

# Off-shell Scalar Supermultiplet in the Unfolded Dynamics Approach

(based on arXiv:1301.2230, N.G. Misuna, M.A. Vasiliev)

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# Outline

- 1 Wess-Zumino model
- 2 Unfolded formulation
- 3 Unfolded scalar supermultiplet
- 4 Lagrangians

# Wess-Zumino model

- Chiral superfield in  $\mathbb{C}^{4|2}$ ,  $\bar{D}_{\dot{\alpha}}\Phi = 0$ :

$$\Phi = C(y) + \sqrt{2}\theta\chi(y) + \theta\bar{\theta}F(y), \quad \{y^m = x^m + i\theta\sigma^m\bar{\theta}; \theta^\mu\}$$

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- General Lagrangian:

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta} K(\Phi, \bar{\Phi}, D^{(n)}\Phi, D^{(n)}\bar{\Phi}) + \left[ \int d^2\theta W(\Phi) + h.c. \right]$$

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- Salam-Strathdee Lagrangian:

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- Wess-Zumino Lagrangian:

$$\begin{aligned} \mathcal{L} = & i\partial_n\bar{\chi}\bar{\sigma}^n\chi + \bar{C}\square C + \bar{F}F + \left[ m \left( CF - \frac{1}{2}\chi\chi \right) + \right. \\ & \left. + g(CCF - \chi\chi C) + kF + h.c. \right] \end{aligned}$$

# Unfolded equation

- Unfolded equations:

$$dW^\Omega(x) + G^\Omega(W(x)) = 0,$$

$$G^\Omega(W^\tau) := \sum_{n=1}^{\infty} f^\Omega_{\tau_1 \dots \tau_n} W^{\tau_1} \dots W^{\tau_n}.$$

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- Gauge symmetries:

$$\delta W^\Omega = d\varepsilon^\Omega - \varepsilon^\tau \frac{\delta G^\Omega(W)}{\delta W^\tau}.$$

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- Nontrivial invariant Lagrangians:

$$\mathcal{L} = H^d(Q), \quad Q = G^\tau(W) \frac{\delta}{\delta W^\tau}.$$

# SUSY-vacuum

- $D = 4 N = 1$  SUSY 1-form connection:

$$\Omega_0 = e^a P_a + \frac{1}{2} \omega^{a,b} M_{ab} + \phi^\alpha Q_\alpha + \bar{\phi}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}}.$$

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- Flat SUSY-background  $d\Omega_0 + \Omega_0 \Omega_0 = 0$ :

$$D^L e^a + 2i\phi^\alpha \bar{\phi}^{\dot{\alpha}} (\sigma^a)_{\alpha\dot{\alpha}} = 0,$$

$$D^L \phi^\alpha = 0, \quad D^L \omega^{a,b} = 0, \quad D^L \bar{\phi}_{\dot{\alpha}} = 0.$$

Lorentz-covariant derivative  $D^L = d + \omega$ .

# Flat superspace

- Transition to superspace  $x^{\underline{m}} \rightarrow z^{\underline{M}} = \{x^{\underline{m}}, \theta^{\underline{\mu}}, \bar{\theta}^{\dot{\mu}}\}$ :

$$e_{\underline{m}}^a(x) dx^{\underline{m}} \rightarrow E_{\underline{M}}^a(z) dz^{\underline{M}}, \quad \omega_{\underline{m}}^{a,b}(x) dx^{\underline{m}} \rightarrow \Omega_{\underline{M}}^{a,b}(z) dz^{\underline{M}},$$

$$\phi_{\underline{m}}^\alpha(x) dx^{\underline{m}} \rightarrow E_{\underline{M}}^\alpha(z) dz^{\underline{M}}, \quad \bar{\phi}_{\underline{m}}^{\dot{\alpha}}(x) dx^{\underline{m}} \rightarrow \bar{E}_{\underline{M}}^{\dot{\alpha}}(z) dz^{\underline{M}}.$$

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- Flat superspace:

$$\begin{aligned} DE^a + 2iE^\alpha \bar{E}^{\dot{\alpha}} (\sigma^a)_{\alpha\dot{\alpha}} &= 0, \\ D\Omega^{a,b} &= 0, \quad DE^\alpha = 0, \quad D\bar{E}_{\dot{\alpha}} = 0. \end{aligned}$$

# Unfolded scalar and spinor

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- Unfolded massless spinor field:

$$\begin{cases} D^L \chi_\alpha^{a(k)} + e_b \chi_\alpha^{a(k)b} = 0, \\ (\bar{\sigma}_b)^{\dot{\alpha}\alpha} \chi_\alpha^{a(k-1)b} = 0. \end{cases}$$

# Scalar supermultiplet

On-shell scalar supermultiplet (Ponomarev, Vasiliev,  
arXiv:1012.2903):

$$\begin{cases} D^L C^{a(k)} + e_b C^{a(k)b} - \sqrt{2} \phi^\alpha \chi_\alpha^{a(k)} = 0, \\ D^L \chi_\alpha^{a(k)} + e_b \chi_\alpha^{a(k)b} - \sqrt{2} i \bar{\phi}^{\dot{\alpha}} (\sigma_b)_{\alpha\dot{\alpha}} C^{a(k)b} = 0. \end{cases}$$

# Scalar supermultiplet in superspace

On-shell scalar supermultiplet (Ponomarev, Vasiliev, arXiv:1012.2903):

$$\begin{cases} DC^{a(k)}(z) + E_b C^{a(k)b}(z) - \sqrt{2} E^\alpha \chi_\alpha^{a(k)}(z) = 0, \\ D\chi_\alpha^{a(k)}(z) + E_b \chi_\alpha^{a(k)b}(z) - \sqrt{2} i \bar{E}^{\dot{\alpha}} (\sigma_b)_{\alpha\dot{\alpha}} C^{a(k)b}(z) = 0. \end{cases}$$

# Off-shell formulation

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- “Dynamical” equations:

$$\bar{D}_{\dot{\alpha}} C^{a(k)} = 0, \quad D_\alpha F^{a(k)} = 0.$$

# Operator Q

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$$Q_\Omega = \Omega^{a,c} \Omega_c{}^{,b} \frac{\partial}{\partial \Omega^{a,b}} + \Omega^{a,b} E_b \frac{\partial}{\partial E^a} + \dots$$

$$\hat{Q} = 2iE^\alpha (\sigma^a)_{\alpha\dot{\alpha}} \bar{E}^{\dot{\alpha}} \frac{\partial}{\partial E^a} + E_a \hat{q}^a + \sqrt{2} E_\alpha \hat{q}^\alpha + \sqrt{2} \bar{E}_{\dot{\alpha}} \hat{\bar{q}}^{\dot{\alpha}},$$

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$$\hat{q}^b = C^{a(k)b} \frac{\partial}{\partial C^{a(k)}} + \chi_\alpha^{a(k)b} \frac{\partial}{\partial \chi_\alpha^{a(k)}} + F^{a(k)b} \frac{\partial}{\partial F^{a(k)}} + h.c.,$$

$$\hat{q}^\alpha = (\chi^\alpha)^{a(k)} \frac{\partial}{\partial C^{a(k)}} - F^{a(k)} \frac{\partial}{\partial \chi_\alpha^{a(k)}} - \dots$$

$$\hat{\bar{q}}^{\dot{\alpha}} = -(\bar{\chi}^{\dot{\alpha}})^{a(k)} \frac{\partial}{\partial \bar{C}^{a(k)}} + \bar{F}^{a(k)} \frac{\partial}{\partial \bar{\chi}_{\dot{\alpha}}^{a(k)}} - \dots$$

# 4-superform Lagrangian

- General solution:

$$\begin{aligned} \mathcal{L} = & E_a E_b \left( \bar{\sigma}^{ab} \right)^{\dot{\alpha}\dot{\beta}} \bar{E}_{\dot{\alpha}} \bar{E}_{\dot{\beta}} L + \frac{\sqrt{2}}{6} \epsilon^{abcd} E_a E_b E_c \bar{E}_{\dot{\alpha}} (\bar{\sigma}_d)^{\dot{\alpha}\alpha} \hat{q}_\alpha L + \\ & + \frac{i\sqrt{2}}{16} E_a E_b E_c E_d \epsilon^{abcd} \hat{q}_\alpha \hat{q}^\alpha L + h.c. \end{aligned}$$

$$L = L(C^{m(k)}, \bar{F}^{m(k)}), \quad \bar{L} = \bar{L}(\bar{C}^{m(k)}, F^{m(k)}) \text{ and } L \neq \hat{q}_a f^a.$$

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$L = L(C^{m(k)}, \bar{F}^{m(k)})$ ,  $\bar{L} = \bar{L}(\bar{C}^{m(k)}, F^{m(k)})$  and  $L \neq \hat{q}_a f^a$ .

- Wess-Zumino Lagrangian:

$$L = i2\sqrt{2} \left( C \bar{F} + kC + \frac{m}{2} C^2 + \frac{g}{3} C^3 \right)$$

# Integral form Lagrangian

- Integral form solution:

$$\begin{aligned} S = & \int \epsilon^{abcd} E_a E_b E_c E_d \delta^2(E_\alpha) \delta^2(\bar{E}_{\dot{\alpha}}) K + \\ & + \left[ \int \delta^2(E_\alpha) \epsilon^{abcd} E_a E_b E_c E_d W(C^{m(k)}, \bar{F}^{m(k)}) + h.c. \right] \end{aligned}$$

$$K \neq \hat{q}^\alpha f_\alpha + h.c., \quad W \neq \hat{q}^a g_a$$

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$$K \neq \hat{q}^\alpha f_\alpha + h.c., \quad W \neq \hat{q}^a g_a$$

- Salam-Strathdee action:  $K = C\bar{C}$ ,  $W = kC + \frac{m}{2}C^2 + \frac{g}{3}C^3$ .

# Conclusion

- ① Superspace off-shell formulation of Wess-Zumino model is built starting from the space-time on-shell formulation.
- ② All superLagrangians in the form of superform and integral form are found.
- ③ Application to maximal SYM and SUGRA?