Two-loop low-energy effective action in three-dimensional Abelian supersymmetric Chern-Simons-matter models

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We compute two-loop low-energy effective actions in Abelian three dimensional Chern-Simons-matter models with $\mathcal{N} = 2$ and $\mathcal{N} = 3$ supersymmetry up to four-derivative order. Calculations are performed with a slowly-varying gauge superfield background. In the massless case the considered models are superconformal. Superconformal symmetry strongly restricts the form of two-loop quantum corrections to the effective actions such that the obtained terms have simpler structure than the analogous ones in the effective action of three-dimensional supersymmetric electrodynamics (SQED)[1].

[1] The talk is based on
I.L. Buchbinder, I.B. Samsonov and BM, *Nucl.Phys.* **B 881** (2014)
I.L. Buchbinder, I.B. Samsonov and BM, *JHEP* **1307** (2013)

Plan

- General motivation
- Three-dimensional $\mathcal{N} = 2$ SQED and review of previous results
- Abelian three-dimensional $\mathcal{N}$=2 Chern-Simons-matter model
- Abelian three-dimensional $\mathcal{N}$=3 Chern-Simons-matter model
- Discussion

The modern interest to three-dimensional supersymmetric field models is partly motivated by recent progress in constructing field theories describing multiple M2 branes. These are three-dimensional superconformal field theories of Chern-Simons gauge fields interacting with matter in a special way such that the superconformal invariance and $\mathcal{N}=6$ (or even $\mathcal{N}=8$) extended supersymmetry are preserved

- $\mathcal{N}=8$ case **Bagger-Lambert & Gustavsson** (BLG) model *Phys.Rev.D (2007,2008)* and *Nucl.Phys.B (2009)*
- $\mathcal{N}=6$ case **Aharony-Bergman-Jafferis-Maldacena** (ABJM) model *JHEP (2008)*

One of the general problems for the BLG and ABJM models is to study the effective action. It is the open question.

Three-dimensional $\mathcal{N}=2$ SQED

The classical action of the $\mathcal{N}=2$ d=3 supersymmetric electrodynamics:

$$
\begin{aligned}
S_{\mathcal{N}=2} \quad &= \quad \frac{1}{e^2}\int d^7z\, G^2 - \int d^7z\, \left(\bar{Q}_+ e^{2V} Q_+ + \bar{Q}_- e^{-2V} Q_-\right) \\
&\quad - \left(m\int d^5z\, Q_+ Q_- + c.c.\right), \qquad (1)
\end{aligned}
$$

where $G = \frac{i}{2}\bar{D}^\alpha D_\alpha V$ is a scalar field strength of vector multiplet V.

The gauge multiplet V:

- real scalar ϕ,
- complex spinor λ_α,
- vector field A^m,
- real auxiliary scalar D.

The matter multiplet $Q_\pm$:

- complex scalar $f_\pm$,
- complex spinor $\psi^\alpha_\pm$,
- complex auxiliary scalar $F_\pm$.

The additional scalar field ϕ arises after dimension reduction the gauge vector field A^m from d=4 to d=3.

$$
\mathbf{d=4} \quad A^m = \begin{pmatrix} A^0 \\ A^1 \\ A^2 \\ A^3 \end{pmatrix} \quad \Rightarrow \quad \mathbf{d=3} \quad A^m = \begin{pmatrix} A^0 \\ A^1 \\ A^2 \end{pmatrix} + \phi \equiv A^3 .
$$

Background field setup

We specify the constraints on the background gauge superfield under considerations:

i) The gauge superfield obeys the $\mathcal{N}=2$ supersymmetric free Maxwell equations,

$$D^\alpha W_\alpha = 0\,, \qquad \bar{D}^\alpha \bar{W}_\alpha = 0\,. \tag{2}$$

ii) Within the derivative expansion of the effective action we look for the leading terms without space-time derivatives of the gauge superfields. Such a long-wave approximation is effectively taken into account by considering the constant background,

$$\partial_m G = 0\,, \quad \partial_m W_\alpha = 0\,, \quad \partial_m \bar{W}_\alpha = 0\,. \tag{3}$$

This approximation suffices to study the Euler-Heisenberg-type effective action which is induced by the $\mathcal{N}=2$ supersymmetric quantum matter fields.

Diagrams

There are two types of diagrams which contribute to two-loop effective action in $\mathcal{N} = 2$ d=3 SQED

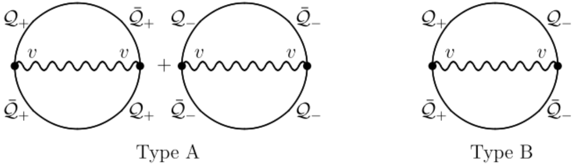

Fig. 1. Two-loop supergraphs in $\mathcal{N} = 2$ supersymmetric electrodynamics.

The propagators

$$\begin{aligned} i\langle Q_+(z)Q_-(z')\rangle &= -mG_+(z,z')\,, \\ i\langle Q_+(z)\bar{Q}_+(z')\rangle &= G_{+-}(z,z') = G_{-+}(z',z)\,, \\ 2i\langle v(z)\,v(z')\rangle &= G_0(z,z')\,, \end{aligned} \tag{4}$$

The photon propagator $G_0(z,z')$ through the heat kernel reads

$$G_0(z,z') = i\int_0^\infty ds\, K_0(z,z'|s)\, e^{-s\epsilon}\,, \qquad K_0(z,z'|s) = \frac{1}{(4i\pi s)^{3/2}} e^{\frac{i\rho^m\rho_m}{4s}} \zeta^2\bar{\zeta}^2\,. \tag{5}$$

Following the work S. M. Kuzenko and I. N. McArthur, *JHEP* **0305** *(2003)* we obtain propagators for matter fields in the model (1).
The chiral propagator $G_+(z,z')$

$$G_+(z,z') = i\int_0^\infty ds\, K_+(z,z'|s)e^{i(m^2+i\epsilon)}\,, \qquad \epsilon \to +0\,, \quad (6)$$

$$K_+(z,z'|s) = \frac{1}{(4i\pi s)^{3/2}}\frac{sB}{\sinh(sB)}e^{isG^2}\mathcal{O}(s)e^{\frac{i}{4}(F\coth(sF))_{mn}\rho^m\rho^n-\frac{1}{2}\bar\zeta^\beta\rho_{\beta\gamma}W^\gamma}\zeta^2 I(z,z'). \quad (7)$$

The chiral-antichiral propagator $G_+(z,z')$

$$G_{+-}(z,z') = i\int_0^\infty ds\, K_{+-}(z,z'|s)e^{i(m^2+i\epsilon)}\,, \qquad \epsilon \to +0\,, \quad (8)$$

$$K_{+-}(z,z'|s) = -\frac{1}{(4i\pi s)^{3/2}}\frac{sB}{\sinh(sB)}e^{isG^2}\mathcal{O}(s)e^{\frac{i}{4}(F\coth(sF))_{mn}\tilde\rho^m\tilde\rho^n+R(z,z')}I(z,z'), \quad (9)$$

where the operator $\mathcal{O}(s) = e^{s(\bar W^\alpha\bar\nabla_\alpha - W^\alpha\nabla_\alpha)}$ and two point functions ζ^α, $\bar\zeta^\alpha$ and ρ^m are the components of the $\mathcal{N}=2$ d=3 supersymmetric interval $\zeta^A = \{\rho^m, \zeta^\alpha, \bar\zeta_\alpha\}$

$$\begin{aligned}
\rho^m &= (x-x')^m - i\gamma^m{}_{\alpha\beta}\zeta^\alpha\bar\theta'^\beta + i\gamma^m{}_{\alpha\beta}\theta'^\alpha\bar\zeta^\beta\,,\\
\zeta^\alpha &= (\theta-\theta')^\alpha\,, \quad \bar\zeta^\alpha = (\bar\theta-\bar\theta')^\alpha\,. \qquad (10)
\end{aligned}$$

The parallel displacement propagator $I(z,z')$ provides the correct gauge transformation properties of the heat kernel and defines as a solution of the equation

$$\zeta^A \nabla_A I(z,z') = 0\,, \quad I(z,z) = 1\,. \tag{11}$$

The covariant derivatives $\nabla_A = (\nabla_m, \nabla_\alpha, \bar{\nabla}^\alpha)$ satisfy the algebra

$$\begin{aligned}
&\{\nabla_\alpha, \bar{\nabla}_\beta\} = -2i(\gamma^m)_{\alpha\beta}\nabla_m + 2i\varepsilon_{\alpha\beta}G\,,\\
&[\nabla_\alpha, \nabla_m] = -(\gamma_m)_{\alpha\beta}\bar{W}^\beta, \qquad [\bar{\nabla}_\alpha, \nabla_m] = (\gamma_m)_{\alpha\beta}W^\beta\\
&[\nabla_m, \nabla_n] = iF_{nm}\,,
\end{aligned} \tag{12}$$

The two-point function $R(z,z')$ is defined as

$$\begin{aligned}
R(z,z') &= -i\zeta\bar{\zeta}G + \frac{7i}{12}\bar{\zeta}^2\zeta W + \frac{i}{12}\zeta^2\bar{\zeta}\bar{W} - \frac{1}{2}\bar{\zeta}^\alpha\tilde{\rho}_{\alpha\beta}W^\beta - \frac{1}{2}\zeta^\alpha\tilde{\rho}_{\alpha\beta}\bar{W}^\beta\\
&\quad + \frac{1}{12}\zeta^\alpha\bar{\zeta}^\beta[\tilde{\rho}^\gamma_\beta D_\alpha W_\gamma - 7\tilde{\rho}^\gamma_\alpha D_\gamma W_\beta]\,.
\end{aligned} \tag{13}$$

$$\tilde{\rho}^m = \rho^m + i\zeta^\alpha\gamma^m_{\alpha\beta}\bar{\zeta}^\beta\,, \qquad D'_\alpha\tilde{\rho}^m = \bar{D}_\alpha\tilde{\rho}^m = 0\,. \tag{14}$$

The one-loop effective action [I. L. Buchbinder et al. *JHEP (2010)*] is defined by the coincide point $z \to z'$ limit of anti-chiral propagator $G_{+-}(z,z)$ (8)

$$\Gamma^{(1)}_{\mathcal{N}=2} = \frac{1}{2\pi}\int d^7z\Big[G\ln(G+\sqrt{G^2+m^2}) - \sqrt{G^2+m^2} + \frac{1}{8\pi}\int d^7z \int_0^\infty \frac{ds}{\sqrt{i\pi s}} e^{is(G^2+m^2)} \frac{W^2\bar{W}^2}{B^2}\left(\frac{\tanh(sB/2)}{sB/2} - 1\right)\Big], \quad (15)$$

where $W_\alpha = \bar{D}_\alpha G$ and $B^2 = \frac{1}{2}D^2W^2$.
The two-loop corrections to effective action have the following superfield structure:

$$\Gamma^{(2)}_{\mathcal{N}=2} = \Gamma_A + \Gamma_B = \frac{e^2}{16\pi^3}\int d^7z\left[\mathcal{L}_1 + W^\alpha \mathcal{L}_{2\alpha}{}^\beta \bar{W}_\beta + (\mathcal{L}_3^{(A)} + \mathcal{L}_3^{(B)})W^2\bar{W}^2\right],$$

Here $\mathcal{L}_1$, $\mathcal{L}_{2\alpha}{}^\beta$ and $\mathcal{L}_3$ are some functions of G and B to be found from direct quantum computations. Note that the contributions of the form $\mathcal{L}_1$ and $\mathcal{L}_2$ are impossible in the d=4 case.

$$
\begin{aligned}
\mathcal{L}_1 &= \int_0^\infty \frac{ds\,dt}{\sqrt{st}} e^{i(s+t)(G^2+m^2)} \frac{B^2}{\sinh(sB)\sinh(tB)} \frac{2\mathrm{arccosh}\sqrt{a/b}}{\sqrt{a(a-b)}}\,, \\
\mathcal{L}_{2\alpha}{}^{\beta} &= -\frac{2iG}{B^2}\int_0^\infty \frac{ds\,dt}{\sqrt{st}} e^{i(s+t)(G^2+m^2)} \frac{B^2}{\sinh(sB)\sinh(tB)} \frac{2\mathrm{arccosh}\sqrt{a/b}}{\sqrt{a(a-b)}} \\
&\quad \times (e^{-sN} - 1 + sN + e^{-tN} - 1 + tN)_\alpha{}^\beta\,, \\
\mathcal{L}_3^{(\mathrm{A})} &= \frac{i}{2}\int_0^\infty \frac{ds\,dt}{\sqrt{st}} e^{i(s+t)(G^2+m^2)} \frac{B^2}{\sinh(sB)\sinh(tB)} \frac{1}{a-b}\left(\frac{2F^-}{b} - \frac{F^+}{a}\right) \\
&+ \int_0^\infty \frac{ds\,dt}{\sqrt{st}} e^{i(s+t)(G^2+m^2)} \frac{B^2}{\sinh sB \sinh tB}\left[\frac{i}{2}(f(s)+f(t) + \frac{a(F^+-F^-)-bF}{a(a-b)}\right. \\
&- \left.\frac{G^2}{B^4}((sB - \sinh sB + tB - \sinh tB)^2 - (\cosh sB + \cosh tB - 2)^2)\right] \frac{2\mathrm{arccosh}\sqrt{}}{\sqrt{a(a-}}
\end{aligned}
$$

$$
\mathcal{L}_3^{(\mathrm{B})} = \frac{4m^2}{B^2}\int_0^\infty \frac{ds\,dt}{\sqrt{st}} e^{i(s+t)(G^2+m^2)} \tanh\frac{sB}{2}\tanh\frac{tB}{2} \frac{\mathrm{arccosh}\sqrt{a/b}}{\sqrt{a(a-b)}}\,.
$$

where

$$
a(s,t) = B\coth(sB) + B\coth(tB)\,, \qquad b(s,t) = s^{-1} + t^{-1} \tag{18}
$$

Besides the standard **Maxwell** kinetic term for gauge $\mathcal{N}=2$ superfield

$$S_{\rm M} = \frac{1}{e^2}\int d^7z G^2 = \frac{1}{g^2}\int d^3x \left(-\frac{1}{4}F^{mn}F_{mn} - \frac{1}{2}\phi\Box\phi - i\bar{\lambda}^\alpha\gamma^m_{\alpha\beta}\partial_m\lambda^\beta + \frac{1}{2}D^2\right)$$

in three-dimensional case we have the **Chern-Simons** one [E.A.Ivanov *Phys.Lett.* **B268** (1991)]

$$S_{CS}^{\mathcal{N}=2} = \frac{ik}{8\pi}{\rm tr}\int_0^1 dt \int d^7z\, \bar{D}^\alpha(e^{-2tV}D_\alpha e^{2tV})e^{-2tV}\partial_t e^{-2tV}\,. \qquad (19)$$

In Abelian case this action reduces to

$$S_{\rm CS} = \frac{k}{2\pi}\int d^7z\, VG = \frac{k}{2\pi}\int d^3x (\frac{1}{2}\varepsilon^{mnp}A_m\partial_n A_p + i\lambda^\alpha\bar{\lambda}_\alpha - 2\phi D)\,, \qquad (20)$$

where k is the Chern-Simons level. In what follows we will consider only Chern-Simons kinetic term for gauge multiplet.

The classical action of the Abelian three-dimensional $\mathcal{N}$=2 Chern-Simons-matter model has the form

$$\begin{aligned} S_{\mathcal{N}=2} &= \frac{k}{2\pi}\int d^7z\, VG - \int d^7z\, \left(\bar{Q}_+ e^{2V} Q_+ + \bar{Q}_- e^{-2V} Q_-\right) \\ &\quad - \left(m\int d^5z\, Q_+Q_- + c.c.\right), \end{aligned} \tag{21}$$

where V is a gauge superfield with superfield strength $G = \frac{i}{2}\bar{D}^\alpha D_\alpha V$ and $Q_\pm$ are chiral matter superfields. Here m is the mass of the chiral superfield and k is the Chern-Simons level. For $m = 0$ this model is superconformal. Using the background field method in $\mathcal{N} = 2$, $d = 3$ superspace (see the review [I. L. Buchbinder, N. G. Pletnev and I. B. Samsonov, *Phys. Part. Nucl.* **44** (2013)]), we split the gauge superfield V into the background V and quantum v parts,

$$V \to V + v\,. \tag{22}$$

Abelian three-dimensional $\mathcal{N}=2$ Chern-Simons-matter model

We choose the following gauge fixing action [L. V. Avdeev, G. V. Grigorev and D. I. Kazakov, *Nucl. Phys.* B **382** (1992)]

$$S_{\rm gf} = \frac{ik\alpha}{8\pi}\int d^7z\, v(D^2+\bar{D}^2)v\,, \tag{23}$$

where α is a real parameter. The quantum action reads

$$\begin{aligned} S^{\mathcal{N}=2}_{\rm quant} &= S_2 + S_{\rm int}\,, \qquad (24)\\ S_2 &= \int d^7z\left(\frac{ik}{4\pi}vHv - \bar{\mathcal{Q}}_+\mathcal{Q}_+ - \bar{\mathcal{Q}}_-\mathcal{Q}_-\right) - \left(m\int d^5z\,\mathcal{Q}_+\mathcal{Q}_- + c.c.\right),\\ S_{\rm int} &= -2\int d^7z\left[(\bar{\mathcal{Q}}_+\mathcal{Q}_+ - \bar{\mathcal{Q}}_-\mathcal{Q}_-)v + (\bar{\mathcal{Q}}_+\mathcal{Q}_+ + \bar{\mathcal{Q}}_-\mathcal{Q}_-)v^2\right] + O(v^3)\,, \end{aligned}$$

where the operator H is

$$H = D^\alpha\bar{D}_\alpha + \frac{\alpha}{2}(D^2+\bar{D}^2)\,. \tag{25}$$

Let us consider the propagator for the superfield v,

$$2i\langle v(z)v(z')\rangle = G(z,z')\,, \tag{26}$$

where the Green's function $G(z,z')$ obeys the equation

$$\frac{ik}{4\pi}HG(z,z') = -\delta^7(z-z')\,. \tag{27}$$

A formal solution to this equation reads

$$G(z,z') = G_1(z,z') + G_2(z,z')\,, \tag{28}$$

where

$$G_1(z,z') = \frac{i\pi}{k}\frac{\bar{D}^\alpha D_\alpha}{\Box}\delta^7(z-z')\,, \tag{29}$$

$$G_2(z,z') = \frac{i\pi}{2k\alpha}\frac{D^2+\bar{D}^2}{\Box}\delta^7(z-z')\,. \tag{30}$$

Our aim is to study the two-loop low-energy effective action in the model (21) in the gauge superfield sector. In general, it is given by a functional of superfield strengths G, W_α, $\bar{W}_\alpha$ and their derivatives, $N_{\alpha\beta} = D_\alpha W_\beta$, $\bar{N}_{\alpha\beta} = \bar{D}_\alpha \bar{W}_\beta$,

$$\Gamma = \int d^7 z\, \mathcal{L}(G, W_\alpha, \bar{W}_\alpha, N_{\alpha\beta}, \bar{N}_{\alpha\beta}, \ldots)\,, \quad (31)$$

where dots stand for higher-order derivatives of the superfield strengths. It is very difficult to find the effective action (31) taking into account all derivatives of the fields. Therefore, to simplify the problem, we restrict ourself to the terms with no more than four space-time derivatives of component fields and it is enough to study **UV** divergences in the model (21).

The one-loop contribution to effective action of the model (21) has the same form (15) like in the $\mathcal{N}=2$ SQED with Maxwell kinetic term (1).

The two-loop effective action is given by the following formal expression

$$\Gamma^{(2)} = \Gamma_{\rm A} + \Gamma_{\rm B}\,, \tag{32}$$

$$\Gamma_{\rm A} = -2\int d^7z\, d^7z' G_{+-}(z,z')G_{-+}(z,z')G(z,z')\,, \tag{33}$$

$$\Gamma_{\rm B} = -2m^2\int d^7z\, d^7z' G_{+}(z,z')G_{-}(z,z')G(z,z')\,. \tag{34}$$

The two terms $\Gamma_{\rm A}$ and $\Gamma_{\rm B}$ are represented by corresponding Feynman graphs in fig. 1.
In case of our chose of background (2) and (3), it is not difficult to demonstrate that the two-loop contributions to the low-energy effective action (33) and (34) are independent of on the gauge fixing parameter α.
To prove this, one can check the vanishing of contributions to the two-loop effective actions (33) and (34) which correspond to the propagator (30).

The part of the effective action Γ_A (33) after representation all the Green's functions in terms of the corresponding heat kernels, reads

$$\Gamma_{\rm A} = -\frac{2\pi}{k}\int d^7z\, d^3\xi \int_0^\infty \frac{ds\, dt\, du}{(4\pi i u)^{3/2}} e^{\frac{i\xi^2}{4u}} e^{i(s+t)m^2} \nabla^\alpha K_{+-}(z,z'|s)\bar{\nabla}_\alpha K_{-+}(z,z'|t)\Big| . \quad (35)$$

Here we integrated by pats the derivatives $D^\alpha \bar{D}_\alpha$ which come from the gauge superfield propagator (29). To find the effective action we need to compute the derivatives of the heat kernels $K_{+-}(z,z'|s)$ and $K_{-+}(z,z'|t)$ (9).

Analogously, the part of the effective action action Γ_B (34) with the gauge superfield propagator (29),

$$\Gamma_{\rm B} = -\frac{2\pi m^2}{k}\int d^7z\, d^3\xi \int_0^\infty \frac{ds\, dt\, du}{(4\pi i u)^{3/2}} e^{\frac{i\xi^2}{4u}} e^{i(s+t)m^2} \nabla^\alpha K_{+}(z,z'|s)\bar{\nabla}_\alpha K_{-}(z,z'|t)\Big| \quad (36)$$

For computing this part of the effective action we need to find the derivatives of the heat kernels $K_{+}(z,z'|s)$ and $K_{-}(z,z'|t)$ (7).

Omitting the details of calculation we demonstrate the result two-loop contribution to the effective action of the model (21)

$$\Gamma^{(2)} = -\frac{15}{256\pi k}\int d^7z \frac{G\,W^2\bar{W}^2}{(G^2+m^2)^3}\,. \tag{37}$$

As have been mentioned above, the model (21) in the massless case, $m=0$, is superconformal. In this case the action (37) has the form

$$\Gamma^{(2)}\big|_{m=0} = -\frac{15}{256\pi k}\int d^7z \frac{G\,W^2\bar{W}^2}{G^5}\,. \tag{38}$$

But this effective action is not $\mathcal{N}=2$ superconformal. Nevertheless, this does not mean any anomaly of the superconformal symmetry. Recall that the expression (38) was derived for the background gauge superfield obeying supersymmetric Maxwell equations (2). One can add some terms with $D^\alpha W_\alpha$ or $\bar{D}^\alpha \bar{W}_\alpha$ to the action (38) to make it superconformal.

$$\Gamma^{(2)}\big|_{m=0} = \frac{15}{128\pi k}\int d^7z \frac{(D^\alpha \bar{D}_\alpha \ln G)^2}{G}\,. \tag{39}$$

The classical action of Abelian three-dimensional $\mathcal{N}=3$ Chern-Simons-matter model

$$
\begin{aligned}
S_{\mathcal{N}=3} &= S^{\rm CS}_{\mathcal{N}=3} + S_{\rm hyper}\,, \qquad (40)\\
S^{\rm CS}_{\mathcal{N}=3} &= \frac{k}{2\pi}\int d^7z\, VG - \frac{ik}{4\pi}\int d^5z\,\Phi^2 - \frac{ik}{4\pi}\int d^5\bar z\,\bar\Phi^2\,,\\
S_{\rm hyper} &= -\int d^7z\,(\bar Q_+ e^{2V} Q_+ + \bar Q_- e^{-2V} Q_-) - \left(\int d^5z\,\Phi Q_+ Q_- + c.c.\right),
\end{aligned}
$$

where Φ is a chiral superfield which is part of the $\mathcal{N}=3$ gauge multiplet (V,Φ). Note that this model is superconformal. One- and two-loop corrections in the model (40) have the form

$$
\Gamma^{(1)}_{\mathcal{N}=3} = -\frac{1}{64\pi}\int d^7z\frac{\sqrt{G^2+\Phi\bar\Phi}}{G^2}[D^\alpha\bar D_\alpha \ln(G+\sqrt{G^2+\Phi\bar\Phi})]^2\,. \qquad (41)
$$

$$
\Gamma^{(2)}_{\mathcal{N}=3} = \frac{15}{128\pi k}\int d^7z\frac{1}{G}[D^\alpha\bar D_\alpha \ln(G+\sqrt{G^2+\Phi\bar\Phi})]^2\,, \qquad (42)
$$

Conclusion

- The two-loop contributions (37) and (42) to effective action of the models (21) and (40) do not contain the **UV** divergences.
- It is interesting to note that the expressions (41) and (42) do not respect full $\mathcal{N}=3$ superconformal group and require $\mathcal{N}=3$ supersymmetrization. The issue of finding $\mathcal{N}=3$ supersymmeteric versions of the actions (42) and (42) deserves a separate study.
- The most natural way to obtain the effective action in the model (40) in explicitly $\mathcal{N}=3$ supersymmetric form is the using of $\mathcal{N}=3$, $d=3$ harmonic superspace approach [B. M. Zupnik and D. V. Khetselius, *Yad. Fiz.* **47** (1988). Quantum aspects of supersymmetric gauge theories in this superspace were studied in [I. L. Buchbinder, E. A. Ivanov, O. Lechtenfeld, N. G. Pletnev, I. B. Samsonov and B. M. Zupnik, *JHEP* **0910** (2009)]. It would be interesting to explore the low-energy effective action in $\mathcal{N}=3$ or $\mathcal{N}=6$ gauge theories using this approach.
- Another interesting problem is to construct two-loop effective action in matter superfield sector of the models (1), (21) and (40).