4D CP violation from pure gauge in six dimensions

Maxim Libanov

INR RAS, Moscow

In collaboration with
J.-M. Frere and S. Mollet

4 June, 2014

QUARKS-2014, Suzdal

Outline

- Introduction
- P, C and CP in 4 and 6 dimensions
- Gauge fields. Hosotani mechanism.
- CP violation induced by BC
  - Examples with $SU(2)$
- Conclusion and perspectives

- Sakharov condition: Matter-antimatter asymmetry requires in 4d both C and CP violation (more general: CS).
- In 4d SM "pure gauge" interactions respect CP . Only the scalar(s) break CP through the phase(s) of the orbitrary Yukawa couplings.

? How C or CP invariance can be broken in theories containing only fermions and their gauge interactions?

- Idea:
  - Extra dimensions: A_4, A_5 components of 6d gauge field A_A look as scalars from 4d point of view
  - If ED space is not simply connected

    ⇒ non trivial holonomies (Wilson lines (WL)) can appear dynamically for non contractibel cycles (like magnetic fluxes through holes)

    ⇒ lead to dinamical gauge symmetry breaking

    Hosotani'83,89

    ⇒ 4d scalars (A_4, A_5) aquire a VEV

    ⇒ could cause CP violation if scalar and pseudo-scalar contributions coexists.

Introduction

- At the classical level, WL are determined by the topology of ED and label degenerate classical vacua.
- Degeneracy disappears due to quantum effects which select the true physical solution.
- These are encoded into the effective potential for WL which depends on topology, matter content and Scherk-Schwarz phases.

NB: Our goal: "proof of concept" -- namely the possibility of CP violation in 4d from pure gauge theory in 6d , but does not propose a realistic model.

Outline

- Introduction
- P, C and CP in 4 and 6 dimensions
- Gauge fields. Hosotani mechanism.
- CP violation induced by BC
  - Examples with SU(2)
- Conclusion and perspectives

P, C and CP in 4 and 6 dimensions

- Some notations:

$$X^A = (x^\mu, y^a) = (t, \vec{x}, \vec{y}), \quad \vec{x} = (x_1, x_2, x_3), \quad \vec{y} = (y_4, y_5)$$

- Γ-matrices:

4d

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma^\mu = (1, \vec{\sigma})$$

$$P^{(4)}_{L,R} = \frac{1 \pm \gamma^5}{2} \Rightarrow$$

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

6d

$$\Gamma^A = \begin{pmatrix} 0 & \Sigma^A \\ \bar{\Sigma}^A & 0 \end{pmatrix}, \quad \Gamma^7 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\Sigma^A = (\gamma^0\gamma^\mu, i\gamma^0\gamma^5, \gamma^0)$$

$$P^{(6)}_{\pm} = \frac{1 \pm \Gamma^7}{2} \Rightarrow$$

$$\Psi = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix}$$

P, C and CP in 4 and 6 dimensions

- C -conjugation:

4d	6d
$\mathcal{C}\psi\mathcal{C}^{-1} = \psi^c = C^{(4)}\gamma^0\psi^* \Rightarrow C^{(4)} \sim \gamma^2\gamma^0$	$\mathcal{C}\Psi\mathcal{C}^{-1} = \Psi^c = C^{(6)}\Gamma^0\Psi^* \Rightarrow C^{(6)} \sim \Gamma^1\Gamma^3\Gamma^5$
even number!	odd number!
$\Downarrow$	$\Downarrow$
$\{\gamma^5, C^{(4)}\gamma^0\} = 0$	$[\Gamma^7, C^{(6)}\Gamma^0] = 0$
$\Downarrow$	$\Downarrow$
$\psi_L^c \leftrightarrow \psi_R^*,\ \ \psi_R^c \leftrightarrow \psi_L^*$	$\Psi_+^c \leftrightarrow \Psi_+^*,\ \ \Psi_-^c \leftrightarrow \Psi_-^*$

NB: In terms of the Lorentz group representations:

4d	6d
$\psi_L \sim \psi_R^*$	$\Psi_+ \sim \Psi_+^*,\ \ \Psi_- \sim \Psi_-^*$

NB: Gauge interactions connect the same chirality $\Rightarrow$ no reason to introduce both chiralities on an equal footing. Chiral Lagrangians:

4d	6d
C -nonivariant	C -invariant

P, C and CP in 4 and 6 dimensions

- P-transformations:

4d

$$\mathcal{P}\psi(t,\vec{x})\mathcal{P}^{-1} = \psi^p = \gamma^0\psi(t,-\vec{x})$$

$$\Downarrow$$

$$\{\gamma^5,\gamma^0\} = 0$$

$$\Downarrow$$

$$\psi_L^p \leftrightarrow \psi_R, \quad \psi_R^p \leftrightarrow \psi_L$$

6d

$$\mathcal{P}\Psi(t,\vec{x},\vec{y})\mathcal{P}^{-1} = \Psi^p = \Gamma^0\Psi(t,-\vec{x},-\vec{y})$$

$$\Downarrow$$

$$\{\Gamma^7,\Gamma^0\} = 0$$

$$\Downarrow$$

$$\Psi_+^p \leftrightarrow \Psi_-, \quad \Psi_-^p \leftrightarrow \Psi_+$$

NB: Chiral Lagrangians:

4d	6d
P-nonivariant	P-noninvariant
$\Downarrow$	$\Downarrow$
CP-ivariant	CP-noninvariant

- In 6d if we introduce only (say) Ψ_+ then we break CP .

  Does it mean that the resulting 4d theory is not CP conserving?

  The answer is NO.

P, C and CP in 4 and 6 dimensions

- We need to find a relation between 4d and 6d CP transformations.
  - From the 4d point of view an analog of γ_5 (which acts on a 6d spinor) is

$$\bar{\gamma}_5 \sim \Gamma_0\Gamma_1\Gamma_2\Gamma_3 \sim \begin{pmatrix} -\gamma_5 & 0 \\ 0 & \gamma_5 \end{pmatrix} \Rightarrow$$

  - $$\Psi = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix} \to \Psi_+ = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix} = \gamma_0 \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \sim \psi_{Dirac}$$
  - C-conjugation: $\Psi_+^{C_6} \sim \gamma_5\gamma_2\Psi_+^*$
  - On the other hand 4d CP_4 should looks like:

$$\Psi_+^{CP_4}(t,\vec{x},\vec{y}) \sim \gamma_0\gamma_2\Psi_+^*(t,-\vec{x},\vec{y}\,') \text{ or } \Psi_+^{P_4}(t,\vec{x},\vec{y}) \sim \gamma_0\gamma_5\Psi_+(t,-\vec{x},\vec{y}\,')$$

  - But $\gamma_0\gamma_5$ is the Lorentz generator of a π-rotation in the $(1-2)$, $(3-5)$ planes. That is, if $\vec{y}\,' = (y_4, -y_5)$ then P_4 is nothing but 6d rotations!

NB: Rotations in the $(4-5)$ plane on an angle θ is 4d chiral rotations

$$\vec{y}\,' = \mathcal{R}(\theta)\vec{y} \to \Psi_+(t,\vec{x},\vec{y}\,') = e^{i\gamma_5\theta/2}\Psi_+(t,\vec{x},\vec{y})$$

- In other words, CP_4 transformations correspond to the transformations

$$\begin{cases} \psi_+(t,\vec{x},\vec{y}) \to \psi_+^{CP_4}(t,\vec{x},\vec{y}) \sim \gamma_0\gamma_2 e^{-i\gamma_5\theta/2}\psi_+^*(t,-\vec{x},\vec{y}\,') \\ \vec{y} \to \vec{y}\,' = \mathcal{R}(\theta)\sigma_3\vec{y} = \mathcal{R}(\theta)\cdot(y_4,-y_5)^T \end{cases}$$

  which are a symmetry of the 6d Lagrangian: 6d C-conjugation + rotations.

- 4d effective theory will be CP violating only if the compactification is incompatible with these symmetries. Or, in other words, if we fail to find a chiral rotation which reabsorbs the phases.

- Example: Consider a flat torus $\mathbb{T}^2$ of radii $R_4 = R_5 = R$. Sch-Sch b.c.

$$\begin{cases} \Psi_+(y_4 + 2\pi R, y_5) = e^{i\beta_4}\Psi_+(y_4, y_5) \\ \Psi_+(y_4, y_5 + 2\pi R) = e^{i\beta_5}\Psi_+(y_4, y_5) \end{cases}$$

- Under the prescribed transformations the b.c. becomes

$$\begin{cases} \Psi_+(y_4 + 2\pi R\cos\theta, y_5 + 2\pi R\sin\theta) = e^{-i\beta_4}\Psi_+(y_4, y_5) \\ \Psi_+(y_4 + 2\pi R\sin\theta, y_5 - 2\pi R\cos\theta) = e^{-i\beta_5}\Psi_+(y_4, y_5) \end{cases}$$

- These b.c. are compatible iff $\beta_4, \beta_5 = 0$ or π
- That is, b.c. break effective 4d CP_4 symmetry as soon as β_4, β_5 are both different from 0 or π.

Outline

- Introduction
- P, C and CP in 4 and 6 dimensions
- **Gauge fields. Hosotani mechanism.**
- CP violation induced by BC
    - Examples with SU(2)
- Conclusion and perspectives

- To be specific in what follows we will work on flat space-time $\mathbb{M}^4 \times \mathbb{T}^2$. $\mathbb{T}^2$ is a flat torus of radii R_4, R_5 (in general, $R_4 \neq R_5$); $i, j, \ldots = 4, 5$.
- Turn off the fermions and turn on non-abelian gauge field A_A.

NB: In general, two kinds of compactification exist: the "non-magnetized" and the "magnetized" one:

- "non-magnetized" -- a non zero field strength is unstable and the only solutions are flat connections.

  Example: SU(N) gauge group and a flat torus $\mathbb{T}^2$ -- this case we will consider in what follows.

  Hosotani'89, Alfaro et al'07
- "magnetized" -- a non zero field strength can be stable and the solution corresponds to a physical flux orthogonal to the ED. The stability is ensured by the quantization of the flux for topological reasons.

  Example: U(N) gauge group and a flat torus $\mathbb{T}^2$.

Gauge fields. Hosotani mechanism.

- B.C. : Because of the translation symmetry on the torus, gauge fields must be periodic up to a gauge transformations:

$$A_A(x, y_i + 2\pi R_i) = T_i(y) A_A(x, y) T_i^{-1}(y) + T_i(y) \partial_A T_i^{-1}(y)$$

where the transition functions $T_i(y)$ must satisfy

$$T_4(y_4, y_5 + 2\pi R_5) T_5(y) = T_5(y_4 + 2\pi R_4, y_5) T_4(y)$$

- The BC, T_i do not fix the symmetry of the effective 4d theory: A_4, A_5 (4d scalars) could acquire "VEV" $\langle A_4 \rangle, \langle A_5 \rangle$ through quantum effects. More precisely, some non-integrable phases become dynamical variables and can lead to effective symmetry breaking in 4d .

NB: Neither "VEV" nor BC (T_i) are gauge invariant concepts:

$$\langle A_i\rangle' = \Omega\langle A_i\rangle\Omega^{-1} + \Omega\partial_i\Omega^{-1}$$

$$T_i'(y) = \Omega(y_i + 2\pi R_i)T_i(y)\Omega(y)^{-1}$$

The true gauge invariant quantities, which label vacuua, are Wilson lines phases (WLP)

Hosotani'89

$$W_{C_i}T_{C_i} = \mathcal{P}\exp\left(\oint_{C_i} dy_j'\langle A_j(y')\rangle\right)T_{C_i}(y)$$

C_i--non-contractible cycles, starting at y, and T_{C_i}, the associated BC.

What is the general form of the WLP **?**

Gauge fields. Hosotani mechanism.

- The compactification is "non-magnitazed": $\langle F_{45}\rangle = 0$. Indeed, Hosotani'89, Alfaro et al'07
  - The Lagrangian for pertubations $\tilde{A}_A$ looks like

    $$\mathrm{Tr}F^2_{AB}[\langle A_M\rangle + \tilde{A}_M] \sim \mathrm{Tr}(\langle F_{AB}\rangle + F_{AB}(\tilde{A}_M) + \ldots)^2 \sim \ldots + gf^{abc}\langle F^a_{45}\rangle \tilde{A}^b_4 \tilde{A}^c_5$$

  - So, $m^2_{bc} \sim gf^{abc}\langle F^a_{45}\rangle$ is the mass matrix for $4d$ scalars $\tilde{A}^b_4, \tilde{A}^c_5$. If $\langle F_{45}\rangle \neq 0$ and the group is simple ($SU(N)$)

    $$m^2_{bc} \neq 0$$

  - But

    $$f^{abc} = -f^{acb} \Rightarrow m^2_{bc} = -m^2_{cb} \Rightarrow \mathrm{Tr}m^2 = 0 \Rightarrow$$

    There should be positive ($m^2 > 0$) and negative ($m^2 < 0$) eigenvalues

    $\Rightarrow$ Nielsen-Olesen instability $\Rightarrow$

    $$\langle F_{45}\rangle = 0$$

    $$\Downarrow$$

Gauge fields. Hosotani mechanism.

- The "VEV" must be pure gauge

$$\langle A_i(y)\rangle = S(y)\partial_i S(y)^{-1}$$

- S must be compiteble with the BC $\Rightarrow$

$$S(y_i + 2\pi R_i) = T_i(y)S(y)V_i^{-1}$$

where V_i constant elements of the gauge group such that

$$[V_4, V_5] = 0$$

- Under the gauge transformation $\Omega = S^\dagger$:

$$S(y) \to 1 \Rightarrow \langle A'_i(y)\rangle = 0, \;\; 1 = T'_i V_i^{-1} \Rightarrow$$

Wilson line phases

$$W_{C_i} T_{C_i} = W'_{C_i} T'_{C_i} = V_i$$

Gauge fields. Hosotani mechanism.

- Therefore, all possible classical vacua can be labeled by constant

$$W_{C_i} T_{C_i} = V_i = \exp(i\alpha_i)$$

where α_i are commuting (hermitian) matrices of $SU(N)$ algebra.

NB: In general, among other, there are two approaches

- One can gauge away "VEVs": $\langle A_{4,5} \rangle = 0$ and leaves with the non-trivial BC

$$T_i = V_i = \exp(i\alpha_i)$$

- One can gauge away BC: $T_i = 1$ and leaves with the non-trivial "VEVs"

$$\langle A_i \rangle = \frac{\alpha_i}{2\pi R_i}$$

We will use the first approach.

Outline

- Introduction
- P, C and CP in 4 and 6 dimensions
- Gauge fields. Hosotani mechanism.
- CP violation induced by BC
  - Examples with SU(2)
- Conclusion and perspectives

- Turn on fermions. BC in the presence of a gauge field for the fermion become

$$\Psi_+(y + 2\pi R_i) = \exp(i\beta_i) T_i \Psi_+(y)$$

or in our gauge $\langle A_{4,5} \rangle = 0$

$$\Psi_+(y + 2\pi R_i) = \exp(i\beta_i) \exp(i\alpha_i) \Psi_+(y)$$

- CP_4 is conserved if BC are symmetric under the transformations

$$\begin{cases} \Psi_+ \to U^* \Psi_+^*, \text{ where } U \text{ -- constant matrix} \\ \vec{y} \to \vec{y}\,' = \mathcal{R}(\theta)\sigma_3 \vec{y} \end{cases}$$

- Under the prescribed transformations BC become

$$\begin{cases} \Psi_+^{CP_4}(y_4 + 2\pi R_4 \cos\theta, y_5 + 2\pi R_4 \sin\theta) = e^{-i\beta_4} \exp\left[-i(U\alpha_4 U^{-1})^*\right] \Psi_+^{CP_4}(\vec{y}) \\ \Psi_+^{CP_4}(y_4 + 2\pi R_5 \sin\theta, y_5 - 2\pi R_5 \cos\theta) = e^{-i\beta_5} \exp\left[-i(U\alpha_5 U^{-1})^*\right] \Psi_+^{CP_4}(\vec{y}) \end{cases}$$

CP violation induced by BC

	θ	β_4	β_5	$U a_4 U^{-1}$	$U a_5 U^{-1}$	
$R_4 \neq R_5$	0	$\{0, \pi\}$	$[0, 2\pi[$	$-a_4 + \frac{2\pi k}{N} T$	$a_5 + \frac{2\pi k'}{N} T$	(1)
$R_4 \neq R_5$	π	$[0, 2\pi[$	$\{0, \pi\}$	$a_4 + \frac{2\pi k}{N} T$	$-a_5 + \frac{2\pi k'}{N} T$	(2)
$R_4 = R_5$	$\pi/2$	$-\beta_5$	$-\beta_4$	$-a_5 + \frac{2\pi k}{N} T$	$-a_4 + \frac{2\pi k'}{N} T$	(3)
$R_4 = R_5$	$3\pi/2$	β_5	β_4	$a_5 + \frac{2\pi k}{N} T$	$a_4 + \frac{2\pi k'}{N} T$	(4)

Transformations that could be identified with an effective CP_4 symmetry in 4d if compatible with BC. Here

$$T = \text{diag}(1, 1, \ldots, 1, 1 - N)$$

$k, k' = 0, \ldots, N-1$ for adjoint fermions

$k, k' = 0$ for fundumental fermions

CP violation induced by BC

Two questions:

- Which patterns can be realized and under which conditions?

  General strategy:

  - We need to compute the effective potential for WLP for each group and representations we want to study

    $$V_{eff} = const\times\left(-V_{eff}^{g+gh} + \sum_{i,R} V_{eff}^{ferm_{i,R}} + \text{possible matter contributions}\right)$$

  - Then find the minima of this potential (α_i) which depend on many parameters: β_i, R_i, matter content, etc.
  - Then check whether CP_4 is conserved or not.

- At which level does CP_4 violation manifest itself (and what could be phenomenologically promising)?
  - Main limitation (without any new mechanism) is the absence of gap between light and heavy sectors: $m \sim 1/R$. A partial (quite inelegant) answer to this issue $m_{light} \sim \beta/R$ while $m_{heavy} \sim 1/R$. If $\beta \ll 1$ then $m_{light} \ll m_{heavy}$
  - CP violation is, even in the Standard Model, a tricky issue to characterize (the Jarlskog determinants providing a partial answer). To prove that CP_4 is violated, the safest way is to provide an "observable". Here we will deal with a single (light) fermion species and the simplest "observable"is then the electric dipole moment (EDM) of the lightest mode.

Example with SU(2)

- Let us consider SU(2) group with one fermion in an adjoint representation.
- Numerical calculations show that, in the interesting regime

$$\beta_{4,5} \in [0, 0.1] \text{ and } 0.9 < r \equiv \frac{R_5}{R_4} < 1,$$

$$a_4 = a_5 = \frac{\pi}{2}\tau_3$$

- It means that the SU(2) is broken into U(1), and we have a neutral fermion with mass

$$m_{light} \simeq \frac{\beta}{R}\left(1 + \frac{\triangle\beta}{\beta} + \triangle r\right), \quad \triangle\beta = \beta_4 - \beta_5, \quad \triangle r = 1 - \frac{R_5}{R_4}$$

- The EDM of this mode is

$$\left|\frac{d_E R}{e^3}\right| \simeq 0.01\left(\triangle r + 4.5\frac{\triangle\beta}{\beta}\right)$$

Example with SU(2)

β	$\Delta\beta/\beta$	Δr	$m_{light}R$	$d_E R/e^3$
10^{-1}	0	10^{-1}	$1.35\ 10^{-1}$	$1.09\ 10^{-3}$
10^{-1}	0	10^{-2}	$1.41\ 10^{-1}$	$0.99\ 10^{-4}$
10^{-1}	0	10^{-3}	$1.41\ 10^{-1}$	$0.98\ 10^{-5}$
10^{-1}	0	10^{-4}	$1.41\ 10^{-1}$	$0.98\ 10^{-6}$
10^{-1}	10^{-1}	0	$1.35\ 10^{-1}$	$4.66\ 10^{-3}$
10^{-1}	10^{-2}	0	$1.41\ 10^{-1}$	$4.50\ 10^{-4}$
10^{-1}	10^{-3}	0	$1.41\ 10^{-1}$	$4.48\ 10^{-5}$
10^{-2}	10^{-1}	0	$1.35\ 10^{-2}$	$4.28\ 10^{-3}$
10^{-3}	10^{-1}	0	$1.35\ 10^{-3}$	$4.28\ 10^{-3}$
10^{-3}	10^{-1}	10^{-1}	$1.27\ 10^{-3}$	$5.71\ 10^{-3}$
10^{-3}	10^{-1}	10^{-2}	$1.33\ 10^{-3}$	$4.41\ 10^{-3}$
10^{-3}	10^{-1}	10^{-3}	$1.34\ 10^{-3}$	$4.29\ 10^{-3}$

Conclusion

- ✓ We made use of the Hosotani mechanism to generate both gauge and CP symmetry breacking through compactification from a 6d model.
- ✓ Our solutions is far from being realistic, they must be seen more as "proof of concept".

Perspectives

- New compactification mechanism (like orbifold or magnetized compactification) might be employed to reach a chiral theory in 4d .
- A mechanism which produces a low energy sector naturally separated from the KK scale would be very welcome. (E.g., one can hope for an effective low energy potential between the remaining scalars, what would provide the lower mass scale).