New solution for the warp factor in the Randall-Sundrum scenario

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Plan of the talk

- Randall-Sundrum (RS) scenario with two branes
- Original RS solution for the warp factor of the metric
- Generalization of the RS solution
- Role of the constant term. Different physical schemes
- Conclusions

Randall-Sundrum scenario

Background metric *(y is an extra coordinate)*

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu - dy^2$$

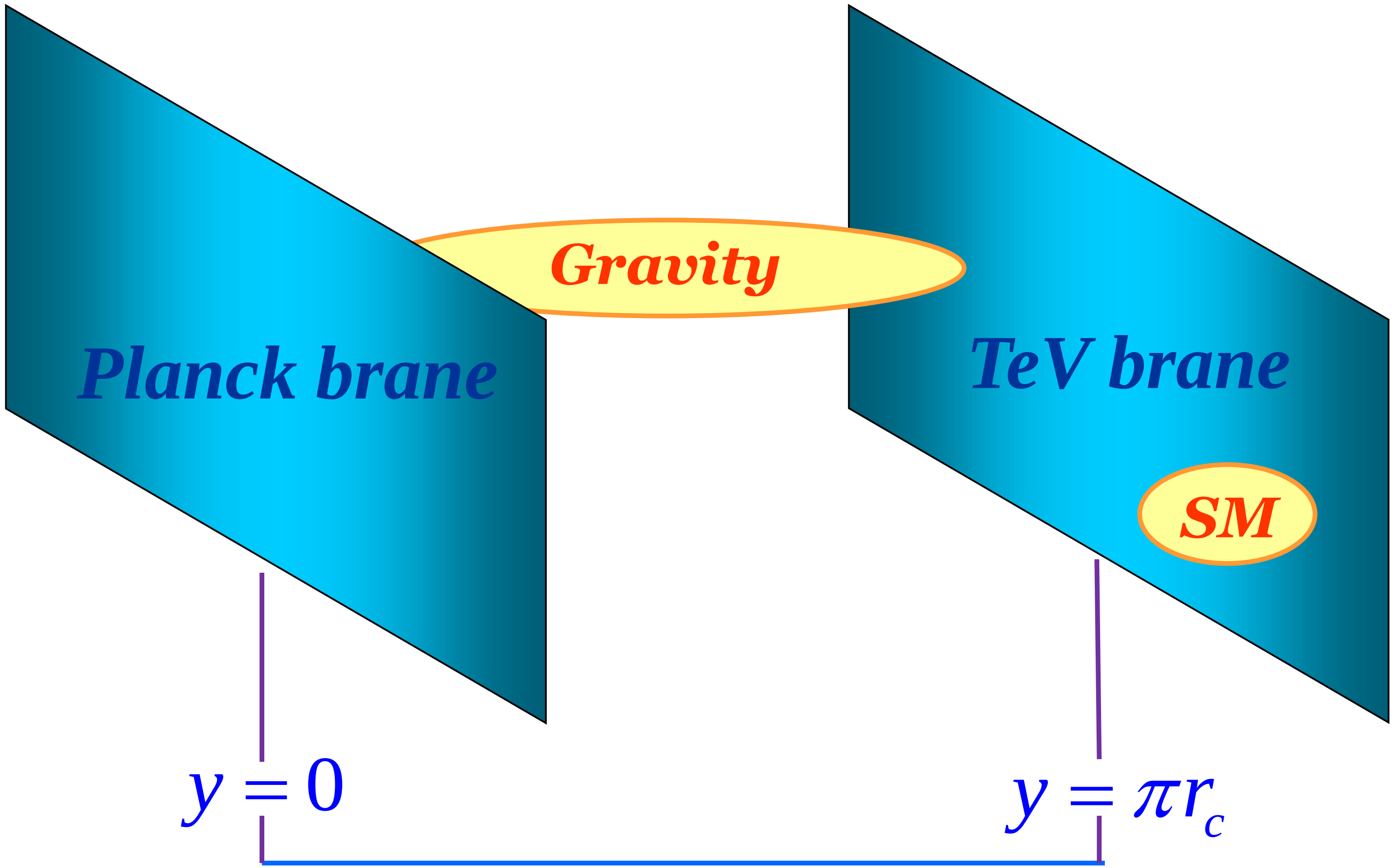
Z_2-symmetry: $(x, y) = (x, -y)$
Periodicity: $(x, y \pm 2\pi r_c) = (x, y)$

→ *orbifold* S^1/Z_2 $0 \le y \le \pi r_c$

Two fixed points: $y=0$ *and* $y=\pi r_c$

→ **two 3-branes**

AdS$_5$ space-time



Five-dimensional action

$$S = S_g + S_1 + S_2$$

$$S_g = \int d^4x \int dy \sqrt{G} \left(2M_5^3 R^{(5)} - \Lambda\right)$$

$$S_{1(2)} = \int d^4x \sqrt{g_{1(2)}} \left(L_{1(2)} - \Lambda_{1(2)}\right)$$

Induced brane metrics

$$g^{(1)}_{\mu\nu} = G_{\mu\nu}(x, y = 0), \quad g^{(2)}_{\mu\nu} = G_{\mu\nu}(x, y = \pi r_c)$$

Einstein-Hilbert's equations

$$\sqrt{|G|}\left(R_{MN} - \frac{1}{2} G_{MN} R\right) = -\frac{1}{4M_5^3}\Big[\sqrt{G}G_{MN}\Lambda + \sqrt{g^{(1)}}\, g^{(1)}_{\mu\nu}\delta^{\mu}_{M}\delta^{\nu}_{N}\delta(y)\Lambda_1 + \sqrt{g^{(2)}}\, g^{(2)}_{\mu\nu}\delta^{\mu}_{M}\delta^{\nu}_{N}\delta(y - \pi r_c)\Lambda_2\Big]$$

Five-dimensional background metric tensor

$$G^{MN} = \begin{pmatrix} g^{\mu\nu} & 0 \\ 0 & -1 \end{pmatrix}$$

$$g^{\mu\nu} = \exp[-2\sigma(y)]\eta^{\mu\nu}$$

$$\rightarrow \quad ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

E-H's equations for the warp function *σ(y)*:

$$\sigma'^2(y) = -\frac{\Lambda}{24M_5^3}$$

$$\sigma''(y) = \frac{1}{12M_5^3}[\Lambda_1\delta(y) + \Lambda_2\delta(\pi r_c - y)]$$

Randall-Sundrum solution

(Randall & Sundrum, 1999)

$$\sigma(y) = \kappa\,|\,y\,| \qquad \Lambda = -24M_5^3\kappa^2, \quad \Lambda_1 = -\Lambda_2 = 24M_5^3\kappa$$

$\longrightarrow$ $\sigma'(y) = \kappa\varepsilon(y)$ *(anti-symmetric in y)*

The RS solution:

- *does not explicitly reproduce the jump on brane $y=\pi r_c$*
- *is not symmetric with respect to the branes, although both branes (at points $y=0$ and $y=\pi r_c$) must be treated on an equal footing*
- *does not include an arbitrary constant*

Generalization of RS solution

For the *open* interval $0 < y < \pi r_c$ **the solution is trivial**

$$\sigma(y) = \kappa y + \text{constant}$$

Let us define: $\Lambda = -24 M_5^3 \kappa^2 \lambda, \quad \Lambda_{1,2} = 12 M_5^3 \kappa \lambda_{1,2}$

Solution of 2-nd equation for $0 \le y \le \pi r_c$

$$\sigma(y) = \frac{\kappa}{4}\left[(\lambda_1 - \lambda_2)\left(|y| - |y - \pi r_c|\right) + (\lambda_1 + \lambda_2)\left(|y| + |y - \pi r_c|\right)\right] + \text{ constant}$$

where $\lambda_1 - \lambda_2 = 2$

Symmetry with respect to the branes

→ ***two possibilities:***

- brane tensions $\Lambda_{1,2}$ have the same sign

→ $\lambda_1 - \lambda_2 = 0$ ***this case cannot be realized***

- brane tensions $\Lambda_{1,2}$ have opposite signs

→ $\lambda_1 + \lambda_2 = 0$

As a result: $\lambda_1 = -\lambda_2 = 1$ $\quad$ $\lambda = \frac{1}{4}\left[\varepsilon(y) - \varepsilon(y - \pi r_c)\right]^2$

The periodicity condition means:

$$\varepsilon(-y - \pi r_c) = -\varepsilon(y + \pi r_c) = -\varepsilon(y - \pi r_c)$$

Expression for the warp function *(A.K., 2013)*

$$\sigma(y) = \frac{\kappa}{2}\left(|y| - |y - \pi r_c|\right) + C$$ **C = constant**

with the fine tuning

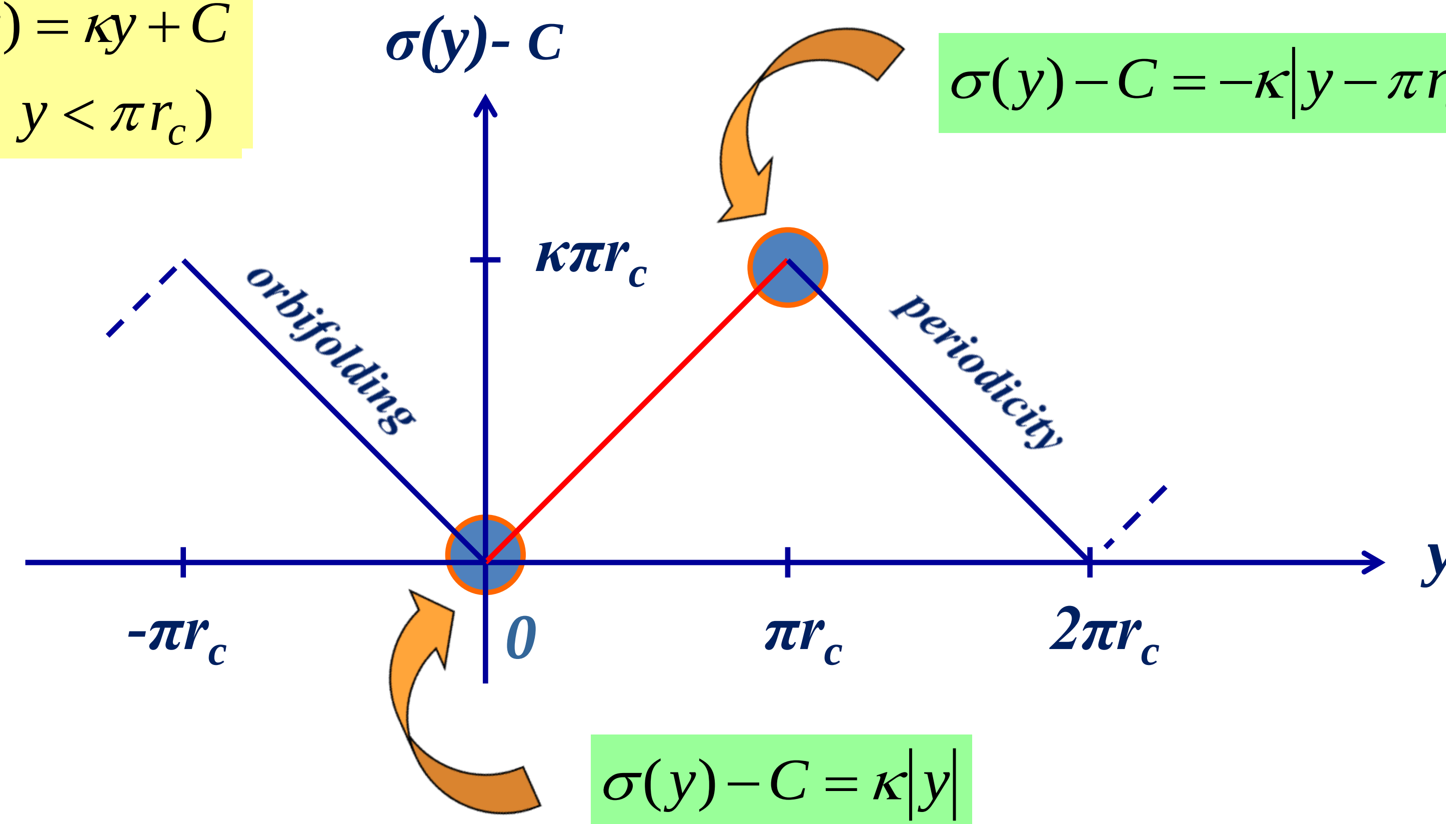
$$\Lambda = -24 M_5^3 \kappa^2, \quad \Lambda_1 = -\Lambda_2 = 12 M_5^3 \kappa$$

Derivative of $\sigma(y)$ at $|y| < \pi r_c$

$$\sigma'(y) = \frac{\kappa}{2}\left[\varepsilon(y) - \varepsilon(y - \pi r_c)\right]$$

is anti-symmetric in y: $\sigma'(y) = -\sigma'(-y) = \kappa\,\mathrm{sign}(y)$

$$\sigma(y) = \kappa y + C$$
$$(0 < y < \pi r_c)$$

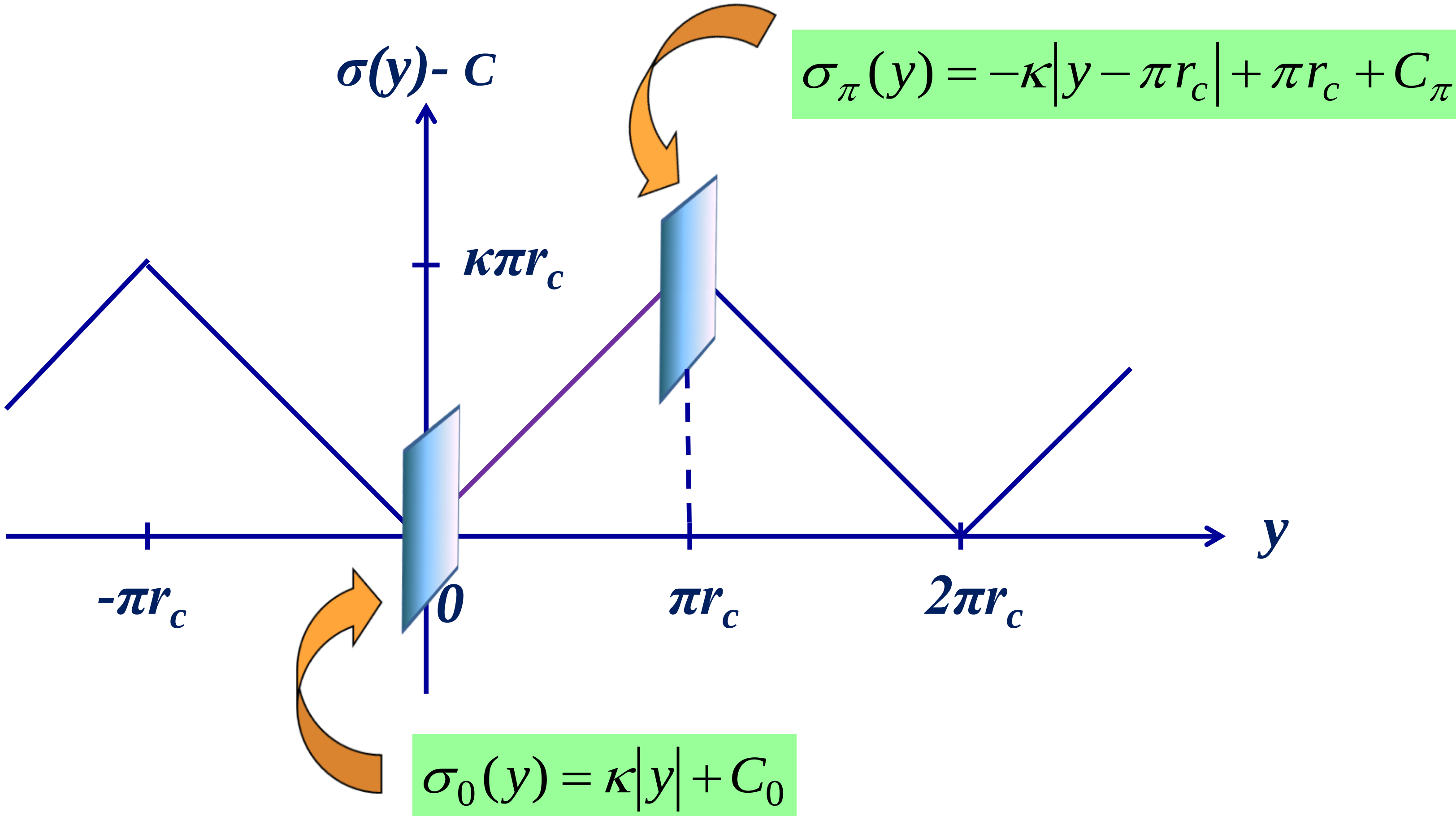


$$\sigma(y) - C = -\kappa|y - \pi r_c| + \pi r_c$$

$$\sigma(y) - C = \kappa|y|$$

$$\sigma''(y) = \kappa[\delta(y) - \delta(y - \pi r_c)] \quad (0 \le y \le \pi r_c)$$

Two equivalent RS-like solutions



Neither of two branes/fixed points is preferable provided orbifold and periodicity conditions are taken into account

If starting from the point y=0 → $\sigma_0(y) = \kappa|y| + C_0$

If starting from the point y=πr$_c$ → $\sigma_\pi(y) = -\kappa|y - \pi r_c| + C_\pi$

Solution symmetric with respect to the branes:

$$\sigma(y) = \frac{1}{2}\left[\sigma_0(y) + \sigma_\pi(y)\right] = \frac{\kappa}{2}\left(|y| - |y - \pi r_c|\right) + C$$

Note that $|-\boldsymbol{y} - \pi r_c| \equiv |y + \pi r_c| = (y \to y - 2\pi r_c) = |\boldsymbol{y} - \pi r_c|$

(consistence with the orbifold symmetry)

Our solution for *σ(y)*:

- ***is symmetric with respect to the branes***

$$y \to |y - \pi r_c|, \quad \kappa \to -\kappa$$

- ***makes the jumps on both branes***

$$\sigma''(y) = \kappa[\delta(y) - \delta(y - \pi r_c)]$$

- ***is consistent with the orbifold symmetry***

$$y \to -y$$

- ***depends on the constant C***

Let us define: $\sigma_1 = \sigma(0), \;\; \sigma_1 = \sigma(\pi r_c)$

note that $\sigma_{1,2} = \sigma_{1,2}(\boldsymbol{C})$ $\quad \Delta\sigma = \sigma_2 - \sigma_1 = 2\pi\kappa r_c$

Hierarchy relation
(depends on C)

$$M_{\mathrm{Pl}}^2 = \frac{M_5^3}{\kappa}\exp(\text{-}2\sigma_1)\left[1\text{-}\exp(\text{-}2\pi\kappa r_c)\right]$$

Masses of KK gravitons
(*x_n are zeros of $J_1(x)$*)

$$m_n = x_n \frac{M_{\mathrm{Pl}}}{\sqrt{\exp(2\pi\kappa r_c) - 1}}\left(\frac{\kappa}{M_5}\right)^{3/2}$$

The very expression for m_n is independent of C, but values of m_n depend on C via M_5 and κ

Interaction Lagrangian on the TeV brane (*massive gravitons only*)

$$L(x) = -\frac{1}{\Lambda_\pi}\sum_{n=1}^{\infty} h_{\mu\nu}^{(n)}(x) T_{\alpha\beta}(x)\eta^{\mu\alpha}\eta^{\nu\beta}$$

Coupling constant

$$\Lambda_\pi = \frac{M_{\mathrm{Pl}}}{\sqrt{\exp(2\pi\kappa r_c)-1}}$$

Parameters M_5, κ, Λ_π depend on constant C

Different values of this constant result in quite diverse physical models ***(A.K., 2014)***

From now on it will be assumed that $\kappa\pi r_c \gg 1$

Different physical schemes

I. $C = \kappa\pi r_c/2$ $\quad$ $\sigma_1 = 0, \quad \sigma_2 = \kappa\pi r_c$

$$M_{\mathrm{Pl}}^2 \cong \frac{M_5^3}{\kappa}$$ *that requires* $$M_5 \sim \kappa \sim M_{\mathrm{Pl}}$$

Masses of KK resonances $\quad m_n \cong x_n \kappa \exp(-\kappa\pi r_c)$

RS1 model *(Randall & Sundrum, 1999)*

Graviton spectrum - heavy resonances, with the lightest one above 1 TeV

II. $C = -\kappa\pi r_c/2$ $\sigma_1 = -\kappa\pi r_c, \quad \sigma_2 = 0$

$$M_{\mathrm{Pl}}^2 \cong \frac{M_5^3}{\kappa} \exp(2\pi\kappa r_c)$$

$\kappa << M_5$ $\kappa r_c \approx 9.5$ for $M_5 = 1\,\mathrm{TeV}, \kappa = 100\,\mathrm{MeV}$

Masses of KK resonances $m_n \cong x_n \kappa$

RSSC model: RS-like scenario with the small curvature of 5-dimensional space-time

For small κ**, graviton spectrum is similar to that of the ADD model**

(Giudice, Petrov & A.K., 2005)

Effective gravity action

$$S_{\text{eff}} = \frac{1}{4}\sum_{n=0}^{\infty}\int d^4x[\partial_\mu h^{(n)}_{\rho\sigma}(x)\partial_\nu h^{(n)}_{\delta\lambda}(x)\eta^{\mu\nu} - m_n^2 h^{(n)}_{\rho\sigma}(x)h^{(n)}_{\delta\lambda}(x)]\eta^{\rho\delta}\eta^{\sigma\lambda}$$

Shift $\sigma \to \sigma - B$ ***is equivalent to the change*** $x^\mu \to x'^\mu = \mathrm{e}^{-B}x^\mu$

Invariance of the action → **rescaling of fields and masses:**

$$h^{(n)}_{\mu\nu} \to h'^{(n)}_{\mu\nu} = \mathrm{e}^{B}h^{(n)}_{\mu\nu}, \quad m_n \to m'_n = \mathrm{e}^{B}m_n$$

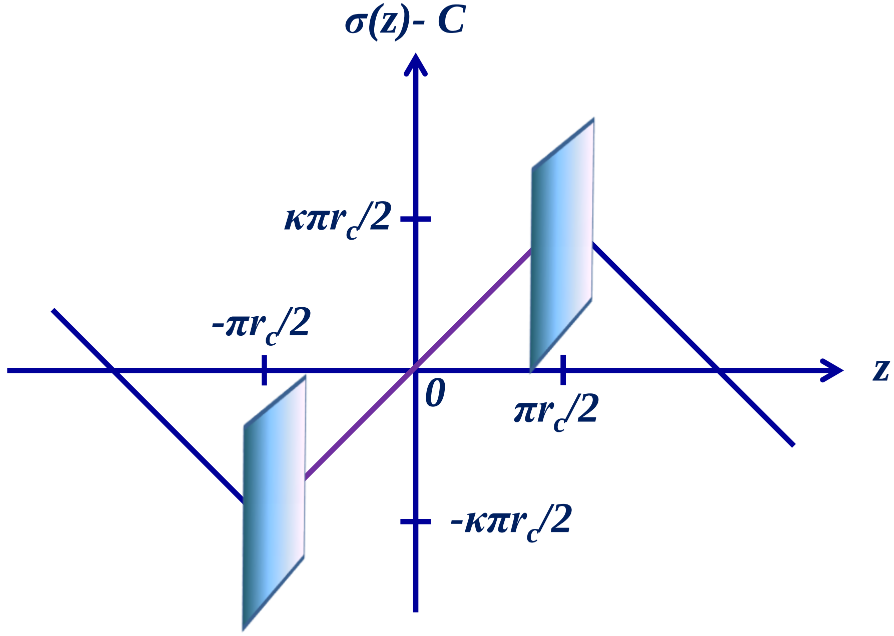
Example: $B = \pi\kappa r_c$ **(RS1 → RSSC)**

$\sigma \to \sigma - \pi\kappa r_c$ → $x_n\kappa\exp(-\kappa\pi r_c) \to x_n\kappa$

III. *C = 0* $\sigma_1 = -\sigma_2 = \kappa\pi r_c/2$

"Symmetric scheme" ***(A.K., 2014)***

In variable z = y - πκr$_c$/2: $\sigma(z) = \frac{\kappa}{2}\left(\left|\frac{\pi r_c}{2}+z\right| - \left|\frac{\pi r_c}{2}-z\right|\right) + C$



$$M_{\rm Pl}^2 \cong \frac{2M_5^3}{\kappa}\sinh(2\pi\kappa r_c)$$

Masses of gravitons $m_n \cong x_n \kappa \exp(-\kappa\pi r_c/2)$

Let $\kappa \ll M_5$ $M_5 = 2\cdot 10^9\,\mathrm{GeV},\ \kappa = 10^4\ \mathrm{GeV}$

$$\begin{cases} \Lambda_\pi = 3\cdot 10^5\,\mathrm{GeV} \\ m_n \cong 3.7\,x_n\,(\mathrm{MeV}) \end{cases}$$

Almost continuous spectrum of the gravitons

Virtual s-channel Gravitons

Scattering of SM fields mediated by graviton exchange in *s*-channel

Processes:

$$pp \to l^+ l^- \,(\gamma\gamma,\, 2\,jets) + X$$

$$e^+ e^- \to f\bar{f}\,(\gamma\gamma), \quad f = l, q$$

Sub-processes:

$$q\bar{q},\; gg \to h^{(n)} \to f\bar{f}, \gamma\gamma$$

Matrix element of sub-process

$$\mathcal{M} = \mathcal{A}\,\mathcal{S}$$

Tensor part of graviton propagator

where $$\mathrm{A} = T^{in}_{\mu\nu} P^{\mu\nu\alpha\beta} T^{f}_{\alpha\beta}$$

Energy-momentum tensors

and $$S(s) = \frac{1}{\Lambda_\pi^2} \sum_{n=1}^{\infty} \frac{1}{s - m_n^2 + i m_n \Gamma_n}$$ ***(process independent)***

Graviton widths: $$\Gamma_n = \eta\, m_n^3 / \Lambda_\pi^2, \;\; \eta \cong 0.1$$

In symmetric scheme *(A.K., 2014)*

$$S(s) = \frac{1}{2\Lambda_\pi^3 \sqrt{s}} \left(\frac{M_5}{\kappa}\right)^{3/2} \frac{J_2(z)}{J_1(z)} \quad \textit{with} \quad z \cong \frac{\sqrt{s}}{\Lambda_\pi} \left(\frac{M_5}{\kappa}\right)^{3/2}$$

(relation of m_n with x_n was used)

For chosen values of parameters:

$$|S(s)| = \frac{\mathrm{O}(1)}{(1\,\mathrm{TeV})\sqrt{s}}$$

TeV physics (*instead of large value of Λ_π*)

Conclusions

- **Generalized RS-like solution for the warp factor σ(y) is derived**
- **New expression:**

  - is symmetric with respect to the branes

  - has the jumps on both branes

  - is consistent with the orbifold symmetry
- **σ(y) depends on arbitrary constant C**
- **Different values of C result in quite diverse physical schemes**
- **Particular solutions are: RS1 model, RSSC model, symmetric scheme**

Thank you
for your attention

Back-up slides

RSSC model vs. ADD model

RSSC model is not equivalent to the ADD model with one ED of the size $R=(\pi\kappa)^{-1}$ up to $\kappa \approx 10^{-20}$ eV

Hierarchy relation for small κ

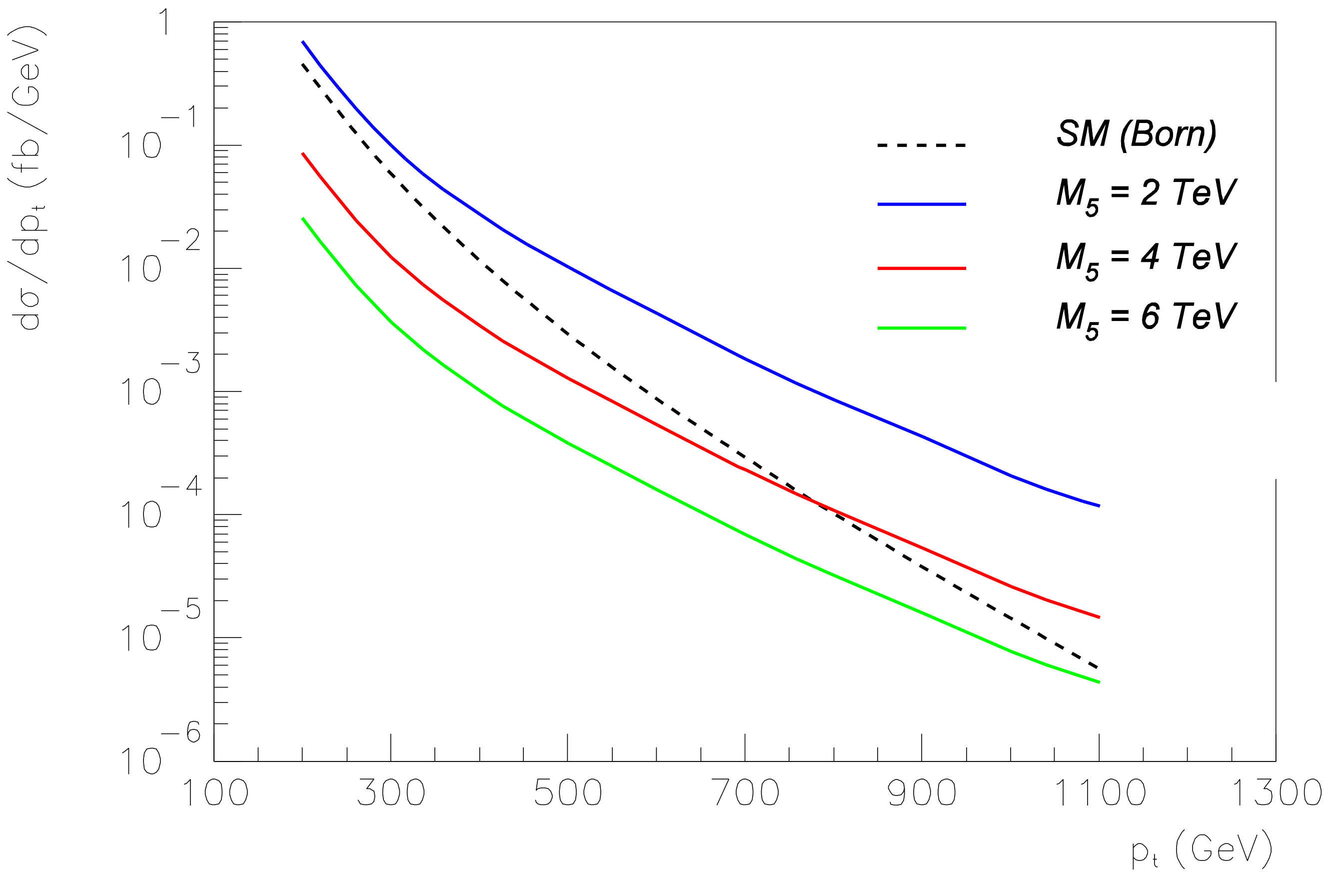
$$M_{\mathrm{Pl}}^2 \cong \frac{M_5^3}{\kappa}\left[\exp(2\pi\kappa r_c)-1\right] \xrightarrow{2\pi\kappa r_c<<1} M_5^3(2\pi r_c)$$

But the inequality $2\pi\kappa r_c << 1$ **means that**

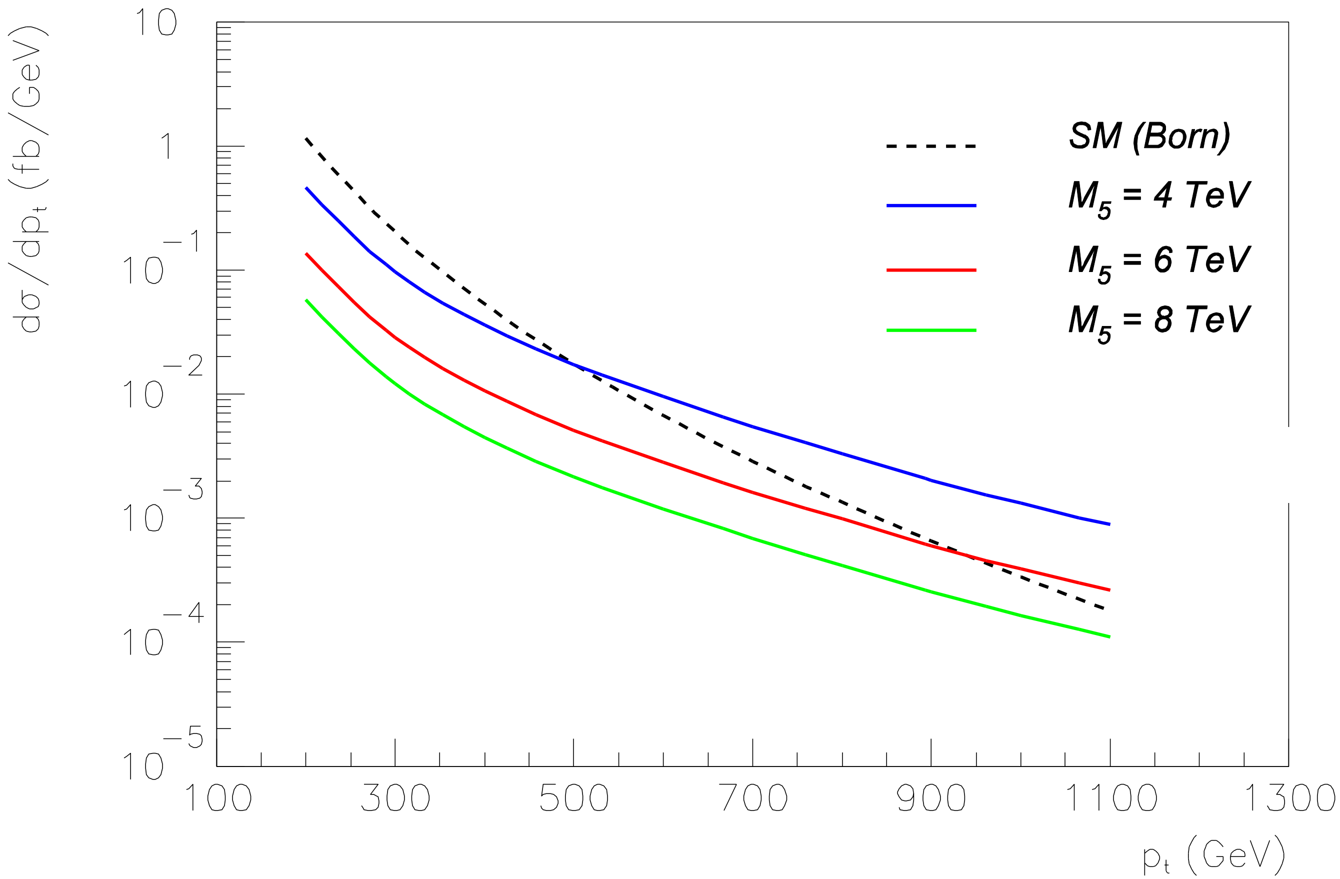
$$\kappa << \frac{M_5^3}{M_{\mathrm{Pl}}^2} \approx 0.17\cdot 10^{-18}\left(\frac{M_5}{1\mathrm{TeV}}\right)^3 \mathrm{eV}$$

Dilepton production at LHC

Pseudorapidity cut: $|\eta| \le 2.4$
Efficiency: 85 %
K-factor:
  1.5 for SM background
  1.0 for signal



Graviton contributions to the process
pp → μ+μ- + X (solid lines)
vs. SM contribution (dashed line) for 7 TeV



Graviton contributions to the process pp → μ+μ- + X (solid lines) vs. SM contribution (dashed line) for 14 TeV

Number of events with $p_t > p_t^{\text{cut}}$

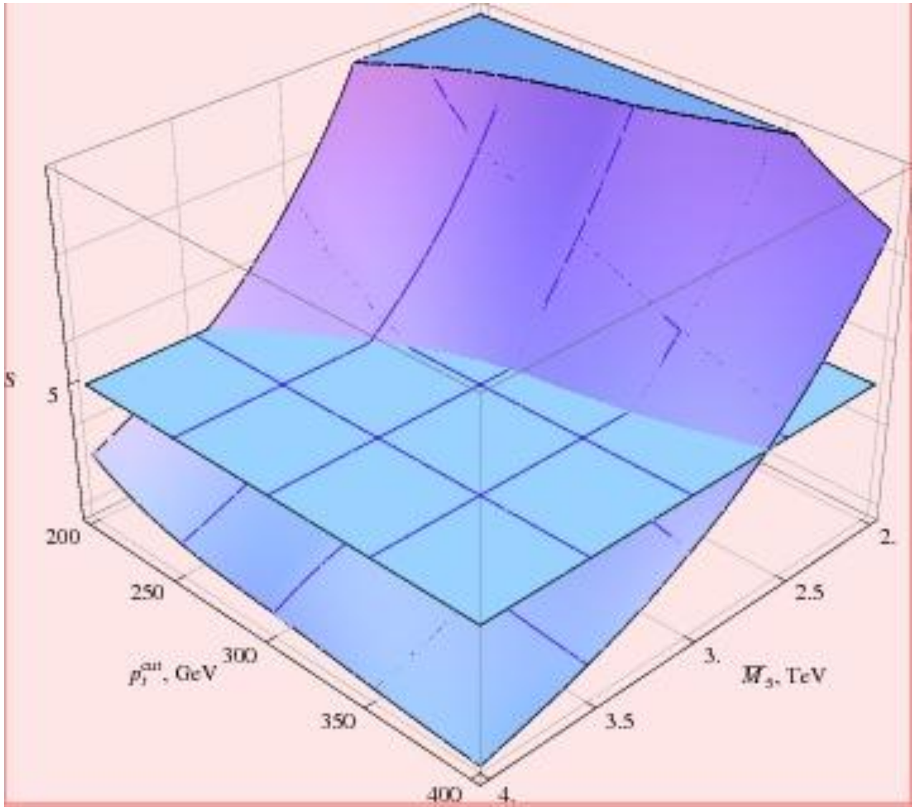
$$N_S = \int_{p_\perp^{\text{cut}}} \mathrm{dp}_\perp \frac{d\sigma(\text{grav})}{\mathrm{dp}_\perp}, \quad N_B = \int_{p_\perp^{\text{cut}}} \mathrm{dp}_\perp \frac{d\sigma(\text{SM})}{\mathrm{dp}_\perp}$$

Interference SM-gravity contribution is negligible

Statistical significance

$$S = \frac{N_S}{\sqrt{N_S + N_B}}$$

$S=5$ → **Lower bounds on M_5 at 95% level**



Statistical significance for the process pp → e^+e^- + X as a function of 5-dimensional reduced Planck scale and cut on electron transverse momentum for 7 TeV (L=5 fb^{-1}) and 8 TeV (L=20 fb^{-1})

No deviations from the CM were seen at the LHC

→ $M_5 > 6.35$ TeV *(A.K., 2013)*
(for 7 + 8 TeV,
$L = 5\ \mathrm{fb}^{-1} + 20\ \mathrm{fb}^{-1}$)

LHC search limits on M_5:
8.95 TeV
(for 13 TeV, $L = 30\ \mathrm{fb}^{-1}$)