Quarks-2014 seminar, Suzdal, Russia

On three-loop renormalization group analysis of the Standard Model:

Three-loop SM RGEs with general Yukawa matrices

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based on
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In collaboration with A.F. Pikelner (JINR) and V.N. Velizhanin (PNPI)

Some motivations

- In the Standard Model fermion masses and famous CKM matrix originate from complex Yukawa matrices after EWSB.
- Three-loop renormalization group equations (RGEs) calculated recently* neglect** flavour mixing.
- RGEs involving complex Yukawa matrices can be important both in top-down and bottom-up approaches to New Physics which pretends to solve Flavour puzzle of the SM.
- Our initial plan was to extend the full matrix two-loop results to the three-loop case

* For the references see below.
** The only exception is *Mihaila,Salomon,Steinhauser'12* (used a trick)

$$\mathcal{L}_{\rm F} = -Y^u_{ij}(\bar{Q}_i\Phi^c)u_{jR} - Y^d_{ij}(\bar{Q}_i\Phi)d_{jR} - Y^e_{ij}(\bar{L}_i\Phi)E_{jR} + \text{h.c.} \qquad (1)$$

Some words on the SM parameters

$$\mathcal{L} = \mathcal{L}_{\rm G} + \mathcal{L}_{\rm H} + \mathcal{L}_{\rm F} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP}.$$

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Gauge

$$g_1, \ g_2, \ g_s$$

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Higgs potential

$\lambda, \; m^2$

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Gauge-fixing

$\xi_W, \; \xi_B, \; \xi_G$

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for cross-checks

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Fadeev-Popov ghosts

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Fadeev-Popov ghosts

for cross-checks

Scalar parameters:

$$a_i = \left(\frac{5}{3}\frac{g_1^2}{16\pi^2}, \frac{g_2^2}{16\pi^2}, \frac{g_s^2}{16\pi^2}, \frac{\lambda}{16\pi^2}\right)$$

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$Y_f Y_f^\dagger$ Right-handed (RH) fermion „propagation“

Some remarks on Flavour in the SM

Not all the parameters of Yukawa matrices are physical („observable“)!

$$\mathcal{L}_{\rm G} + \mathcal{L}_{\rm H} + \cancel{\mathcal{L}_{\rm F}} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP}$$

Example from quark sector:

is invariant under global flavour symmetry

$$U(3)_Q \times U(3)_u \times U(3)_d$$

SU(2) compatible rotation of LH quarks

Independent rotations of RH quarks

NB: PMNS matrix is not taken into account

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The symmetry is broken by Yukawa interactions down to $U(1)_B$

Broken generators of the symmetry can be used to „get rid“ of unphysical parameters.

36 – 26 = 6 + 3 + 1

NB: PMNS matrix is not taken into account

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$$36 - 26 = 6 + 3 + 1$$

$Y^u, \ Y^d$

Broken generators

Masses

Mxing angles V_{CKM}

Phase

Some remarks on Flavour in the SM

On masses and mixing in quark sector

Yukawa matrices can be diagonalized by bi-unitary transformations

$$U_L Y^u U_R^\dagger = Y^u_{\rm diag}, \qquad D_L Y^d D_R^\dagger = Y^u_{\rm diag}$$

$$\begin{aligned} \tilde{u}_L &= U_L\, u_L, \\ \tilde{u}_R &= U_R\, u_R, \\ \tilde{d}_L &= D_L\, d_L, \\ \tilde{d}_R &= D_R\, d_R \end{aligned}$$

After electroweak symmety breaking (EWSB) one has diagonal quark mass matrices

$$M^f_{\rm diag} = Y^f_{\rm diag} v/\sqrt{2}$$

$$\langle \Phi \rangle = v/\sqrt{2}$$

Among 6 phases only 1 is observable

$$V_{CKM} = U_L\, D_L^\dagger$$

CKM matrix is invariant

$$U_L \to U_L \bar{U}$$
$$D_L \to D_L \bar{U}$$

$$\bar{U}^{-1} = \bar{U}^\dagger$$

„mass basis“ „flavour basis“

Some history (3-loop SM RGE calculations)

- Gauge coupling beta-functions
  - *Tarasov,Vladimirov,Zharkov'80*
  - *Curtright'80*
  - *Jones'80*
  - *Steinhauser'98*
  - *Pickering,Gracey,Jones'01*

- *Mihaila, Salomon, Steinhauser'12*
  - By simple fermion loop counting the full result with complex Yukawa matrices was deduced from calculated expression involving diagonal coupling

- *Bednyakov, Pikelner, Velizhanin'12*

* Some extended history of SM RGE calculations can be found in the cited Refs

Some history (3-loop SM RGE calculations)

- Higgs potential parameters
  - *Chetyrkin, Zoller'12*
    - Only contributions due to top-Yukawa and strong coupling constants
  - *Chetyrkin, Zoller'13*
    - Full result in the SM with diagonal Yukawa couplings
  - *Bednyakov, Pikelner, Velizhanin'13*
    - An independent calculation with diagonal Yukawa couplings
  - *Bednyakov,Pikelner,Velizhanin'14* (this talk)
    - Full result with general Yukawa matrices

* Some extended history of SM RGE calculations can be found in the cited Refs

Some history (3-loop SM RGE calculations)

- Yukawa couplings
  - *Chetyrkin, Zoller'12*
    - top-Yukawa and strong coupling contributions
  - *Bednyakov, Pikelner, Velizhanin'13*
    - Strong and EW couplings + top, bottom and tau Yukawa couplings
  - *Bednyakov,Pikelner,Velizhanin* (new result - this talk)
    - Full result with general Yukawa matrices

* Some extended history of SM RGE calculations can be found in the cited Refs

Some technical details: renormalization

- We are interested in renormalization constants of certain dimensionally regularized 2-, 3-, and 4-point Green functions Γ at 1, 2, and 3 loops

$$\Gamma_{\text{Ren}}\left(\frac{Q^2}{\mu^2}, a_i\right) = \lim_{\varepsilon\to 0} Z_\Gamma\left(\frac{1}{\varepsilon}, a_i\right)\Gamma_{\text{Bare}}\left(Q^2, a_{i,\text{Bare}}, \varepsilon\right),$$

$$a_{i,\text{Bare}} = Z_{a_i}\left(1/\varepsilon, a_j\right) a_i$$

$$\left(\frac{\partial}{\partial \ln \mu^2} + \beta_{a_i}\frac{\partial}{a_i} + \gamma_\Gamma\right)\Gamma_{\text{ren}}\left(\frac{Q^2}{\mu^2}, a_i\right) = 0$$

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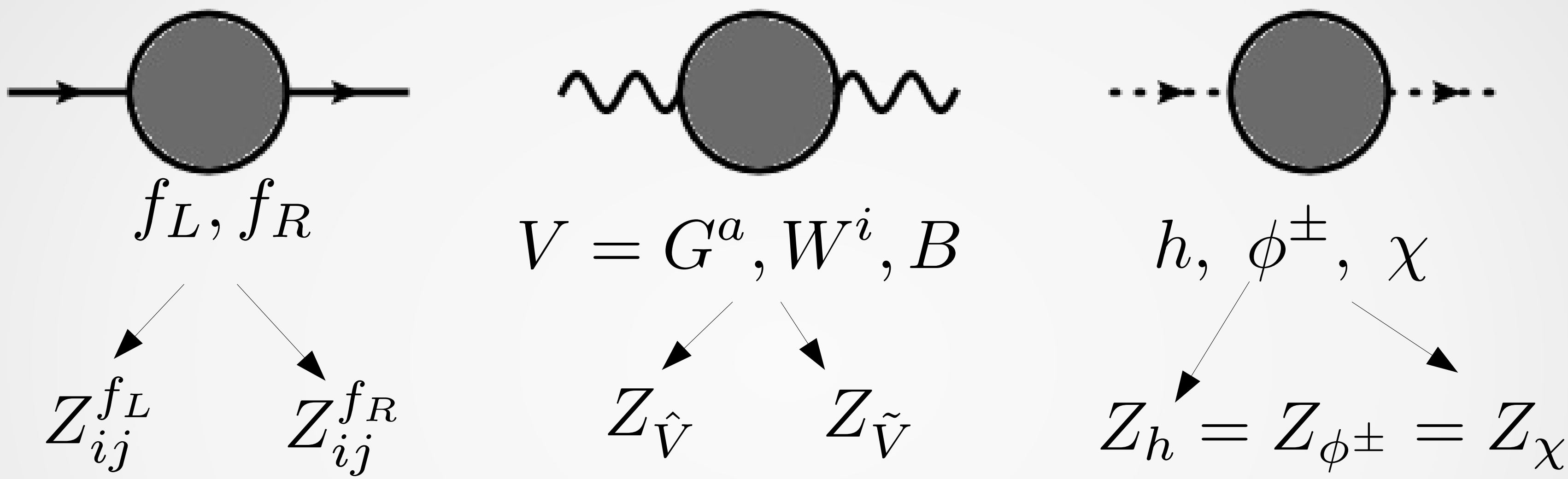
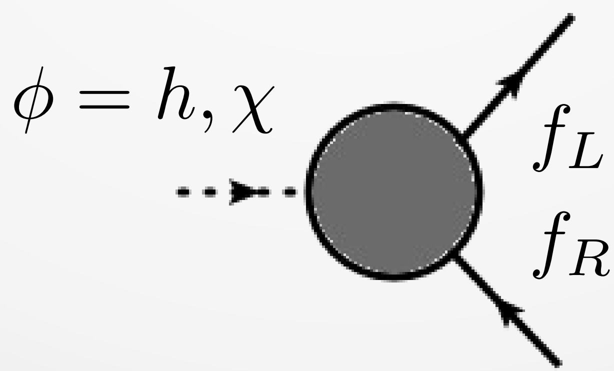
- Modified Minimal Subtraction $\overline{\rm MS}$ renormalization scheme is utilized allowing to deduce RG functions solely from ultraviolet divergencies of the corresponding Green functions. One can modify InfraRed structure (InfraRed Rearrangement) to simplify caclculations.

Vladimirov'80

Some technical details: renormalization



$$\left(Z_{\bar{f}_L f_R \phi} Y_f\right)_{ij}$$

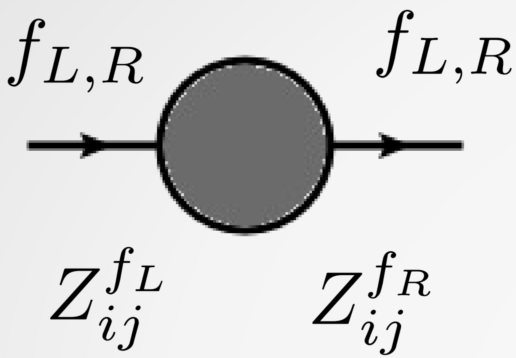


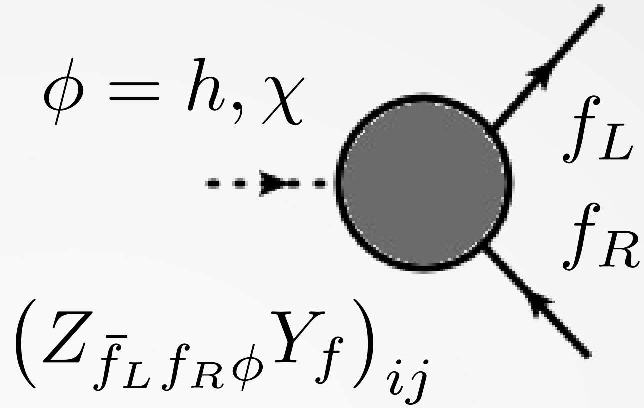
We use „unbroken“ massless SM and background field gauge to simplify our calculations

$$Z_{g_2} = Z_{\hat{W}}^{-1/2},$$
$$Z_{g_s} = Z_{\hat{G}}^{-1/2}$$

$$Z_{\bar{f}_L f_R \phi} Y_f = Z_{f_L}^{1/2\dagger}(Y_f + \Delta Y_f) Z_{f_R}^{1/2} Z_\phi^{1/2}$$

Some technical details: renormalization





$$\Gamma^{(2)}_{f,\mathrm{Ren}}\left(\frac{k^2}{\mu^2}, a_i, Y_f\right) = \left[Z_f^{1/2}\right]^\dagger \Gamma^{(2)}_{f,\mathrm{Bare}}\left(k^2, a_{i,\mathrm{Bare}}, Y_{f,\mathrm{Bare}}, \epsilon\right)\left[Z_f^{1/2}\right]$$

$$\Gamma^{(3)}_{\bar{f}' f\phi,\mathrm{Ren}}\left(\frac{k_i^2}{\mu^2}, a_i, Y_f\right) = \left[Z_{f'}^{1/2}\right]^\dagger \Gamma^{(3)}_{\bar{f}' f\phi,\mathrm{Bare}}\left(k_i^2, a_{i,\mathrm{Bare}}, Y_{f,\mathrm{Bare}}, \epsilon\right)\left[Z_f^{1/2}\right] Z_\phi^{1/2}.$$

recurrsively obtain matrix renormalizationconstants in perturbation theory.

NB: Only $Z_f^{1/2\dagger} Z_f^{1/2}$ can be determined from the requirement of $\Gamma^{(2)}_{f,\mathrm{Ren}}$ finitness in $\varepsilon \to 0$ limit

Some technical details: beta-functions

$$a_{k,\mathrm{Bare}}\mu^{-2\epsilon} = Z_{a_k} a_k(\mu) = a_k + \sum_{n=1}^{\infty} c_k^{(n)} \frac{1}{\epsilon^n},$$

$$\beta_i(a_k) = \left.\frac{d a_i(\mu,\epsilon)}{d\ln\mu^2}\right|_{\epsilon=0}, \qquad \beta_i = \beta_i^{(1)} + \beta_i^{(2)} + \beta_i^{(3)} + \ldots$$

$$\mu^{-\epsilon} Y_{f,\mathrm{Bare}} = Y_f + \Delta Y_f = Y_f + \sum_{n}^{\infty} \frac{\Delta Y_f^n}{\varepsilon^n}$$

$$\beta_{Y_f} Y_f \equiv \left.\frac{d Y_f(\mu,\epsilon)}{d\ln\mu^2}\right|_{\epsilon=0}, \qquad \beta_{Y_f} = \beta_{Y_f}^{(1)} + \beta_{Y_f}^{(2)} + \beta_{Y_f}^{(3)} + \ldots$$

Beta-functions can be obtained in perturbation theory given the requirement that bare quantities do not depend on the renormalization scale μ

Some remarks: no external quarks

In *Mihaila, Salomon, Steinhauser'12* and *Bednyakov,Pikelner,Velizhanin'12* a trick was used to obtain gauge-coupling beta-functions dependeing on general complex Yukawa matrices from the simple diagonal case.

Counting fermion traces with at least two couplings to the Higgs field one can get the full result by the substitutions similar to

$$n_Y^2 y_u^2 y_d^2 \to \mathrm{tr} Y_u Y_u^\dagger \mathrm{tr} Y_d Y_d^\dagger \qquad n_Y y_u^2 y_d^2 \to \mathrm{tr} Y_u Y_u^\dagger Y_d Y_d^\dagger$$

Here n_Y is number of the above-mentioned fermon loops

$y_u, \; y_d$ are diagonal Yukawa couplings (matrix elements)

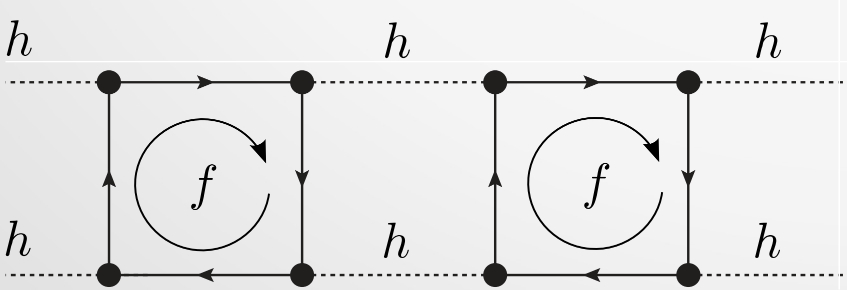
However, this trick is only applicable for gauge couplings since Yukawa matrices contribute starting from the two-loop level.

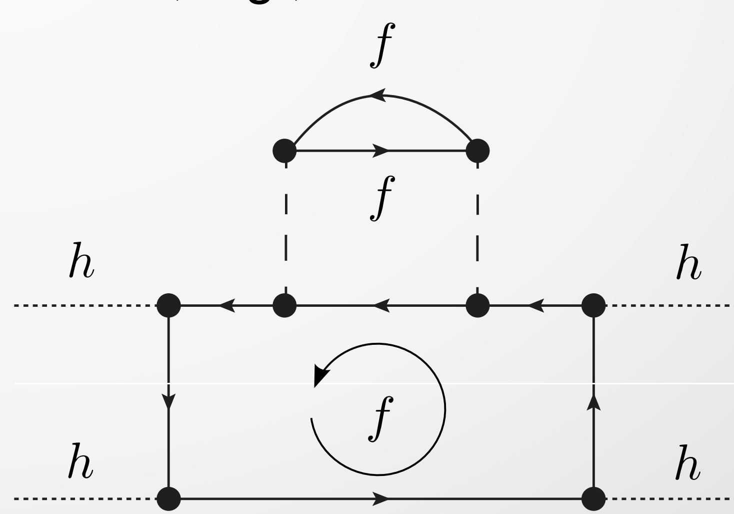
Some remarks: no external quarks

In *Mihaila, Salomon, Steinhauser'12* and *Bednyakov,Pikelner,Velizhanin'12* a trick was used to obtain gauge-coupling beta-functions dependeing on general complex Yukawa matrices from the simple diagonal case.

For the case of Higgs potential parameters the trick fails since it does not allow to distinguish contributions, e.g., of the form

Due to this one needs to take into account flavour indices explicitly





Some techical details: computer setup

From Lagrangian...

LanHep package by A. Semenov
was used to obtain computer-readable
Feynman rules for the SM with general Yukawa matrices

```
Let q1 = {u,d}, q2 = {c,s}, q3 = {t,b}.
Let qq = {q1,q2,q3}, uR = {u,c,t}, dR={d,s,b}

let YuM = { { Yu11, Yu12, Yu13},
            { Yu21, Yu22, Yu23},
            { Yu31, Yu32, Yu33} }.
lterm  -YuM*QQ*(1+g5)/2*i*tau2*uR*PP + AddHermConj.
```

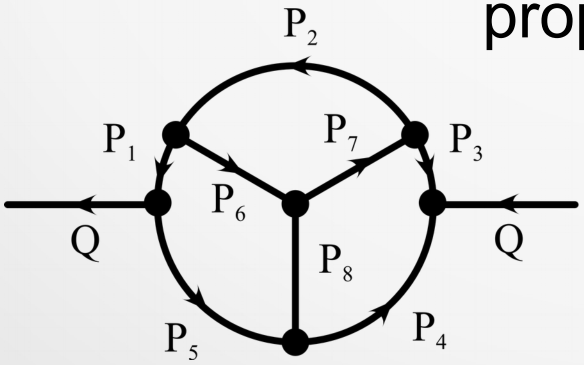
LanHep introduces exlicit generation indices in the output.

Some techical details: computer setup

...to Green functions...

FeyArts 3 by T. Hahn and **DIANA** (QGRAF) by M. Tentyokov (P. Noguera) were used to generate Feynman diagrams and analytic expressions for the required Green functions

BE



An important step:
propagator momenta distribution!

A. Pikelner

<u>Standard codes for Feynman integral evaluation can be used</u>

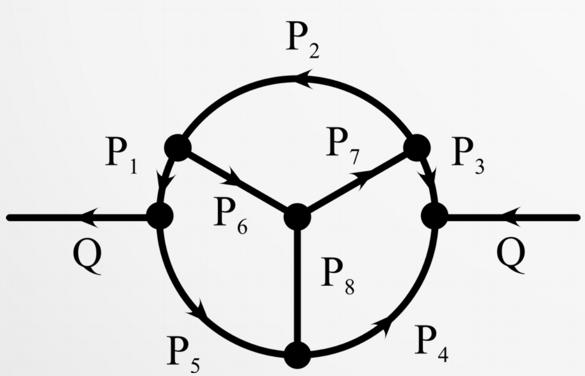
Some techical details: computer setup

…,Feynman Integrals...

<u>Massless</u> propagators

<u>MINCER</u> (Gorishni, Larin, Surguladze, Tkachov; Vermasseren)

BE



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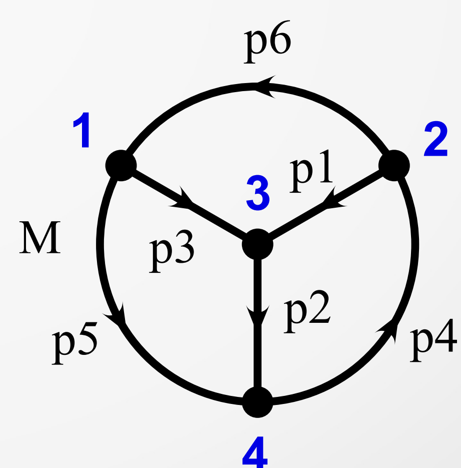


<u>MATAD</u> (Steinhauser)/<u>BAMBA</u>(Velizhanin)

Massive „<u>bubbles</u>“ with auxilary mass on internal lines

Some techical details: computer setup

...divergencies and beta-functions

With MINCER setup no auxilary parameters are introduced so one can calculate bare quantities and carry out renormalization procedure at the level of SM parameters

$$\Gamma_{\mathrm{Ren}}\left(\frac{Q^2}{\mu^2}, a_i\right) = \lim_{\varepsilon\to 0} Z_\Gamma\left(\frac{1}{\varepsilon}, a_i\right)\Gamma_{\mathrm{Bare}}\left(Q^2, a_{i,\mathrm{Bare}}, \varepsilon\right),$$

Recursive application

With BUBBLES setup an auxilary parameter M is introduced so one should use explicti counter-term insertions to obtain the result of incomplete R-operation

DIANA Feynman Rules were modifed to generate the required counter terms (with account of generation indices)

Some issues: „infinite“ RGE functions

The Yukawa beta-function are obtained from $\dot{A} \equiv dA/dt \equiv dA/d\ln\mu^2$

$$0 = \mu^\epsilon \dot{Y}_{f,\mathrm{Bare}} = \left(-\frac{\epsilon}{2}Y_f + \beta_{Y_f} Y_f + \dot{\Delta} Y_f\right) + \frac{\epsilon}{2}\left(Y_f + \Delta Y_f\right)$$

In our first attempt Yukawa matrix beta-functions extracted from this expression were not finite in the limit $\epsilon \to 0$

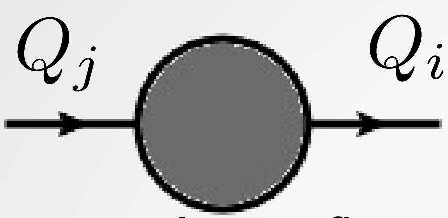
So-called t'Hooft pole equations were not satisfied

Similar problem arises for matrix anomalous dimensions of quark fields!

We made a lot of cross-checks: gauge-fixing parameter independence, two-loop results were reproduced, two independent IRR techniques were used...

The problem appears only at three loops..

Some issues: „square root“ ambiguity



It turns out that the problem lies in quark field renormalization

In our first attempt we use perturbation theory to carry out square root operation of hermition matrix

$$Z_f \equiv Z_f^{1/2\dagger} Z_f^{1/2}$$

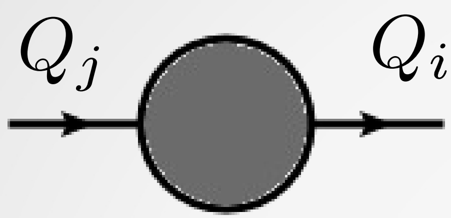
the resulting expression $\tilde{Z}^{1/2}$ was also hermitian

$$Z_f = \tilde{Z}_f^{1/2} \tilde{Z}_f^{1/2}, \qquad \tilde{Z}_f^{1/2\dagger} = \tilde{Z}_f^{1/2\dagger}$$

but the pole equations require introduction of non-hermitian contributions to high poles in ε, which correspond to the introduction addition unitary Factors $\bar{Z}^{1/2}$ so that

$$Z_f^{1/2} = \bar{Z}_f^{1/2} \tilde{Z}_f^{1/2}$$

Some issues: „square root“ ambiguity



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$$Q_{L,\mathrm{Bare}} = Z_Q^{1/2} Q_L$$

$$u_{R,\mathrm{Bare}} = Z_u^{1/2} u_R$$

$$d_{R,\mathrm{Bare}} = Z_d^{1/2} d_R$$

Left-handed form SU(2) doublet

Right-handed SU(2) singlets

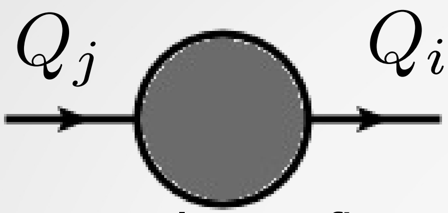
$Z_f^{1/2}$ required for calculation of Yukawa matrix beta-function should be extracted from the corresponding self-energy counter-term

$$i\hat{p} P_L \left[\left(Z_{f_L}^{1/2\dagger} Z_{f_L}^{1/2} \right)_{ij} - \delta_{ij} \right] + (L \to R)$$

Hermitian matrix, invariant under

$Z_f^{1/2} \to \bar{Z}_f^{1/2} Z_f^{1/2}$ with $\bar{Z}_f^{-1} = \bar{Z}_f^{\dagger}$ being unitary

Some issues: „square root“ ambiguity



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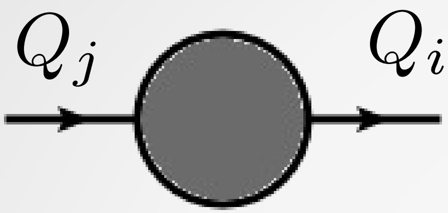
$$Z_f^{1/2} = \bar{Z}_f^{1/2} \tilde{Z}_f^{1/2}$$

Some issues: „square root“ ambiguity



It turns out that the problem lies in quark field renormalization

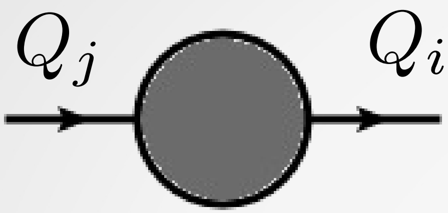
$$h = \frac{1}{16\pi^2}$$

The pole equations = (finitness of corresponding anomalous dimension) require introduction of the following unitary factors

$$\begin{aligned}
\bar{Z}^{1/2}_{Q_L} &= 1 - \frac{a_1 h^2}{320}\left(\frac{1}{6\epsilon^2} - \frac{1}{\epsilon^3}\right)\left(Y_u Y_u^\dagger Y_d Y_d^\dagger - Y_d Y_d^\dagger Y_u Y_u^\dagger\right) \\
&+ \frac{h^3}{64}\left(\frac{1}{6\epsilon^2} + \frac{1}{\epsilon^3}\right)\left(Y_u Y_u^\dagger Y_u Y_u^\dagger Y_d Y_d^\dagger - Y_d Y_d^\dagger Y_u Y_u^\dagger Y_u Y_u^\dagger - (u \leftrightarrow d)\right), \\
\bar{Z}^{1/2}_{u_R} &= 1 - \frac{h^3}{32}\left(\frac{1}{6\epsilon^2} - \frac{1}{\epsilon^3}\right)\left(Y_u^\dagger Y_u Y_u^\dagger Y_d Y_d^\dagger Y_u - Y_u^\dagger Y_d Y_d^\dagger Y_u Y_u^\dagger Y_u\right), \\
\bar{Z}^{1/2}_{d_R} &= 1 - \frac{h^3}{32}\left(\frac{1}{6\epsilon^2} - \frac{1}{\epsilon^3}\right)\left(Y_d^\dagger Y_d Y_d^\dagger Y_u Y_u^\dagger Y_d - Y_d^\dagger Y_u Y_u^\dagger Y_d Y_d^\dagger Y_d\right),
\end{aligned}$$

This factors allow us to make Yukawa matrix beta-functions finite!

Some issues: „square root“ ambiguity



What about the remaining ambiguity?

The factors presented above do not alter first poles in ε and, thus, have no effect on beta-functions and anomalus dimensions...

What if we introduce arbitrary unitary factors $\mathcal{Z}_f$, which
1) compatible with SU(2) symmetry
2) contain also single poles in ε
3) satisfy pole equations by itself, i.e.

$$\gamma'_f \equiv -\mathcal{Z}_f^\dagger \dot{\mathcal{Z}}_f, \qquad \mathcal{Z}_f^\dagger = \mathcal{Z}_f^{-1}$$ are finite

These factors **definetely** modify RGEs for general Yukawa matrices!

However, they can be eliminated by SU(2) compatible rotations of bare quark fields, leading to the same observable masses and CKM matrix!

$$\{U, D\}_{L,\mathrm{bare}} \to \{U, D\}_{L,\mathrm{bare}} \mathcal{Z}_L$$

Some results:Yukawa matrix beta-function

$$\beta^{(1)}_{Y_u} = -4a_s + \frac{3}{2}\left(\mathrm{tr}[Y_d] + \mathrm{tr}[Y_u]\right) + \frac{3}{4}(Y_u - Y_d),$$

$$\begin{aligned}\beta^{(2)}_{Y_u} &= 3\hat{\lambda}^2 - 6\hat{\lambda}Y_u + \frac{11}{8}Y_{dd} - \frac{1}{2}Y_{du} - \frac{1}{8}Y_{ud} \\ &+ \frac{3}{4}\left(Y_{uu} + \mathrm{tr}[Y_{ud}]\right) + \frac{15}{8}Y_d(\mathrm{tr}[Y_d] + \mathrm{tr}[Y_u]) \\ &+ 8a_s\left(Y_u - Y_d\right) + 10a_s\left(\mathrm{tr}[Y_d] + \mathrm{tr}[Y_u]\right) + a_s^2\left(\frac{40}{9}n_G - \frac{202}{3}\right) \\ &- \frac{27}{8}\left(\mathrm{tr}[Y_{dd}] + \mathrm{tr}[Y_{uu}] + \left(\mathrm{tr}[Y_d] + \mathrm{tr}[Y_u]\right)Y_u\right),\end{aligned}$$

$$\beta^{(3)}_{Y_u} = -18\hat{\lambda}^3 + \frac{43}{16}Y_{udu} + \frac{75}{32}Y_{uud} + \frac{83}{32}Y_{duu} - \frac{37}{16}Y_{dud} + \ldots$$

EW couplings neglected

Number of generations

Avaliable on request, will be published in the near future

where $\underbrace{Y_{ff'\ldots}}_{n} \equiv h^n Y_f Y_f^\dagger Y_{f'} Y_{f'}^\dagger \ldots$

$$\hat{\lambda} \equiv \frac{\lambda}{16\pi^2}$$

Some remarks on CKM renormalization

RGEs for CKM matrix elements are given by

see *Naculich'93*, *Kielanowski et al'08*

$$\dot{V}_{CKM} = \dot{U}_L U_L^\dagger V_{CKM} + V_{CKM} D_L \dot{D}_L^\dagger$$

We also have

$$U_L Y^u U_R^\dagger = Y^u_{\rm diag}, \qquad D_L Y^d D_R^\dagger = Y^u_{\rm diag}$$

$$\begin{aligned} U_L \frac{d}{dt}\left(Y^u Y^{u\dagger}\right) U_L^\dagger &= \frac{d}{dt}\left(Y^u_{\rm diag}\right)^2 + \left[U_L \dot{U}_L^\dagger, \left(Y^u_{\rm diag}\right)^2\right] \\ &= \left(Y^u_{\rm diag}\right)^2 U_L \beta^\dagger_{Y_u} U_L^\dagger + U_L \beta_{Y_u} U_L^\dagger \left(Y^u_{\rm diag}\right)^2 \end{aligned}$$

Diagonal entries of the equation gives RGE for Yukawa couplings in mass basis, and the non-diagonal ones fix $\left(U_L \dot{U}_L^\dagger\right)_{ij}, \; i \neq j$

Diagnonal entries can be set to zero by appropriate field rephasing leaving $Y^u_{\rm diag}$ invariant

Some conclusions and outlook

• Our plans dedicated to the extension of available two-loop results* for beta-fuctions and anomalous dimension of fundamental SM couplings to the three-loop case are fulfilled!

• Some ambiguities in RGEs for matrix Yukawa couplings are clarified

•The results can be found online (as ancillary files of arXiv preprints)

•RGE for diagonal Yukawa couplings in mass basis and CKM matrix are almost completed

* see *Luo,Xiao'02* and references therein.

Thank you for your attention!