{(3\beta H\dot{\varphi})^2}.$$

$$\mathcal{P}_\zeta(k) \approx \langle\zeta(x)^2\rangle$$

The same equation can be solved by Green's function method [Anber & Sorbo, PRD 81, 043534 (2010); N. Barnaby et al JCAP 1104, 009 (2011)] and the result is compatible within a factor of ~ 2.





The curvature perturbation power spectrum as a function of k for different values of ξ at CMB scales.

The contribution from vacuum fluctuations of the inflaton is included. It is dominant on the CMB scales and calculated using standard "slow-roll" formula

$$\mathcal{P}_{\mathcal{R}}(k) \approx \left(\frac{H^2}{2\pi|\dot{\phi}|}\right)^2$$

Primordial Black Holes

The idea of possible black hole production in the early Universe was stated by Zeldovich & Novikov (1967) and Hawking (1971). Primordial density perturbations could collapse and form a PBH.

The classical PBH formation criterion in the radiation-dominated epoch is formulated in terms of smoothed density contrast at horizon crossing [Carr & Hawking, MNRAS 168, 399 (1974)]: $\delta > \delta_c \approx 1/3,$

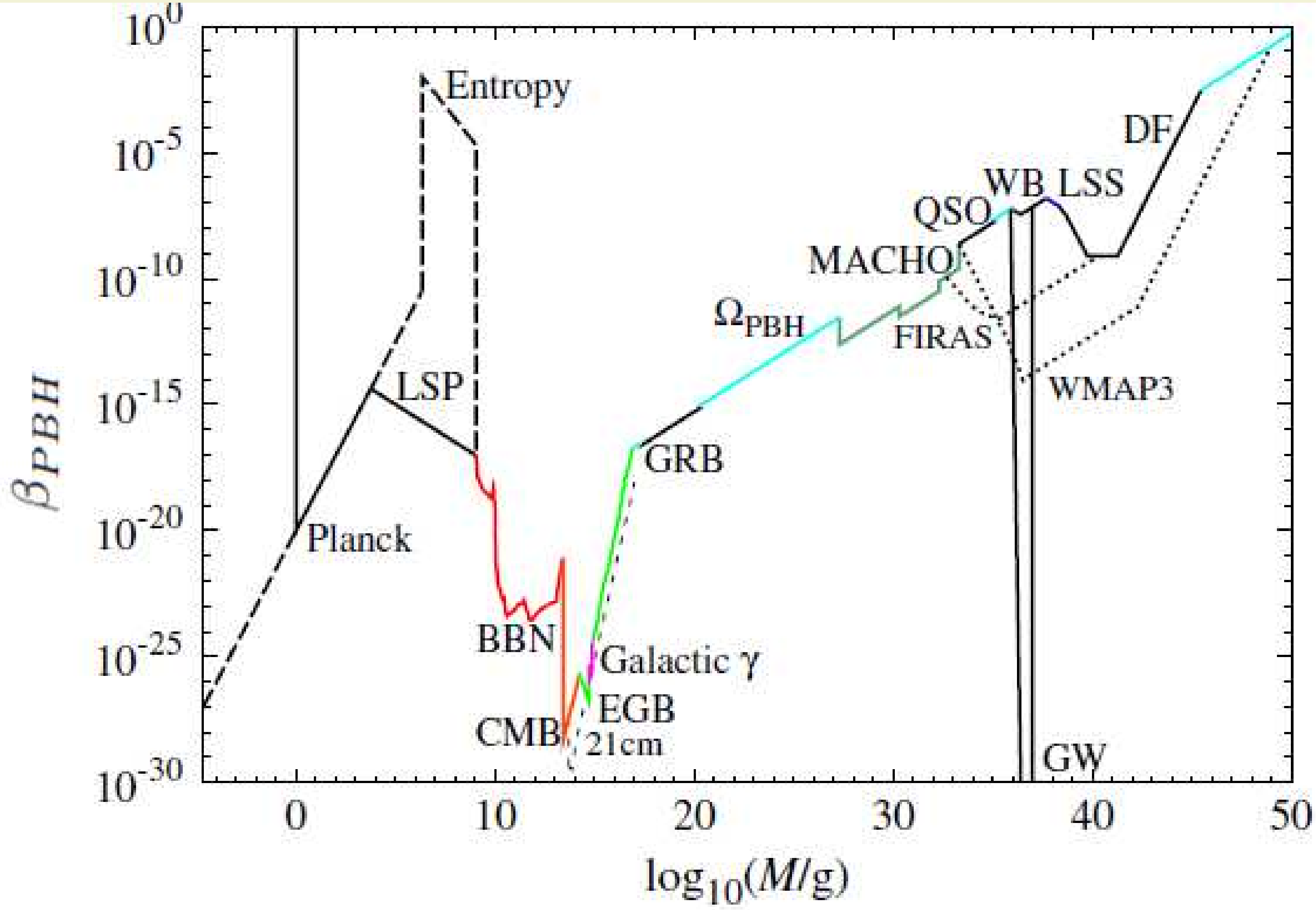
using well-known relations, this can be translated to a PBH formation threshold in terms of the curvature perturbation $\zeta_c = \frac{9}{4}\,\delta_c \approx 0.75.$

Or, for somewhat larger $\delta_c \approx 0.45$ (see e.g. Musco et al, CQG 26, 235001 (2009)], $\zeta_c \approx 1.$

The process of Hawking (1974) – PBH evaporation with $T_H = 1.06 \left(\frac{M}{10^{13}\mathrm{g}}\right)^{-1} \mathrm{GeV}.$

and evaporation time ~ $10^{-28}(M/1\mathrm{g})^3$ s. PBHs with initial mass ~ 10^{15} g, have a lifetime equal to present Universe’s age.

The available constraints on PBH abundance



Constraints on energy density fraction of the Universe contained in PBHs at moment of their formation, β_{PBH}, as a function of PBH (or horizon) mass.

From: Carr et al, PRD 81, 104019 (2010).

PBH mass spectra

The energy density fraction of the Universe contained in collapsed objects of initial mass larger than *M* in Press-Schechter (1974) formalism is given by

$$\frac{1}{\rho_i}\int\limits_M^\infty \tilde{M} n(\tilde{M}) d\tilde{M} = \int\limits_{\zeta_c}^\infty p_\zeta(\zeta) d\zeta = P(\zeta > \zeta_c; R(M), t_i),$$

where function *P* is the probability that in the region of comoving size *R* the smoothed value of ζ will be larger than the PBH formation threshold value, *n(M)* is the mass spectrum of the collapsed objects. For it,

$$n(M) = 2\frac{\rho_i}{M}\left|\frac{\partial P}{\partial R}\right|\frac{dR}{dM}$$

Using

$$M = \frac{4\pi}{3}\rho_i (a_i R)^3 \qquad M_i \approx \frac{4\pi}{3} t_i^3 \rho_i \approx \frac{4\pi}{3}(H_c^{-1})^3 \rho = \frac{4\pi M_P^2}{H_c} \qquad M_h = M_i^{1/3} M^{2/3}$$

and the approximation that mass of PBH is proportional to horizon mass,

$$M_{BH} = f_h M_h = f_h M_i^{1/3} M^{2/3}, \qquad f_h \approx (1/3)^{1/2} = \mathrm{const}$$

we obtain

$$n_{BH}(M_{BH}) = n(M)\frac{dM}{dM_{BH}} = \left(\frac{4\pi}{3}\right)^{-1/3}\left|\frac{\partial P}{\partial R}\right|\frac{f_h \rho_i^{2/3} M_i^{1/3}}{a_i M_{BH}^2}.$$

PBH Constraints on the Model

For a derivation of the PBH constraints, we need an expression for the probability distribution function (PDF) of the curvature perturbation field.

In our case, the simple assumption we can use is the following: ζ -field is distributed as a square of some Gaussian field χ :

$$\zeta = A(\chi^2 - \langle \chi^2 \rangle)$$

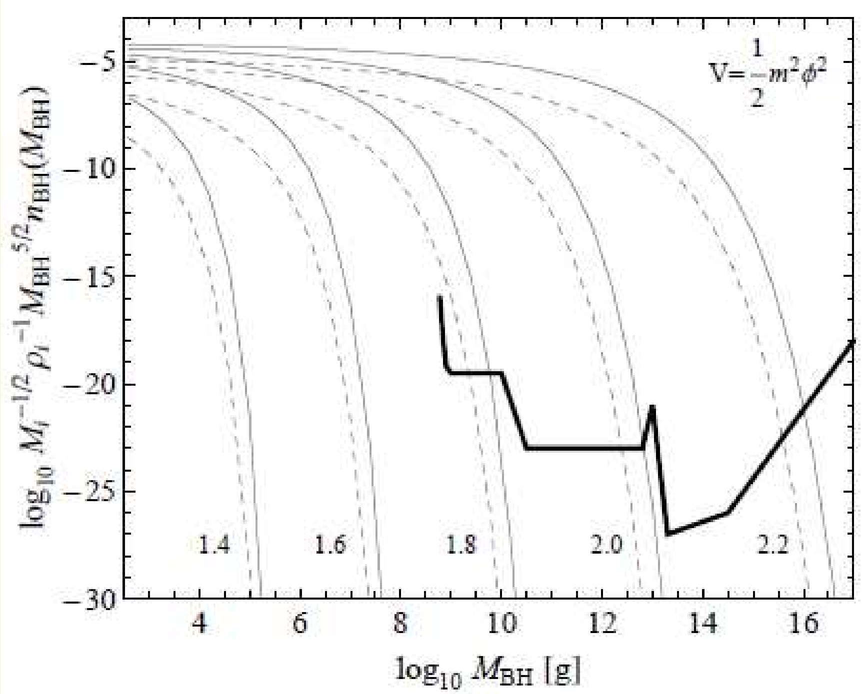
This is done having in mind the fact that particular solution for perturbations of the inflaton field is bilinear in the field *A*,

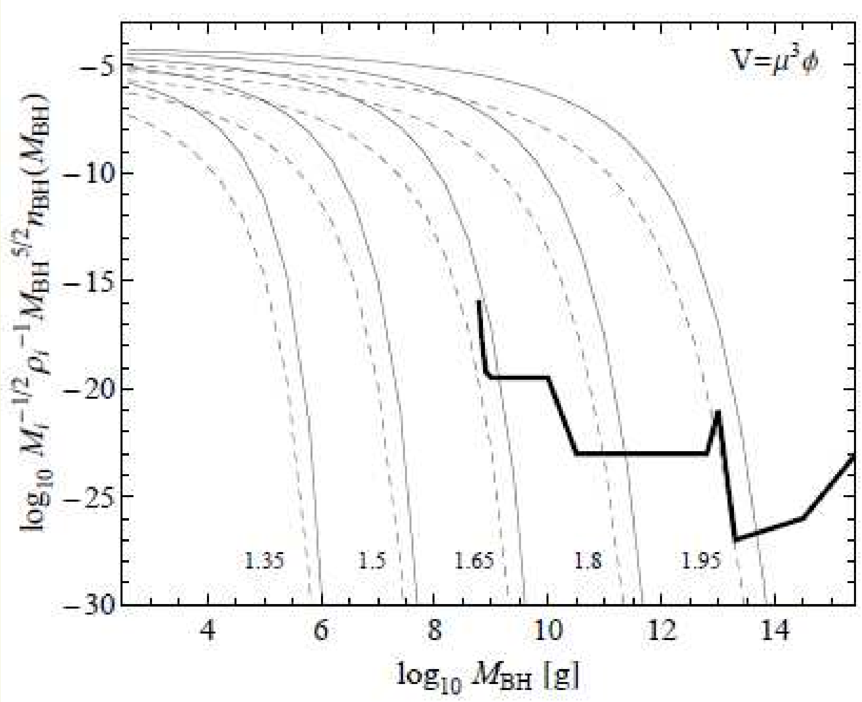
$$\delta\ddot{\varphi} + 3\beta H \delta\dot{\varphi} - \frac{\nabla^2}{a^2}\delta\varphi + V''\delta\varphi = \frac{\alpha}{f}\left[\vec{E}\cdot\vec{B} - \langle\vec{E}\cdot\vec{B}\rangle\right]$$

In this case

$$p_\zeta(\zeta) = \frac{1}{A\sqrt{\frac{\zeta}{A} + \langle\chi^2\rangle}}\, p_\chi\left(\sqrt{\frac{\zeta}{A} + \langle\chi^2\rangle}\right)$$

$$p_\chi(\chi) = \frac{1}{\sigma_\chi\sqrt{2\pi}} e^{-\frac{\chi^2}{2\sigma_\chi^2}}, \qquad \sigma_\chi^2 \equiv \langle\chi^2\rangle$$





Comparing the number of PBHs that are produced in the model with the existing cosmological constraints on their possible number (thick line), we can put a limit for quadratic potential

$$\xi_{CMB} < 1.75 \quad (\alpha/f < 26 M_P^{-1})$$

The corresponding limit for the linear potential is a bit stronger,

$$\xi_{CMB} < 1.65 \quad (\alpha/f < 35 M_P^{-1})$$

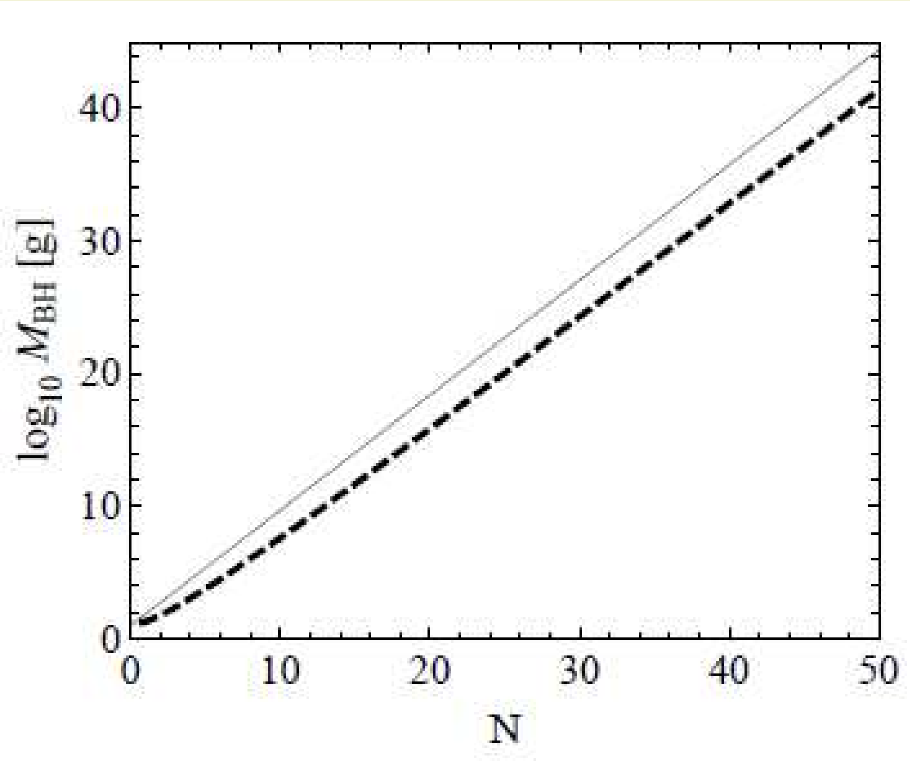
From data on the CMB scales [see Pajer & Peloso, Class.Quant.Grav. 30, 214002 (2013)],

$\xi_{CMB} <$ *(2.1 – 2.4)*.

In work Linde et al, Phys.Rev. D87 (2013) 103506 the following constraint from PBH production was obtained for this model (the case of quadratic potential was considered):

$$\xi_{CMB} < 1.5.$$

In our approach, we calculated PBH constraints using full machinery of Press-Schechter formalism rather than the simple integral over the PDF of the curvature field.



The mass of the produced PBH depending on the number N of inflation e-folds. Dashed line is calculation using the formula that we use

$$M_{BH} = \frac{f_h M_{eq} k_{eq}^2}{a_e^2} \frac{e^{2N}}{H(N)^2},$$

Solid line is calculated using

$$M_{BH} = 10 e^{2N} \mathrm{g}$$

(Linde et al, PRD 87 (2013) 103506).

Conclusions

❖ We have calculated the primordial curvature perturbation power spectrum in the particular inflationary model in which an axion-like pseudoscalar field is coupled to a gauge field

❖ PBH mass spectrum was calculated using Press-Schechter formalism for such model, under two different assumptions on the model potential form and taking into account the non-Gaussianity of perturbations

❖ Limits on the model's parameters were obtained from known constraints on PBH abundance

Contact: pklimai@gmail.com