Baryon as dyonic instanton

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Baryon

Baryon current

$$\hat{J}_0^V \langle \bar{q}^\alpha \gamma_0 q^\alpha \rangle \neq 0$$

Chiral perturbation theory

Chiral lagrangian

$$L_\chi = \frac{f_\pi^2}{4} Tr(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2} Tr\left([U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2\right)$$

Skyrmion

$$U_0(\mathbf{x}) = \cos f(r) \mathbf{1} + i \sin f(r) \frac{x^a t^a}{r}$$

Baryon charge

$$B = \int \frac{1}{24\pi} \epsilon_{ijk} \langle (U\partial_i U^\dagger)(U\partial_j U^\dagger)(U\partial_k U^\dagger) \rangle$$

AdS/CFT correspondence

$$\bar{q}\gamma_\mu t^a q \leftrightarrow V_\mu^a$$
$$\bar{q}\gamma^5\gamma_\mu t^a q \leftrightarrow A_\mu^a$$
$$\bar{q}^\alpha q^\beta \leftrightarrow X^{\alpha\beta}$$

Holographic model of QCD

$$S = \frac{N_c}{4\pi^2}\int d^3x dt dz \left\{ \frac{1}{z}\left(-\frac{1}{16}\right)(F_L^2 + F_R^2) + \frac{1}{z^3}(DX)^2 + \frac{1}{z^5}\ 3|X|^2 \right\}$$

$$S_{CS} = \frac{N_c}{24\pi^2}\int \frac{3}{2}\left\{ \hat{L}\ tr(F_L\tilde{F}_L) - \hat{R}\ tr(F_L\tilde{F}_L) \right\}$$

Chiral condensate

$$|X| = mz + \sigma z^3$$

Baryon charge

$$Q_B = \frac{1}{16\pi^2} \int d^3x dz \epsilon^{\mu\nu\lambda\rho} \left[F_L^{\mu\nu} F_L^{\lambda\rho} - F_R^{\mu\nu} F_R^{\lambda\rho} \right].$$

$$Q_5 = \frac{1}{16\pi^2} \int d^3x dz \epsilon^{\mu\nu\lambda\rho} \left[F_L^{\mu\nu} F_L^{\lambda\rho} + F_R^{\mu\nu} F_R^{\lambda\rho} \right].$$

Dyonic instanton

Topological solution in d=5 N=1 SYM

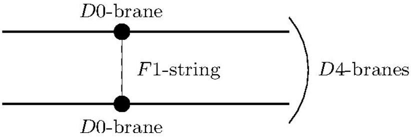
$$A_\mu \sim \frac{\rho^2}{x^2(x^2+\rho^2)} \eta^a_{\mu\nu} x_\nu \frac{\sigma^a}{2}$$

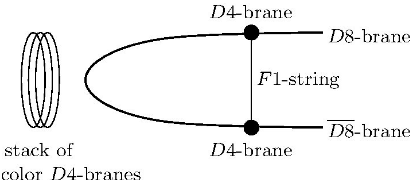
$$\phi = \mathbf{v} \frac{x^2}{x^2+\rho^2} \frac{\sigma^3}{2}$$

Instanton radius

$$\rho^2 = \frac{1}{4\pi^2} \frac{Q}{\mathbf{v}}$$

Analogy with Sakai-Sugimoto





Cylindric ansatz

$$
\begin{aligned}
L_j^a &= -\frac{1+\xi_2(r,z)+\eta_2(r,z)}{r}\epsilon_{jak}\frac{x_k}{r} + \frac{\xi_1(r,z)+\eta_1(r,z)}{r}\left(\delta_{ja}-\frac{x_j x_a}{r^2}\right) \\
&\quad + \left(V_r(r,z)+A_r(r,z)\right)\frac{x_j x_a}{r^2}, \\
R_j^a &= -\frac{1-\xi_2(r,z)+\eta_2(r,z)}{r}\epsilon_{jak}\frac{x_k}{r} + \frac{\xi_1(r,z)-\eta_1(r,z)}{r}\left(\delta_{ja}-\frac{x_j x_a}{r^2}\right) \\
&\quad + \left(V_r(r,z)-A_r(r,z)\right)\frac{x_j x_a}{r^2}, \\
L_5^a &= \left(V_z(r,z)+A_z(r,z)\right)\frac{x_a}{r}, \\
R_5^a &= \left(V_z(r,z)-A_z(r,z)\right)\frac{x_a}{r}, \\
X &= \chi_1(r,z)\frac{\mathbf{1}}{2} + i\chi_2(r,z)\frac{\tau^a x^a}{r},
\end{aligned}
$$

Cylindric ansatz

$$V_j^a = -\frac{1+\eta_2(r,z)}{r}\epsilon_{jak}\frac{x_k}{r} + \frac{\xi_1(r,z)}{r}\left(\delta_{ja} - \frac{x_j x_a}{r^2}\right) + V_r(r,z)\frac{x_j x_a}{r^2},$$

$$A_j^a = -\frac{\xi_2(r,z)}{r}\epsilon_{jak}\frac{x_k}{r} + \frac{\eta_1(r,z)}{r}\left(\delta_{ja} - \frac{x_j x_a}{r^2}\right) + A_r(r,z)\frac{x_j x_a}{r^2},$$

$$V_5^a = V_z(r,z)\frac{x_a}{r},$$

$$A_5^a = A_z(r,z)\frac{x_a}{r},$$

$$X = \chi_1(r,z)\frac{\mathbf{1}}{2} + i\chi_2(r,z)\frac{\tau^a x^a}{r},$$

Scalar field phases

$$\eta_1 = \phi \sin(\theta) \cos(\alpha), \qquad \eta_2 = \phi \cos(\theta) \cos(\beta),$$
$$\xi_2 = \phi \sin(\theta) \sin(\alpha), \qquad \xi_1 = \phi \cos(\theta) \sin(\beta),$$
$$\chi_1 = \chi \cos(\gamma), \qquad \chi_2 = \chi \sin(\gamma).$$

It is possible to choose the gauge

$$\alpha = \beta = \omega$$

Action

$$
\begin{aligned}
E = \frac{N_c}{6\pi}\int dr\ dz \Bigg\{ & \frac{2}{z}(\partial_z\phi)^2 + \frac{2}{z}(\partial_r\phi)^2 \\
& + \frac{2}{z}\phi^2\Big[(\cos(2\omega)A_z + \partial_z\theta)^2 + (\cos(2\theta)V_z + \partial_z\omega)^2 + (\sin(2\omega)A_z + \sin(2\theta)V_z)^2 \\
& \quad + (\cos(2\omega)A_r + \partial_r\theta)^2 + (\cos(2\theta)V_r + \partial_r\omega)^2 + (\sin(2\omega)A_r + \sin(2\theta)V_r)^2 \Big] \\
& + \frac{r^2}{z}(\partial_z A_r - \partial_r A_z)^2 + \frac{r^2}{z}(\partial_z V_r - \partial_r V_z)^2 \\
& + \frac{1}{r^2 z}(1-\phi^2)^2 + \frac{1}{r^2 z}\phi^4 \sin(2\theta)^2 \sin(2\omega)^2 \\
& + \frac{3r^2}{z^3}(\partial_z\chi)^2 + \frac{3r^2}{z^3}\chi^2(\partial_z\gamma - A_z)^2 \\
& + \frac{3r^2}{z^3}(\partial_r\chi)^2 + \frac{3r^2}{z^3}\chi^2(\partial_r\gamma - A_r)^2 - \frac{9r^2}{z^5}\chi^2 \\
& + \frac{6}{z^3}\chi^2\phi^2 \left[\cos(\gamma)^2\sin(\theta)^2 + \sin(\gamma)^2\cos(\theta)^2 + 2\cos(\gamma)\sin(\gamma)\cos(\theta)\sin(\theta)\cos(2\omega)\right] \Bigg\}
\end{aligned}
$$

Potentials

$$\sin(2\theta)^2 \sin(2\omega)^2 = 0 \quad \Rightarrow \quad \theta = 0, \frac{\pi}{2} \;\bigvee\; \omega = 0, \frac{\pi}{2}$$

$$\cos(\gamma)^2 \sin(\theta)^2 + \sin(\gamma)^2 \cos(\theta)^2 + 2\cos(\gamma)\sin(\gamma)\cos(\theta)\sin(\theta)\cos(2\omega) = 0$$

$$\begin{aligned} \cos(2\omega) = 1 \quad &\Rightarrow \quad \gamma = -\theta + \pi n \\ \cos(2\omega) = -1 \quad &\Rightarrow \quad \gamma = \theta + \pi n \\ \sin(2\theta) = 0 \quad &\Rightarrow \quad \gamma = \theta + \pi n \end{aligned}$$

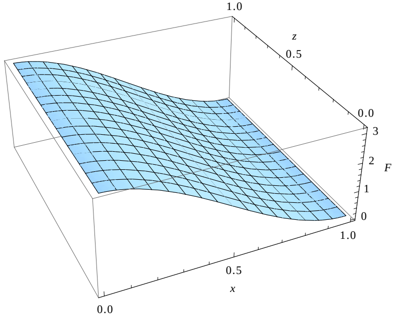
Baryon charge

$$Q_B = \frac{1}{2\pi}\Bigg\{ \int_0^{z_m} dz \Big[(\varphi^2 - 1) A_z + \phi^2 V_z \sin(2\theta)\sin(2\omega) + \varphi^2 \cos(2\omega)\partial_z\theta \Big]_{r=0}^{r\to\infty} - (r \leftrightarrow z)$$

$$Q_A = \frac{1}{2\pi}\Bigg\{ \int_0^{z_m} dz \Big[(\varphi^2 - 1) V_z + \phi^2 A_z \sin(2\theta)\sin(2\omega) + \varphi^2 \cos(2\theta)\partial_z\omega \Big]_{r=0}^{r\to\infty} - (r \leftrightarrow z)$$

Chiral phase

θ



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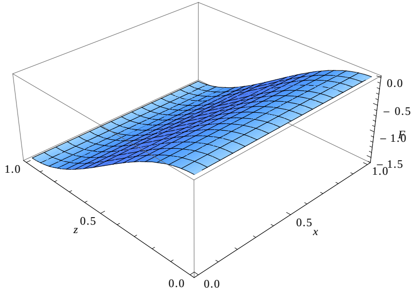
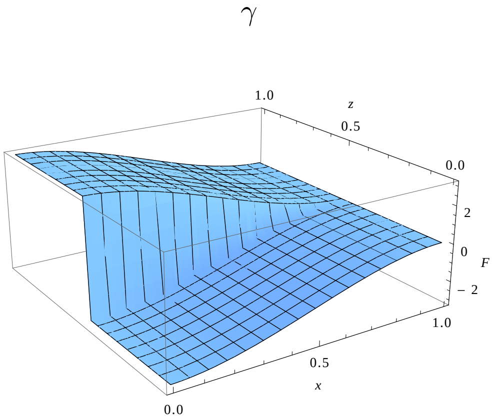
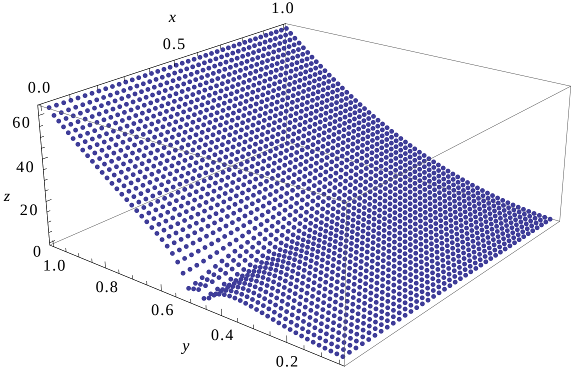


Chiral phase



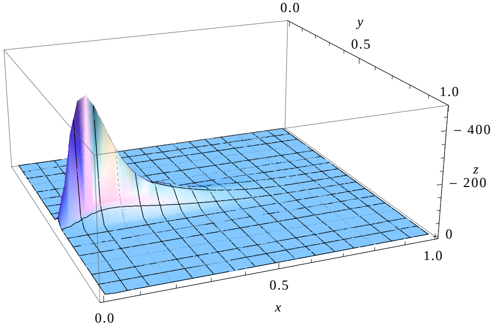
Chiral condensate



The asymptotic on the boundary

Hedgehog skyrmion

$$X \sim \cos(\gamma)\mathbf{1} + i\sin(\gamma)\frac{\tau^a r^a}{r}, \qquad \gamma \in (0, -\pi)$$



Thank you for your attention!