SUPERSYMMETRIC SYSTEMS WITH BENIGN GHOSTS

Suzdal, June 4 2014

based on
D. Robert + A.S., JMP, **49**:042014 (2008)
and
A.S., arXiv:1306.6066 [hep-th]

MOTIVATION:

Problems with causality in quantum and classical gravity.

Dream solution:

[A.S., 2005]

- our Universe as a soap film in a flat higher dimensional bulk. The TOE is a field theory in this bulk.
- To be renormalizable, it should involve higher derivatives

DANGER: the ghosts

• GHOSTS = instability (rather *absence*) of vacuum

• inherent for higher-derivative theories.

Conventional system

$$E = \frac{\dot{q}^2}{2} + V(q)$$

can have a classical and/or quantum bottom

• Consider the Pais-Uhlenbeck oscillator.

$$L \;=\; \frac{1}{2}(\ddot{q} + \Omega^2 q)^2 \;.$$

Then

$$E \;=\; \ddot{q}(\ddot{q} + \Omega^2 q) - \dot{q}(q^{(3)} + \Omega^2 \dot{q}) - \frac{1}{2}(\ddot{q} + \Omega^2 q)^2$$

can be as negative as one wishes.

• Common lore: negative residues in propagators break unitarity.

NOT TRUE!!

• No problem in free theories

• Interactions may lead to collapse and breaking of unitarity. Like falling into the center in the attractive potential $\sim 1/r^2$.

• If quantum theory is sick, so is its classical counterpart. If classical theory is benign, so is its quantum counterpart

- Adding simple-minded nonlinear terms $\sim q^4$ or $\sim q^2\dot{q}^2$ to the Pais-Uhlenbeck Lagrangian does lead to collapse.
- Islands of stability

Benign nonlinear SQM system with ghosts
[D. Robert + A.S., 2006]

$$S = \int dt dx d\bar{\theta} d\theta \left[\frac{i}{2} \bar{\mathcal{D}}\Phi \frac{d}{dt} \mathcal{D}\Phi + V(\Phi) \right] ,$$

with the real (0+1)-dimensional superfield

$$\Phi = \phi + \theta\bar{\psi} + \psi\bar{\theta} + D\theta\bar{\theta}$$

- An extra time derivative.

The Hamiltonian

$$H = pP - DV'(\phi) + \text{fermion term}$$

is not positive definite.

Integrals of motion:

1. $E \equiv H$
2. $N = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

- Exactly solvable.
- Take

$$V(\Phi) = -\frac{\omega^2\Phi^2}{2} - \frac{\lambda\Phi^4}{4}\,,$$

- Explicit solutions

$$\phi(t) \;=\; \phi_0\,\mathrm{cn}[\Omega t|m]$$

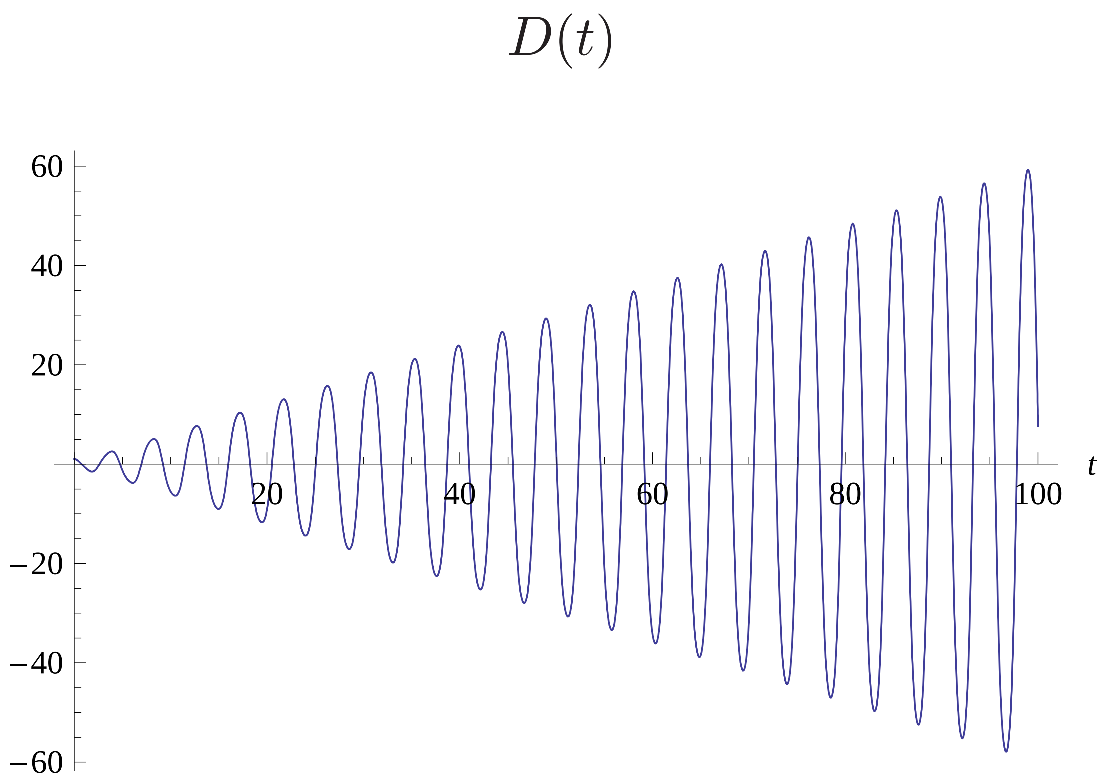
with

$$\alpha = \frac{\omega^4}{\lambda N}, \quad \Omega = [\lambda N(4+\alpha)]^{1/4}, \quad m = \frac{1}{2}\left[1 - \sqrt{\frac{\alpha}{4+\alpha}}\right],$$

$$\phi_0 \;=\; \left(\frac{N}{\lambda}\right)^{1/4}\sqrt{\sqrt{4+\alpha}-\sqrt{\alpha}}$$

$$D(t) \propto \dot{\phi}(t)\int^t \frac{dt'}{\dot{\phi}^2(t')}$$

- $\phi(t)$ is bounded.
- $D(t)$ grows linearly.



$D(t)$
60
40
20
−20
−40
−60
20
40
60
80
100
t

Quantum problem

- is also exactly solvable.
- If $\lambda = 0$, one can define $\{x_\pm, p_\pm\}$ such that

$$H_B = \frac{p_+^2 + \omega^2 x_+^2}{2} - \frac{p_-^2 + \omega^2 x_-^2}{2} \ .$$

with $E_{n_+, n_-} = \omega(n_+ - n_-)$.

- Infinite degeneracy at each level.

In interacting case:

- Still an infinity of zero energy states
- Other states form a continuous spectrum, $E_{\mathrm{cont}} \in [-\infty, -\omega] \cup [\omega, \infty]$.
- Explicit expressions for wave functions exist.
- Evolution operator is unitary.

UNUSUAL ALGEBRAIC STRUCTURES

full canonical Hamiltonian

$$H = pP - DV'(x) + \bar{\psi}\bar{\chi} - V''(x)\chi\psi \ .$$

is not Hermitian

- reality of spectrum $\longrightarrow$ crypto-Hermiticity !

Supercharges

$$\begin{aligned} Q &= \psi[p + iV'(x)] - \bar{\chi}(P - iD) \ , \\ \bar{Q} &= \bar{\psi}(P + iD) - \chi[p - iV'(x)] \ . \end{aligned}$$

and the extra pair

$$\begin{aligned} T &= \psi[p - iV'(x)] + \bar{\chi}(P + iD) \ , \\ \bar{T} &= \bar{\psi}(P - iD) + \chi[p + iV'(x)] \ . \end{aligned}$$

Mixed model

$$L = \int d\bar{\theta} d\theta \left[\frac{i}{2} (\bar{\mathcal{D}} X) \frac{d}{dt} (\mathcal{D} X) + \frac{\gamma}{2} \bar{\mathcal{D}} X \mathcal{D} X + V(X) \right] .$$

Physics is similar to the model with $\gamma = 0$, but

- not integrable anymore
- No linear growth for $D(t)$

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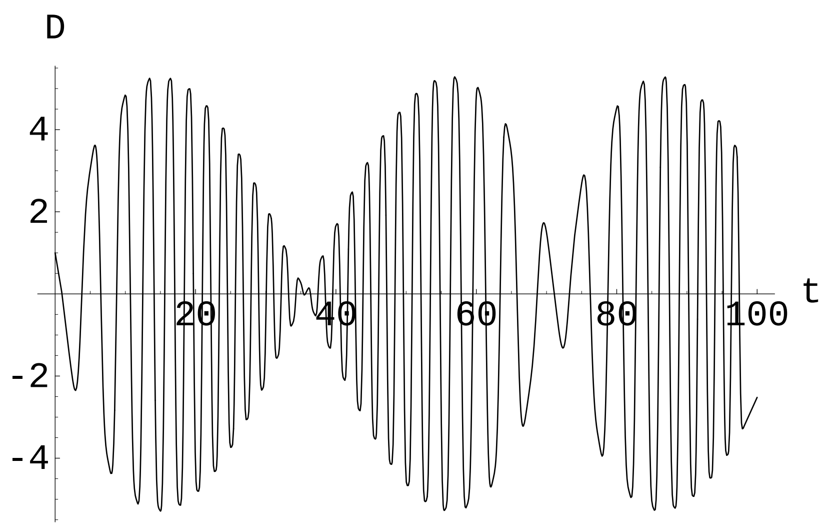


Figure 1: The function $D(t)$ for a deformed system ($\omega = 0, \lambda = 1, \gamma = .1$).

algebra is deformed

- Let $H = H_0 - \gamma F/2$ and

$$F = \psi\bar{\psi} - \chi\bar{\chi}, \qquad F_+ = \bar{\chi}\psi, \qquad F_- = \bar{\psi}\chi$$

then

$$\begin{aligned}
[F_\pm, F] = \mp 2F_\pm, \quad [F_+, F_-] = F\ ,\\
[Q, H_0 = -\frac{\gamma}{2}Q, \ [\bar{Q}, H_0] = \frac{\gamma}{2}\bar{Q},\\
[T, H_0] = \frac{\gamma}{2}T, \ [\bar{T}, H_0] = -\frac{\gamma}{2}\bar{T}\ ,\\
[Q, F] = -Q, \quad [\bar{Q}, F] = \bar{Q},\\
[T, F] = T, \quad [\bar{T}, F] = -\bar{T}\ ,\\
[Q, F_-] = \bar{T}, \ [\bar{Q}, F_+] = -T,\\
[T, F_-] = -\bar{Q}, \quad [\bar{T}, F_+] = Q\ ,\\
\{Q, \bar{Q}\} = 2H_0 - \gamma F, \quad \{T, \bar{T}\} = 2H_0 + \gamma F,\\
\{Q, T\} = 2\gamma F \quad \{\bar{Q}, \bar{T}\} = 2\gamma F
\end{aligned}$$

Spectrum

In noninteracting case ($\lambda = 0$),
$E_{n_+,n_-} = \omega_+ n_+ - \omega_- n_-$
with

$$\omega_\pm = \frac{\gamma}{2}\left(\sqrt{1+\tau^2} \mp 1\right) \qquad , \tau = \frac{2\omega}{\gamma}.$$

• The same as for the Pais-Uhlenbeck oscillator,

$$L = \frac{1}{2}\left(\ddot{q}^2 - (\omega_+^2 + \omega_-^2)\dot{q}^2 + \omega_+^2\omega_-^2 q^2\right)$$

• pure point, dense everywhere

• similarly - in the interacting case.

(1+1) FIELD THEORY.

- Let Φ depend on t and x. Choose

$$S = \int dt dx d\bar{\theta} d\theta \left[-2i\mathcal{D}\Phi\partial_+\mathcal{D}\Phi + V(\Phi) \right] ,$$

where $\partial_\pm = (\partial_t \pm \partial_x)/2$ and

$$\mathcal{D} = \frac{\partial}{\partial\theta} + i\theta\partial_-, \qquad \bar{\mathcal{D}} = \frac{\partial}{\partial\bar{\theta}} - i\bar{\theta}\partial_+$$

Bosonic Lagrangian

$$\mathcal{L}_B = \partial_\mu\phi\partial_\mu D + DV'(\phi)$$

Equations of motion

$$\begin{aligned} \Box\phi + \omega^2\phi + \lambda\phi^3 &= 0 \\ \Box D + D(\omega^2 + 3\lambda\phi^2) &= 0 . \end{aligned}$$

Two integrals of motion:

$$E = \int dx \left[\dot{\phi}\dot{D} + \phi' D' + D\phi(\omega^2 + \lambda\phi^2)\right]$$

(positive or negative)

$$N = \int dx \left[\frac{1}{2}\left(\dot{\phi}^2 + (\phi')^2\right) + \frac{\omega^2\phi^2}{2} + \frac{\lambda\phi^4}{4}\right]$$

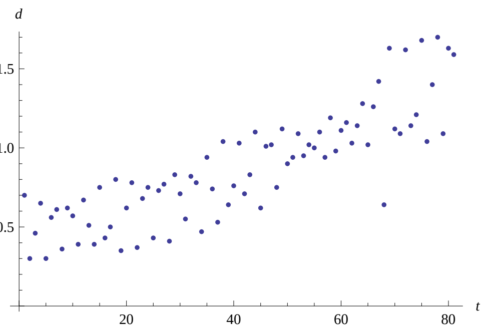
(positive definite)

- Stochasticity. Solved numerically.

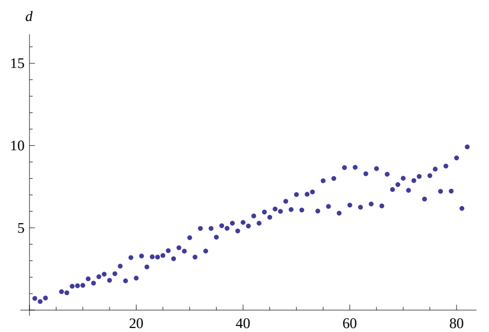
Initial conditions

$$\begin{aligned} \phi(x, t=0) &= Ce^{-x^2}, \quad \text{with} \quad C = 1, 3, 5 \\ D(x, t=0) &= \cos \pi x / L \end{aligned}$$

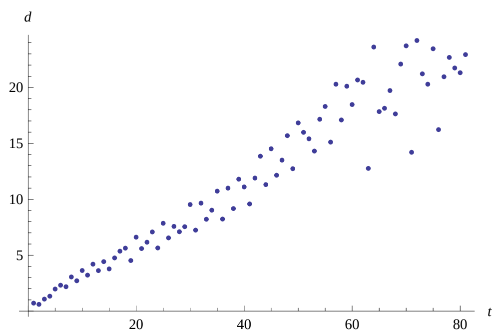
($L = 10 -$ length of the box)



(a) $C = 1$



(b) $C = 3$



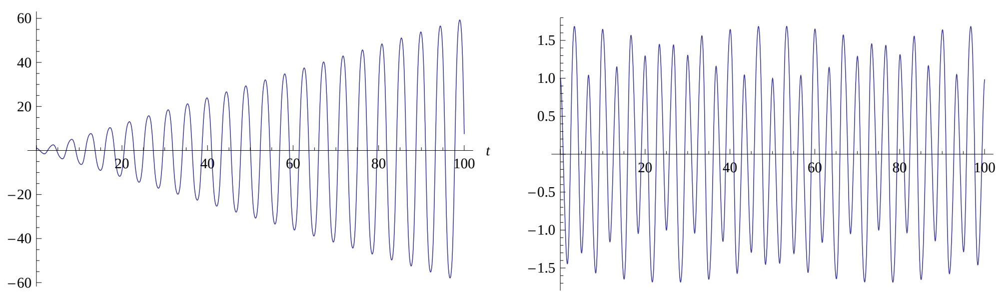
(c) $C = 5$

Figure 2: Dispersion $d = \sqrt{\langle D^2 \rangle_x}$ as a function of time for different values of C.

IMPLICATIONS FOR INFLATION ?

- Homogeneous classical field needed.
- Let $\phi(t)$ be homogeneous. Then different Fourier modes of $D(x,t)$ decouple.

ONLY ZERO MODE GROWS !



(a) $k=0$ (b) $k=1$

- $D(x,t)$ becomes more and more homogeneous.