

Breaking of isotopic invariance in the decay $\eta(1405/1475) \rightarrow 3\pi$

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- 5 Discussion and conclusion

Introduction

- Feasible mechanisms of isospin braking
- Discussing $\eta(1405/1475)$ decay mechanisms
- Estimating $\iota \rightarrow \pi^+ \pi^- \pi^0$ branching ratio
- Discussion and conclusion

Introduction

The BESIII Collaboration studied the decays $J/\psi \rightarrow \gamma\pi^+\pi^-\pi^0$ and $J/\psi \rightarrow \gamma\pi^0\pi^0\pi^0$. The resonance structure in the 3π mass spectrum with the mass $M \sim 1400$ MeV and width $\Gamma \sim 50$ MeV was observed to which associated is the narrow structure in the $\pi^+\pi^-$ and $\pi^0\pi^0$ mass spectra in vicinity of 990 MeV with the width about 10 MeV.

$\pi^+ \pi^-$ mass spectrum

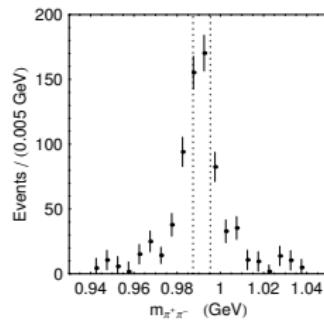


Figure: The $\pi^+ \pi^-$ mass spectrum in the decay $J/\psi \rightarrow \gamma \pi^+ \pi^- \pi^0$

- BESIII was observed the isospin-breaking decay $\eta(1405/1475) \rightarrow 3\pi$.
- The structure observed in the $\pi\pi$ mass spectrum is clearly associated with the K^+K^- and $K^0\bar{K}^0$ thresholds:

$$2m_{K^\pm} = 0.987354 \text{ MeV},$$

$$2m_{K^0} = 0.995228 \text{ MeV}.$$

- One should study and analyze the mechanisms explaining the decay $\eta(1405/1474) \rightarrow 3\pi$.
- In what follows the shorthand notation will be used:

$$\eta(1405/1475) \equiv \iota.$$

$K^* \bar{K} + c.c.$ mechanism

- $K^* \bar{K} + c.c.$ mechanism (triangle diagram):

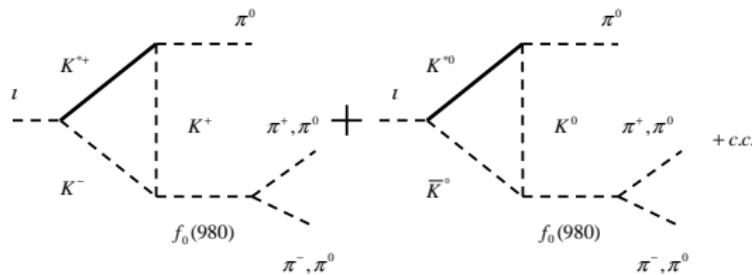


Figure: Triangle diagram

$a_0 - f_0$ mixing

- $a_0 - f_0$ mixing:

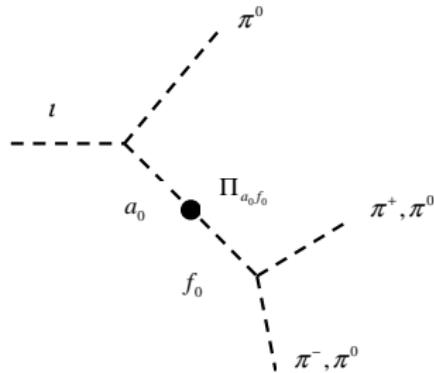


Figure: $a_0 - f_0$ mixing

Status and decay modes of $\eta(1405/1475)$

- Experimental status of $\iota \equiv \eta(1405/1475)$ is still unclear: is it the single state which differently reveals in different channels or there are two resonances $\eta(1405)$ and $\eta(1475)$?
- Allowed dominant decay modes are $\eta\pi\pi$ and $K\bar{K}\pi$ with the still contradictory branching ratios.
- What about coupling constants?

Spectrum of $K^+ K^- \pi^0$

Some guesses about coupling constants can be obtained using the data on the decay $J/\psi \rightarrow \gamma K^+ K^- \pi^0$.

- Spectrum of the state $K^+ K^- \pi^0$ in the decay $J/\psi \rightarrow \gamma K^+ K^- \pi^0$:

$$\frac{d\Gamma}{dm} = A \frac{m^2 \Gamma_{\iota \rightarrow K^+ K^- \pi^0}(m) \left(1 - \frac{m^2}{m_{J/\psi}^2}\right)^3}{(m^2 - m_\iota^2)^2 + m^2 \Gamma_\iota^2(m)}$$

- m is invariant mass of the state $K^+ K^- \pi^0$.
- $\Gamma_\iota(m) = \Gamma_{\iota \rightarrow K\bar{K}\pi}(m) + \Gamma_{\iota \rightarrow \eta\pi\pi}(m)$.

Diagrams for $\iota \rightarrow K\bar{k}\pi$ decay

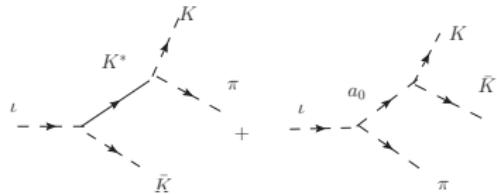


Figure: Diagrams for $\iota \rightarrow K\bar{k}\pi$ decay. All possible isotopic modes for intermediate $K^*\bar{K} + c.c$ states are included.

Amplitudes of decay $i \rightarrow K^+ K^- \pi^0$

$$\begin{aligned}
 M_{\iota \rightarrow K^+ K^- \pi^0} = & \frac{g_\iota a_0 \pi g_{a_0 K K}}{D_{a_0}(m_0^2)} + g_\iota K^* \kappa g_{K^* K \pi^0} \times \\
 & \left[\frac{m_0^2 - m_-^2 - (m_\iota^2 - m_K^2)(m_K^2 - m_\pi^2)/m_+^2}{m_{K^*}^2 - m_+^2 - im_{K^*} \Gamma_{K^*}} + \right. \\
 & \left. \frac{m_0^2 - m_+^2 - (m_\iota^2 - m_K^2)(m_K^2 - m_\pi^2)/m_-^2}{m_{K^*}^2 - m_-^2 - im_{K^*} \Gamma_{K^*}} \right],
 \end{aligned}$$

$$m_+^2 = (p_{K^+} + p_{\pi^0})^2,$$

$$m_-^2 = (p_{K^-} + p_{\pi^0})^2,$$

$$m_0^2 = (p_{K^+} + p_{K^-})^2.$$

Inverse propagator of scalar mesons

- Inverse propagator of scalar meson $S = a_0, f_0$:

$$D_S(m^2) = m_S^2 - m^2 - \Pi_{SS}(m^2).$$

- Polarization operator due to pseudoscalar loops:

$$\begin{aligned} \Pi_{SS}(m^2) &= \sum_{P_1 P_2} g_{SP_1 P_2}^2 \left[G_{SPP}(m^2, m_{P_1}, m_{P_2}) - \right. \\ &\quad \left. \text{Re} G_{SPP}(m_S^2, m_{P_1}, m_{P_2}) \right] \end{aligned}$$

Pseudoscalar loop

$$G_{SPP}(m^2, m_{P_1}, m_{P_2}) = \frac{1}{16\pi^2 m^2} \left[m_+ m_- \ln \frac{m_{P_1}}{m_{P_2}} + F(m^2) \right], \text{ where}$$

$$F(m^2) = \sqrt{(m_+^2 - m^2)(m_-^2 - m^2)} \ln \frac{\sqrt{m_+^2 - m^2} + \sqrt{m_-^2 - m^2}}{\sqrt{m_+^2 - m^2} - \sqrt{m_-^2 - m^2}}$$

at $m^2 < m_-^2$;

$$F(m^2) = \sqrt{(m_+^2 - m^2)(m^2 - m_-^2)} \left(2 \arctan \sqrt{\frac{m_+^2 - m^2}{m^2 - m_-^2}} - \pi \right)$$

at $m_-^2 \leq m^2 \leq m_+^2$;

Pseudoscalar loop cont'd

and

$$F(m^2) = \sqrt{(m^2 - m_-^2)(m^2 - m_+^2)} \left(i\pi - \ln \frac{\sqrt{m^2 - m_-^2} + \sqrt{m^2 - m_+^2}}{\sqrt{m^2 - m_-^2} - \sqrt{m^2 - m_+^2}} \right)$$

at $m^2 > m_+^2$.

- $m_{\pm} = m_{P_1} \pm m_{P_2}$, $m_{P_1} \geq m_{P_2}$.

Determining ι parameters

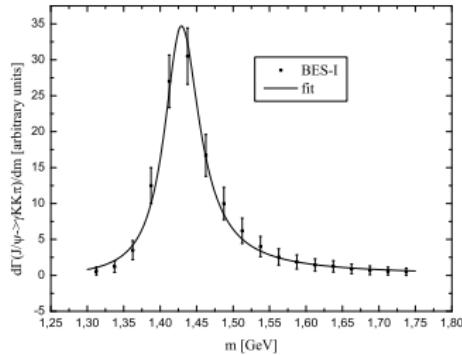


Figure: Mass spectrum of $K^+ K^- \pi^0$ in the decay $J/\psi \rightarrow \gamma K^+ K^- \pi^0$

- The resonance parameters of ι obtained from fit: $m = 1.429$ GeV, $g_{\iota a_0 \pi} = 2.1$ GeV, $g_{\iota K^* K} = -0.2$.

Branching ratios

- Ratio of branching fractions calculated from fitting parameters:

$$\frac{B(\iota \rightarrow K\bar{K}\pi)}{B(\iota \rightarrow \eta\pi\pi)} \approx 0.2$$

- Experiment:

$$\frac{B(\iota \rightarrow K\bar{K}\pi)}{B(\iota \rightarrow \eta\pi^+\pi^-)} \approx 9$$

- Skipping this discrepancy for future resolution, let's consider the calculated number as the first indication on negligible character $\iota \rightarrow K^*\bar{K} + c.c.$ mode.

Triangle diagram as the source of isospin breaking

- $\text{Img}_{\nu f_0 \pi}^{\text{eff}}$ is given by the Cutkosky rule:

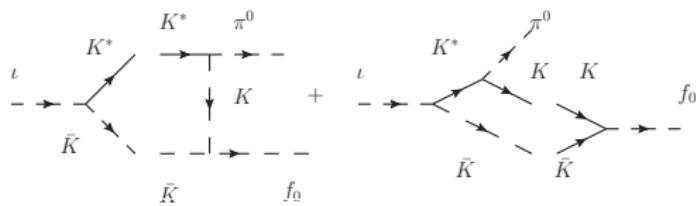


Figure: Contributions to imaginary part of $g_{\nu f_0 \pi}^{\text{eff}}$

$K\bar{K}$ discontinuity

$$\text{Disc}_{K\bar{K}} = \frac{g_{\iota K^* K} g_{K^* K \pi^0} g_{f_0 K\bar{K}}}{16\pi\mu|\mathbf{p}'_\pi|} \left\{ 4|\mathbf{p}'_\pi||\mathbf{p}'_K| + \left[s + m_\pi^2 + 2m_K^2 - m^2 - 2\mu^2 + \frac{(s - m_K^2)(m_K^2 - m_\pi^2)}{m^2} \right] \ln \frac{a_{K\bar{K}} + 1 - i\varepsilon}{a_{K\bar{K}} - 1 - i\varepsilon} - \frac{(s - m_K^2)(m_K^2 - m_\pi^2)}{m^2} \ln \frac{a_{K\bar{K}}^{(0)} + 1}{a_{K\bar{K}}^{(0)} - 1} \right\},$$

$a_{K\bar{K}} \equiv a_{K\bar{K}}(m^2) = -\frac{2E'_\pi E'_K + m_K^2 + m_\pi^2 - m^2}{2|\mathbf{p}'_\pi||\mathbf{p}'_K|}$; $E'_\pi, E'_K, (|\mathbf{p}'_\pi|, |\mathbf{p}'_K|)$ are energies (momenta) of pion and kaon respectively, in the f_0 rest frame; $a_{K\bar{K}}^{(0)} = a_{K\bar{K}}(m^2 = 0)$.

$K^* \bar{K}$ discontinuity

$$\text{Disc}_{K^* \bar{K}} = \frac{g_{\iota K^* K} g_{K^* K \pi^0} g_{f_0 K \bar{K}}}{16\pi\sqrt{s}|\mathbf{p}_\pi|} \left\{ -4|\mathbf{p}_\pi||\mathbf{p}_{\bar{K}}| \left(1 + \frac{s - m_K^2}{m^2} \right) + \left[s + m_\pi^2 + 2m_K^2 - m^2 - 2\mu^2 + \frac{(s - m_K^2)(m_K^2 - m_\pi^2)}{m^2} \right] \times \ln \frac{a_{K^* \bar{K}} + 1 + i\varepsilon}{a_{K^* \bar{K}} - 1 + i\varepsilon} \right\},$$

$a_{K^* \bar{K}} = \frac{2E_{f_0} E_{\bar{K}} - \mu^2}{2|\mathbf{p}_\pi||\mathbf{p}_{\bar{K}}|}$; E_{f_0} , $E_{\bar{K}}$, ($|\mathbf{p}_{f_0}| = |\mathbf{p}_\pi|$, $|\mathbf{p}_{\bar{K}}|$) are energies (momenta) of f_0 and anti-kaon respectively, in the ι rest frame.

Singularity of triangle diagram, fixed μ, m

- There is the kinematic region $\sqrt{s} > m + m_K$, $m > m_K + m_\pi$, $\mu > 2m_K$, where the particles in the loop are on mass shell:

$$s_{\pm} = m_\pi^2 + \mu^2 \left\{ 1 + \frac{1}{2m_K^2} \left[m^2 - m_K^2 - m_\pi^2 \right. \right.$$

$$\left. \left. \pm \sqrt{\lambda(m^2, m_K^2, m_\pi^2) \left(1 - \frac{4m_K^2}{\mu^2} \right)} \right] \right\},$$

$$\lambda(m_1^2, m_2^2, m_3^2) = m_1^4 + m_2^4 + m_3^4 - 2(m_1^2 m_2^2 + m_1^2 m_3^2 + m_2^2 m_3^2),$$

where s , m^2 , μ^2 are the squared masses of virtual ν , K^* , f_0 , respectively. $\sqrt{s_-} = 1.405$ GeV, $\sqrt{s_+} = 1.498$ GeV at $m = 0.895$ GeV, $\mu = 1$ GeV.

Singularity of triangle diagram, fixed s, m

$$\begin{aligned}\mu_{\pm}^2 &= \frac{1}{2m^2} \left[(s - m^2 - m_K^2)(s - m_\pi^2) - 4s\mathbf{p}_{\bar{K}}^2 \right] \pm \frac{2\sqrt{s}|\mathbf{p}_{\bar{K}}||\mathbf{p}_K^{(0)}|}{m}, \\ |\mathbf{p}_{\bar{K}}| &= \frac{1}{2\sqrt{s}} \sqrt{[s - (m - m_K)^2][s - (m + m_K)^2]}, \\ |\mathbf{p}_K^{(0)}| &= \frac{1}{2m} \sqrt{[m^2 - (m - m_K)^2][m^2 - (m + m_K)^2]}.\end{aligned}$$

- For example, at $m = 0.895$ GeV and $\sqrt{s} = 1.405$ GeV one obtains $\mu_- = 0.995$ GeV, $\mu_+ = 1.084$ GeV.

Contributions to imaginary part

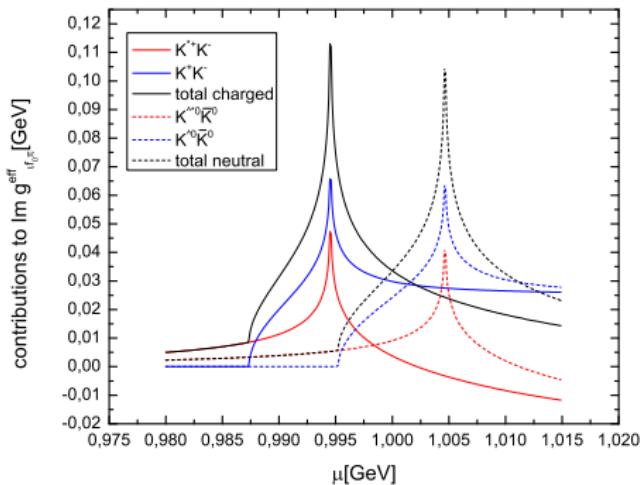


Figure: Contributions to $\text{Im } g_{\nu_0\pi}^{\text{eff}}$

Singularity of triangle diagram, fixed s, μ

$$(m^2)_{\pm} = m_K^2 + \frac{1}{2}(s + m_\pi^2 - \mu^2) \pm \sqrt{\frac{1}{2} \left(1 - \frac{4m_K^2}{\mu^2} \right) [s - (\mu - m_\pi)^2][s - (\mu + m_\pi)^2]}$$

If, for example, $\mu = 1$ GeV, $m_K = 0.496$ GeV, then

- $\sqrt{s} = 1.405$ GeV $\Rightarrow m_+ = 0.895$ GeV, $m_- = 0.827$ GeV.
- $\sqrt{s} = 1.429$ GeV $\Rightarrow m_+ = 0.916$ GeV, $m_- = 0.845$ GeV.
- For comparison: $(m_{K^*} - \frac{\Gamma_{K^*}}{2}, m_{K^*} + \frac{\Gamma_{K^*}}{2}) = (0.869, 0.919)$ GeV.

Lines of triangle singularity

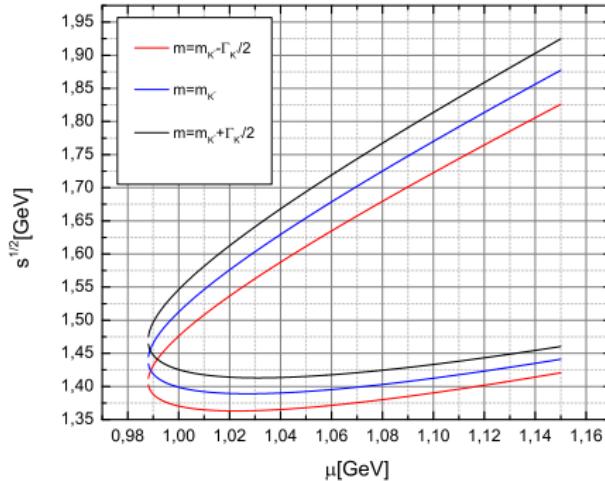


Figure: Lines of triangle singularity at different m .

Smoothing triangle singularity

Finite width of intermediate K^* is achieved by the substitution

$$g_{\iota f_0 \pi}^{\text{eff}}(m_{K^*}^2) \rightarrow \frac{2}{\pi} \int_{m_K + m_\pi}^{\infty} \frac{g_{\iota f_0 \pi}^{\text{eff}}(m^2) m_{K^*} \Gamma_{K^*}}{(m^2 - m_{K^*}^2)^2 + m_{K^*}^2 \Gamma_{K^*}^2} mdm.$$

This results in the suppression by more than order of magnitude of $\Gamma_{\iota \rightarrow \pi^+ \pi^- \pi^0}$ as compared with "alive" K^* .

Estimating the $a_0 - f_0$ mixing contribution

- Partial width of the decay $\iota \rightarrow \pi^+ \pi^- \pi^0$:

$$\Gamma_{\iota \rightarrow \pi^+ \pi^- \pi^0} = \frac{g_{\iota a_0 \pi^0}^2 g_{f_0 \pi^+ \pi^-}^2}{16(2\pi)^3 m_\iota^3} \int_{2m_\pi}^{m_\iota - m_\pi} dm \times \\ \left| \frac{\Pi_{a_0 f_0}(m^2)}{D_{a_0}(m^2) D_{f_0}(m^2) - \Pi_{a_0 f_0}^2(m^2)} \right|^2 \times \\ \sqrt{(m^2 - 4m_\pi^2)[m_\iota^2 - (m - m_\pi)^2][m_\iota^2 - (m + m_\pi)^2]}$$

$$\Pi_{a_0 f_0}(m^2)$$

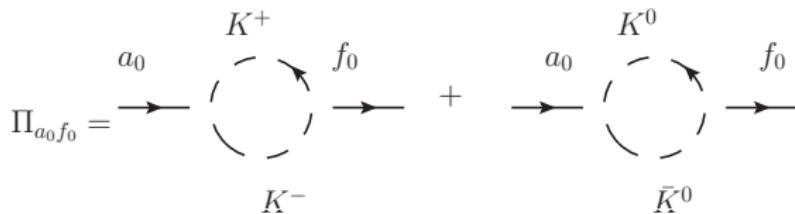


Figure: Loop contributions to $\Pi_{a_0 f_0}$

$$\Pi_{a_0 f_0}(m^2) = g_{f_0 K \bar{K}} g_{a_0 K \bar{K}} \left[G_{SPP}(m^2, m_{K^+}, m_{K^+}) - G_{SPP}(m^2, m_{K^0}, m_{K^0}) \right]$$

$\Pi_{a_0 f_0}(m^2)$ cont'd

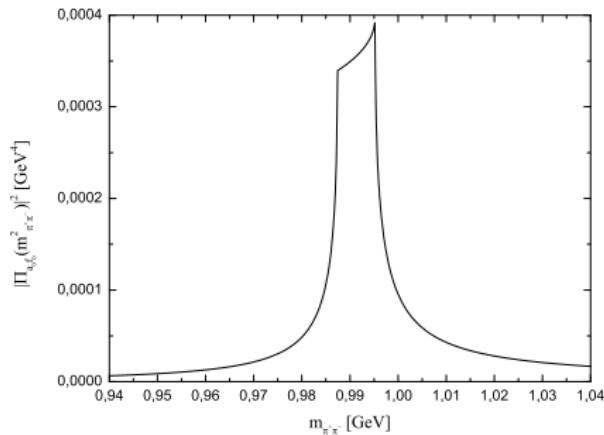


Figure: Resonance-like structure is due to the factor

$$\left| \sqrt{1 - 4m_{K^+}^2/m_{\pi^+\pi^-}^2} - \sqrt{1 - 4m_{K^0}^2/m_{\pi^+\pi^-}^2} \right|^2$$

$\pi^+ \pi^-$ spectrum

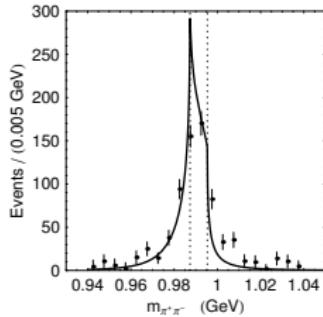


Figure: $\pi^+ \pi^-$ spectrum in the decay $\iota \rightarrow \pi^+ \pi^- \pi^0$.

- Theory: $\frac{B(\iota \rightarrow \pi^+ \pi^- \pi^0)}{B(\iota \rightarrow K\bar{K}\pi)} \approx 3 \times 10^{-3}$.
- Experiment: $\frac{B(\iota \rightarrow \pi^+ \pi^- \pi^0)}{B(\iota \rightarrow K\bar{K}\pi)} \approx 5 \times 10^{-3}$.

Conclusion

- The model of $a_0 - f_0$ mixing,
 $\eta(1405/1475) \rightarrow a_0 \pi^0 \rightarrow f_0 \pi^0 \rightarrow \pi^+ \pi^- \pi^0$, qualitatively explains large violation of isospin symmetry.
- But the problem still remains: the mode $K\bar{K}\pi$ is suppressed as compared with the $\eta\pi\pi$ one, in contradiction with experiment.
- Feasible resolution might be in taking into account the final state $\pi\pi$ interaction and/or a careful reanalysis of relevant data in order to isolate the regions of the phase space pertinent to the state $\eta(1405/1475)$.

Thanks!