

On massive spin 3/2 interactions in frame-like formalism

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Outline

① Introduction

② Frame-like gauge invariant approach

- Frame-like formulation
- Cubic vertexes

③ Massive spin 3/2

- Free fields
- Electromagnetic coupling
- Gravitational coupling

④ Summary

The talk is based on [arXiv:1405.7781](https://arxiv.org/abs/1405.7781)
with I.L. Buchbinder and Yu.M. Zinoviev

Fermionic higher spin fields

- Free fields: Lagrangian formulation for massless Fang and Fronsdal (1978) and massive fields Singh and Hagen (1974)
- Interacting massless (gauge) fields: Full non-linear theory as extended supersymmetric gauge theory Fradkin and Vasiliev (1987)

$$\begin{array}{lll} \text{Spin } 3/2 & \rightarrow & \text{Extended Poincaré (AdS) superalgebras} \\ \text{Spin } \geq 3/2 & \rightarrow & \text{Extended Higher Spin superalgebras} \end{array}$$

Gauge invariant principles

- Interacting massive fields: Classification of cubic vertices in light-cone approach Metsaev (2007). Superstring theory.
Gauge-invariant description and cubic vertices as first step.

Frame-like gauge invariant approach

Frame-like formulation

Frame-like formualtion for spins higher $s \geq 3/2$ is just generalization of frame formulation for gravity in terms of tetrad $\bar{e}_\mu{}^a$ and Lorentz connection $\bar{\omega}_\mu{}^{a,b}$

- Field content

$$\begin{aligned}\bar{e}_\mu{}^a, \bar{\omega}_\mu{}^{a,b} &\Rightarrow \Phi_\mu{}^{a_1 \dots a_{s-1}, b_1 \dots b_k}, \quad 0 \leq k \leq s-1 \quad \text{for bosons} \\ \psi_\mu &\Rightarrow \Psi_\mu{}^{a_1 \dots a_{s-3/2}, b_1 \dots b_k}, \quad 0 \leq k \leq s-3/2 \quad \text{for fermions}\end{aligned}$$

Physicly extra fields $\Phi_\mu{}^{a_1 \dots a_{s-1}, b_1 \dots b_k} \sim \partial^k \Phi_\mu{}^{a_1 \dots a_{s-1}}$

- Gauge transformations

$$\delta \Phi_\mu{}^{a_1 \dots a_{s-1}, b_1 \dots b_k} = \partial_\mu \xi^{a_1 \dots a_{s-1}, b_1 \dots b_k} + \dots$$

- Gauge invariant field strength (curvatures)

$$\mathcal{R}_{\mu\nu}{}^{a_1 \dots a_{s-1}, b_1 \dots b_k} = \partial_{[\mu} \Phi_{\nu]}{}^{a_1 \dots a_{s-1}, b_1 \dots b_k} + \dots$$

- Lagrangian for free fields

$$\mathcal{L}_0 = \sum \mathcal{R} \wedge \mathcal{R}$$

Frame-like gauge invariant approach

Frame-like description

Spin-2 example (linearized gravity in AdS)

- Field content: $\bar{e}_\mu{}^a = e_\mu{}^a + h_\mu{}^a \quad \bar{\omega}_\mu{}^{a,b} = w_\mu{}^{a,b} + \omega_\mu{}^{a,b}$

$$\begin{array}{ll} e_\mu{}^a, w_\mu{}^{a,b} & - \quad \text{fixed (non-dynamical) fields of AdS space} \\ h_\mu{}^a, \omega_\mu{}^{a,b} & - \quad \text{dynamical spin-2 fields} \end{array}$$

- Gauge transformations

$$\delta_0 h_\mu{}^a = D_\mu \hat{\xi}^a + \hat{\eta}_\mu{}^a \quad \delta_0 \omega_\mu{}^{a,b} = D_\mu \hat{\eta}^{a,b} - \lambda^2 e_\mu{}^{[a} \hat{\xi}^{b]}$$

where D_μ is AdS covariant derivative

$$D_\mu \xi^a = \partial_\mu \xi^a + w_\mu{}^{ab} \xi^b, \quad D_{[\mu} D_{\nu]} \xi^a = \lambda^2 e_{[\mu}{}^a \xi_{\nu]}$$

- Gauge invariant field strength (Torsion and Curvature)

$$T_{\mu\nu}{}^a = D_{[\mu} h_{\nu]}{}^a - \omega_{[\mu,\nu]}{}^a \quad R_{\mu\nu}{}^{a,b} = D_{[\mu} \omega_{\nu]}{}^{a,b} - \lambda^2 e_{[\mu}{}^{[a} h_{\nu]}{}^{b]}$$

- Lagrangian for free fields **MacDowell and Mansouri (1977)**

$$\mathcal{L}_0 = a_0 \left\{ \frac{\mu\nu\alpha\beta}{abcd} \right\} R_{\mu\nu}{}^{ab} R_{\alpha\beta}{}^{cd}, \quad \left\{ \frac{\mu\nu\alpha\beta}{abcd} \right\} = e^{[\mu}{}_a e^{\nu}{}_b e^{\alpha}{}_c e^{\beta]}{}_d$$

Frame-like gauge invariant approach

Cubic vertices

General setup in gauge-invariant description

- The Lagrangian and gauge transformations are represented as a series in powers of some coupling constant κ

$$\mathcal{L} = \mathcal{L}_0 + \kappa \mathcal{L}_1 + \dots, \quad \delta = \delta_0 + \kappa \delta_1 + \dots$$

where

$$\mathcal{L}_1 = \Phi \Phi \Phi, \quad \delta_1 \Phi = \Phi \xi$$

- Condition of gauge invariance $\delta \mathcal{L} = 0$ requires

$$\kappa^0 \quad \delta_0 \mathcal{L}_0 = 0$$

$$\kappa^1 \quad \delta_0 \mathcal{L}_1 + \delta_1 \mathcal{L}_0 = 0$$

...

Cubic vertexes

Additional features in frame-like formulation

- Cubic vertex in terms of linearized curvatures **Vasiliev (2011)**

$$\mathcal{L}_1 = \mathcal{R}\mathcal{R}\mathcal{R} + \mathcal{R}\mathcal{R}\Phi + \mathcal{R}\Phi\Phi$$

- There exist quadratic deformation for all curvatures $\mathcal{R} \rightarrow \hat{\mathcal{R}} = \mathcal{R} + \Delta\mathcal{R}$ such that deformed curvatures transform covariantly

$$\Delta\mathcal{R} = \Phi\Phi, \quad \delta_1\Phi = \Phi\xi \quad \Rightarrow \quad \delta\hat{\mathcal{R}} = \delta_0\Delta\mathcal{R} + \delta_1\mathcal{R} = \mathcal{R}\xi$$

- Non-trivial interacting Lagrangian

$$\mathcal{L} = \hat{\mathcal{R}}\hat{\mathcal{R}} + \mathcal{R}\mathcal{R}\Phi \quad \rightarrow \quad \delta\mathcal{L} = \mathcal{R}\mathcal{R}\xi = 0$$

Massive spin 3/2

Free fields

Frame-like gauge invariant formulation

- Field content

ψ_μ — spin-vector master field

ϕ — spinor auxiliary Stueckelberg field

- Free Lagrangian and gauge transformation

$$\begin{aligned}\mathcal{L}_0 = & -\frac{i}{2} \left\{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \right\} \bar{\psi}_\mu \Gamma^{abc} D_\nu \psi_\alpha + \frac{i}{2} e^\mu{}_a \bar{\phi} \gamma^a D_\mu \phi + \\ & + 3im e^\mu{}_a \bar{\psi}_\mu \gamma^a \phi - \frac{3M}{2} \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} \bar{\psi}_\mu \Gamma^{ab} \Psi_\nu - M \bar{\phi} \phi\end{aligned}$$

$$\delta_0 \psi_\mu = D_\mu \xi + \frac{iM}{2} \gamma_\mu \xi \quad \delta_0 \phi = 3m \xi$$

where $M^2 = m^2 + \lambda^2$. In $m = 0$ we have massless spin-3/2 and massive spin-1/2

Notations $\Gamma^{ab} = 1/2 \gamma^{[a} \gamma^{b]}$, $\left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} = e^{[\mu}{}_a e^{\nu]}{}_b$

Conventions $D_{[\mu} D_{\nu]} \xi = \lambda^2 / 2 \Gamma_{\mu\nu} \xi$, $\lambda^2 = -\Lambda/3$

Massive spin 3/2

Free fields

Frame-like gauge invariant formulation

- Gauge invariant strength (curvatures)

$$\begin{aligned}\Psi_{\mu\nu} &= D_{[\mu}\psi_{\nu]} + \frac{m}{6}\Gamma_{\mu\nu}\phi + \frac{iM}{2}\gamma_{[\mu}\psi_{\nu]} \\ \Phi_\mu &= D_\mu\phi - 3m\psi_\mu + \frac{iM}{2}\gamma_\mu\phi\end{aligned}$$

- Free Lagrangian through curvature

$$\mathcal{L}_0 = c_1 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} \bar{\Psi}_{\mu\nu}\Gamma^{abcd}\Psi_{\nu\alpha} + i c_2 \left\{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \right\} \bar{\Psi}_{\mu\nu}\Gamma^{abc}\Phi_\alpha + c_3 \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} \bar{\Phi}_\mu\Gamma^{ab}\Phi_\nu$$

The requirement to reproduce the original Lagrangian partially fixes the parameters

$$3c_3 = -8c_1, \quad 32c_1M = 1 - 12c_2m$$

One-parameter ambiguity is related with the identity:

$$\left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} D_\mu (\bar{\Psi}_{\nu\alpha}\Gamma^{abcd}\Phi_\beta) = 0$$

Massive spin 3/2

Electromagnetic coupling

For e/m gauge group $SO(2) \simeq U(1)$ we turn $\psi_\mu, \phi \rightarrow \psi_\mu^i, \phi^i$ where $i = 1, 2$

- Minimal coupling $D_\mu \rightarrow D_\mu + e_0 \varepsilon^{ij} A_\mu$

$$\delta_0 \mathcal{L}_0 = -3ie_0 \varepsilon^{ij} e^\mu{}_a \bar{\psi}_\mu{}^i \Gamma^{abc} F^{bc} \xi^j$$

- Non-minimal interaction

$$\begin{aligned} \mathcal{L}_1 &= 3\alpha_1 \varepsilon^{ij} \left\{ \frac{\mu\nu}{ab} \right\} \bar{\psi}_\mu{}^i (2F^{ab} + \Gamma^{abcd} F^{cd}) \psi_\nu{}^j - \\ &\quad -\alpha_2 i \varepsilon^{ij} e^\mu{}_a \bar{\psi}_\mu{}^i (2F^{ab} \gamma^b - \Gamma^{abc} F^{bc}) \phi^j + \frac{M\alpha_2}{3m} \varepsilon^{ij} \bar{\phi}^i (\Gamma F) \phi^j \end{aligned}$$

$$\begin{aligned} \delta_1 \psi_\mu{}^i &= i\alpha_1 \varepsilon^{ij} (\Gamma F) \gamma_\mu \xi^j, & \delta \phi^i &= \alpha_2 \varepsilon^{ij} (\Gamma F) \xi^j \\ \delta_1 A_\mu &= -24\alpha_1 \varepsilon^{ij} \bar{\psi}_\mu{}^i \xi^j - 2i\alpha_2 \varepsilon^{ij} \bar{\phi}^i \gamma_\mu \xi^j \end{aligned}$$

One relation holds $4M\alpha_1 + 2m\alpha_2 = e_0$

Massive spin 3/2

Electromagnetic coupling

- Deformations for curvatures $\Delta \mathcal{R}$

$$\begin{aligned}\Delta \Psi_{\mu\nu}{}^i &= e_0 \varepsilon^{ij} A_{[\mu} \psi_{\nu]}{}^j, & \Delta \Phi_\mu{}^i &= e_0 \varepsilon^{ij} A_\mu \phi^j \\ \Delta F_{\mu\nu} &= \varepsilon^{ij} (a_1 \bar{\psi}_{[\mu}{}^i \psi_{\nu]}{}^j + i a_2 \bar{\psi}_{[\mu}{}^i \gamma_{\nu]} \phi^j + a_3 \phi^i \Gamma_{\mu\nu} \phi^j)\end{aligned}$$

- Deformed curvatures transform

$$\begin{aligned}\delta \hat{\Psi}_{\mu\nu}{}^i &= e_0 \varepsilon^{ij} F_{\mu\nu} \xi^j, & \delta \hat{\Phi}_\mu{}^i &= 0 \\ \delta \hat{F}_{\mu\nu} &= \varepsilon^{ij} [2a_1 \bar{\Psi}_{\mu\nu}{}^i \xi^j - i a_2 \bar{\Phi}_{[\mu}{}^i \gamma_{\nu]} \xi^j]\end{aligned}$$

In this one relation holds $18ma_3 = -ma_1 - 6Ma_2$

- Interacting Lagrangian

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4} \hat{F}^{\mu\nu} \hat{F}_{\mu\nu} + c_1 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} \hat{\Psi}_{\mu\nu}{}^i \Gamma^{abcd} \hat{\Psi}_{\alpha\beta}{}^i \\ & + i c_2 \left\{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \right\} \hat{\bar{\Psi}}_{\mu\nu}{}^i \Gamma^{abc} \hat{\Phi}_\alpha{}^i + c_3 \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} \hat{\bar{\Phi}}_\mu{}^i \Gamma^{ab} \hat{\Phi}_\nu{}^i\end{aligned}$$

Requirement of gauge invariance gives

$$a_1 = -12\alpha_1 = -96e_0 c_1, \quad a_2 = 2\alpha_2 = 12e_0 c_2$$

Massive spin 3/2

Gravitational coupling ($\frac{3}{2} - \frac{3}{2} - 2$)

General scheme

- Let us for a while note

$$\begin{aligned}\psi &\equiv (\psi_\mu, \phi), \quad \Psi \equiv (\Psi_{\mu\nu}, \Phi_\mu) - \text{fields and curvatures for spin-3/2} \\ \omega &\equiv (\omega_\mu^{ab}, h_\mu^a), \quad R \equiv (R_{\mu\nu}^{ab}, T_{\mu\nu}^a) - \text{fields and curvatures for spin-2}\end{aligned}$$

- Deformations to curvatures $\hat{\mathcal{R}} = \mathcal{R} + \Delta\mathcal{R}$

$$\Delta\Psi = \omega\psi, \quad \Delta R = \psi\psi$$

- Corrections to gauge transformation

$$\delta_1\psi = \omega\xi \oplus \psi\hat{\xi}, \quad \delta_1\omega = \psi\xi$$

- Curvature transformations $\hat{\mathcal{R}} = \delta_1\mathcal{R} + \delta_0\Delta\mathcal{R} = \mathcal{R}\xi$

$$\delta\hat{\Psi} = \Psi\hat{\xi} \oplus R\xi, \quad \delta\hat{R} = \Psi\xi$$

- Interacting Lagrangian

$$\mathcal{L} = \hat{R}\hat{R} \oplus \hat{\Psi}\hat{\Psi} \oplus \Psi\Psi\omega \Rightarrow \delta\mathcal{L} = R\Psi\xi \oplus \Psi\Psi\hat{\xi} = 0$$

Massive spin 3/2

Gravitational coupling

Deformations for massive spin-3/2

- Deformations to curvatures

$$\begin{aligned}\Delta \Psi_{\mu\nu} &= g_0(\omega_{[\mu}{}^{ab}\Gamma_{ab}\psi_{\nu]} + 2Mi h_{[\mu}{}^a\gamma_a\psi_{\nu]} - \frac{2m}{3}h_{[\mu}{}^a\Gamma_{\nu]}{}^a\phi) \\ \Delta \Phi_\mu &= g_0(\omega_\mu{}^{ab}\Gamma_{ab}\phi + 2Mi h_\mu{}^a\gamma_a\phi)\end{aligned}$$

- Corrections to gauge transformation

$$\begin{aligned}\delta_1 \psi_\mu &= -g_0(\Gamma^{ab}\psi_\mu\hat{\eta}_{ab} + 2iM\gamma^a\psi_\mu\hat{\xi}_a - \frac{2m}{3}\Gamma_\mu{}^a\phi\hat{\xi}_a \\ &\quad - \omega_\mu{}^{ab}\Gamma_{ab}\xi - 2iMh_\mu{}^a\gamma_a\xi) \\ \delta_1 \phi &= -g_0(\Gamma^{ab}\phi\hat{\eta}_{ab} + 2iM\gamma^a\phi\hat{\xi}_a)\end{aligned}$$

- Curvature transformations

$$\begin{aligned}\delta \hat{\Psi}_{\mu\nu} &= -g_0(\Gamma^{ab}\Psi_{\mu\nu}\hat{\eta}_{ab} + 2iM\gamma^a\Psi_{\mu\nu}\hat{\xi}_a + \frac{2m}{3}\Gamma_{[\mu}{}^a\Phi_{\nu]}\hat{\xi}_a \\ &\quad - R_{\mu\nu}{}^{ab}\Gamma_{ab}\xi - 2iMT_{\mu\nu}{}^a\gamma_a\xi) \\ \delta \hat{\Phi}_\mu &= -g_0(\Gamma^{ab}\Phi_\mu\eta_{ab} + 2iM\gamma^a\Phi_\mu\hat{\xi}_a)\end{aligned}$$

Massive spin 3/2

Gravitational coupling

Deformations for massless spin-2

- Deformations to curvatures

$$\begin{aligned}\Delta R_{\mu\nu}^{ab} &= b_1 \bar{\psi}_{[\mu} \Gamma^{ab} \psi_{\nu]} + i b_2 e_{[\mu}^{[a} \bar{\psi}_{\nu]} \gamma^{b]} \phi + \\ &\quad + i b_3 \bar{\psi}_{[\mu} \Gamma_{\nu]}^{ab} \phi + b_4 e_{[\mu}^a e_{\nu]}^b \bar{\phi} \phi + b_5 \bar{\phi} \Gamma_{\mu\nu}^{ab} \phi \\ \Delta T_{\mu\nu}^a &= i b_6 \bar{\psi}_{[\mu} \gamma^a \psi_{\nu]} + b_7 e_{[\mu}^a \bar{\psi}_{\nu]} \phi + b_8 \bar{\psi}_{[\mu} \Gamma_{\nu]}^a \phi + i b_9 \bar{\phi} \Gamma_{\mu\nu}^a \phi\end{aligned}$$

- Corrections to gauge transformation

$$\begin{aligned}\delta_1 \omega_\mu^{ab} &= 2 b_1 \bar{\psi}_\mu \Gamma^{ab} \xi - i b_2 e_\mu^{[a} \bar{\phi} \gamma^{b]} \xi - i b_3 \bar{\phi} \Gamma_\mu^{ab} \xi \\ \delta_1 h_\mu^a &= 2 i b_6 \bar{\psi}_\mu \gamma^a \xi + b_7 e_\mu^a \bar{\phi} \xi + b_8 \bar{\phi} \Gamma_\mu^a \xi\end{aligned}$$

- Curvature transformations

$$\begin{aligned}\delta \hat{R}_{\mu\nu}^{ab} &= 2 b_1 \bar{\Psi}_{\mu\nu} \Gamma^{ab} \xi + i b_2 e_{[\mu}^{[a} \bar{\Phi}_{\nu]} \gamma^{b]} \xi - i b_3 \bar{\Phi}_{[\mu} \Gamma_{\nu]}^{ab} \xi \\ \delta \hat{T}_{\mu\nu}^a &= 2 i b_6 \bar{\Psi}_{\mu\nu} \gamma^a \xi - b_7 e_{[\mu}^a \bar{\Phi}_{\nu]} \xi + b_8 \bar{\Phi}_{[\mu} \Gamma_{\nu]}^a \xi\end{aligned}$$

Only b_1 is not fixed

Massive spin 3/2

Gravitational coupling $(\frac{3}{2} - \frac{3}{2} - 2)$

- Interacting Lagrangian

$$\begin{aligned}\mathcal{L} = & c_0 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} \hat{R}_{\mu\nu}^{ab} \hat{R}_{\alpha\beta}^{cd} \\ & + c_1 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} \hat{\bar{\Psi}}_{\mu\nu} \Gamma^{abcd} \hat{\Psi}_{\nu\alpha} + i c_2 \left\{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \right\} \hat{\bar{\Psi}}_{\mu\nu} \Gamma^{abc} \hat{\Phi}_\alpha + c_3 \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} \hat{\bar{\Phi}}_\mu \Gamma^{ab} \hat{\Phi}_\nu \\ & + i c_4 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} \bar{\Psi}_{\mu\nu} \Gamma^{abc} \Phi_\alpha h_\beta^d + c_5 \left\{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \right\} \bar{\Phi}_\mu \Gamma^{ab} \Phi_\nu h_\alpha^c\end{aligned}$$

In dimensions $d > 4$ we should add one more Abelian vertex

$$\left\{ \begin{smallmatrix} \mu\nu\alpha\beta\gamma \\ abcde \end{smallmatrix} \right\} \bar{\Psi}_{\mu\nu} \Gamma^{abcd} \Psi_{\alpha\beta} h_\gamma^e$$

- Condition of gauge invariance for Lagrangian

$$b_1 = \frac{6c_1}{c_0} g_0, \quad c_4 = 4c_2 g_0, \quad c_5 = 4c_3 g_0$$

Using frame-like gauge invariant approach we show that like massless one

- Massive spin 3/2 theory can be rewritten in terms of gauge invariant curvatures
- Cubic vertexes can be constructed as curvature deformation procedure