Strings $\rightarrow$ $\mathcal{F}(\Box)$ $\rightarrow$ Cosmology
or
What does gravity learn from string field theory?

Alexey Koshelev

Vrije Universiteit Brussel,

Quarks, June 2-8, 2014

Outline

- **Problems to address**
- **Strings, SFT, p-adic strings and all of that**
- $\mathcal{F}(\Box)$ **physics and Ostrogradski instability**
- **Non-local gravity as GR completion**
- **Scalar reformulation of the non-local gravity**
- **Various limits**
- **Conclusion and open questions**

Problems

- **Cosmology and gravity do require graceful resolutions of singularities**
- **The major issues are initial singularity (Big Bang) and black hole singularities**
- **The initial singularity problem is the problem with most cosmological solutions because they hit a singularity approaching the time when Universe had begun**
- **Standard ways to avoid an initial singularity meet the more serious problem of ghosts**
- **These ghosts are related to higher derivatives in for example $f(R)$ models which may feature non-singular bouncing solutions**
- **Is there a way around?**

Strings

- **Strings step down from point-like objects and due to the presence of infinite dimensional conformal symmetry group in 2 dimensions feature a number of spectacular properties**

- **SFT is the non-perturbative description of strings. UV completeness is one of the successes of SFT**
  Witten;Aref'eva,Medvedev,Zubarev;Preitschopf,Thorn,Yost;...

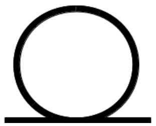
- **Closed strings contain gravitation as one of the excitations and therefore we should in principle be able to find out the corrections to Einstein's gravity**

Hint for finiteness

Low level example action from SFT:

$$L \sim \frac{1}{2}\varphi(\Box - m^2)\varphi + \frac{\lambda}{4}\left(e^{-\beta\Box}\varphi\right)^4$$

Simplest one-loop graph:



$$\sim \quad \frac{e^{m^2\beta}}{\beta}\mathrm{Ei}_2(m^2\beta) = \frac{1}{\beta} + \left(\frac{\gamma}{2} + \ln(m^2\beta)\right)m^2 + O(m^4)$$

Clearly UV divergences have gone.
Moreover, IR divergences are eliminated for massless fields as well.

$\mathcal{F}(\Box)$ physics

The Lagrangian to understand is

$$S = \int d^D x \left(\frac{1}{2}\varphi \mathcal{F}(\Box)\varphi - \lambda v(\varphi) + \ldots \right)$$

$\mathcal{F}(\Box) = \sum_{n\geq 0} f_n \Box^n$, **i.e. it is an analytic function.**

Canonical physics has $\mathcal{F}(\Box) = \Box - m^2$, **i.e.** $L = \frac{1}{2}\varphi\Box\varphi - \frac{m^2}{2}\varphi^2$

Ostrogradski statement says that higher (> 2) derivatives in the Lagrangian are equivalent to either ghosts or tachyons, or both.

$\mathcal{F}(\Box) = \Box - m^2 + f_2\Box^2$ **is an example.**

Anothe example: p-adic strings – an effective theory capturing properties of the SFT scattering amplitudes

$$L \sim -\frac{1}{2}\varphi p^{-\frac{\Box}{2m^2}}\varphi + \frac{1}{p+1}\varphi^{p+1}$$

Vladimirov,Volovich,Zelenov,Dragovich,Khrennikov,Brekke,Freund,Olson, Witten,. . .

$\mathcal{F}(\Box)$ physics (continued)

Terminology in $(-,+,+,+)$ signature:

$$\begin{aligned}
\textbf{good:}\ & L = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - m^2\phi^2 \\
\textbf{tachyon:}\ & L = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + m^2\phi^2 \\
\textbf{ghost:}\ & L = +\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - m^2\phi^2 \\
\textbf{ghost and tachyon:}\ & L = +\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + m^2\phi^2
\end{aligned}$$

Tachyons can be treated by saying magic words about non-perturbative minima while ghosts are really dangerous beasts.

Ghosts lead to a very rapid vacuum decay.

$\mathcal{F}(\Box)$ physics (continued)

A word "finite" was not explicitly in the cited statement and it appears to be crucial.

So, lets go "infinite", which implicitly means create a non-local Lagrangian.

Two examples are in order

$$\mathcal{F}(\Box) = (\Box - m^2)e^{-\beta\Box}$$
$$\mathcal{F}(\Box) = e^{-\beta\Box} - m^2$$

Roots of an equation

$$\mathcal{F}(\sigma) = 0$$

is the key to understand the physics here.

The *ghost-free* condition requires no more than one root σ exists.

Non-local gravity

The following Lagrangian describes the modification of gravity expected from the closed String Field Theory.

$$S=\int d^4x\sqrt{-g}\left(\frac{M_P^2 R}{2}+\frac{\lambda}{2}R\mathcal{F}(\Box)R-\Lambda+\ldots\right) \text{ here } M_P^2=\frac{1}{8\pi G_N}$$

Biswas,Koivisto,Mazumdar,Siegel,Dragovich,Vernov,AK,...

One of the equations of motion (trace) is

$$\lambda\sum_{n=1}^{\infty}f_n\sum_{l=0}^{n-1}\left(\partial_\mu\Box^l R\partial^\mu\Box^{n-1-l}R+2\Box^l R\Box^{n-l}R\right)+6\lambda\Box\mathcal{F}(\Box)R-M_P^2R=-4\Lambda$$

The ghost-free condition in Minkowski space ($\Lambda=0$) requires that the following equation

$$6\lambda\sigma\mathcal{F}(\sigma)-M_P^2=0$$

has no more than one root σ, i.e.

$$6\lambda\sigma\mathcal{F}(\sigma)-M_P^2=(\kappa\sigma-m^2)e^{\gamma(\sigma)}$$

Non-local gravity (continued)

It is really not obvious but the above mentioned equation has the following explicit analytic solutions

- **We need $\mathcal{F}'(\beta) = 0$ and some radiation**

$$a = a_0 \cosh(\beta t) \;\Rightarrow\; H = \beta \tanh(\beta t)$$

- **We need $\mathcal{F}'(\beta) = 0$ and NO radiation**

$$a = a_0 \exp\left(\frac{\beta}{2} t^2\right) \;\Rightarrow\; H = \beta t$$

- **We need some more special condition on $\mathcal{F}(\sigma)$ and NO radiation**

$$a = a_0 t^p \;\Rightarrow\; H = \frac{p}{t}$$

The two first solutions are the manifestly non-singular bouncing solutions moreover with a de Sitter late time asymptotic for the first one

Raychaudhuri Equation

It is the essential tool to analyze the geodesics congruence

$$R_{\mu\nu}\xi^\mu\xi^\nu = \kappa\left(T_{\mu\nu}\xi^\mu\xi^\nu + \frac{1}{2}T + \tau_{\mu\nu}\xi^\mu\xi^\nu + \frac{1}{2}\tau\right)$$

Here $\xi_\alpha\xi^\alpha = -1$, T represents matter and τ represents the gravity modification

$R_{\mu\nu}\xi^\mu\xi^\nu < 0$ represents a regular space-time while normal matter (satisfying the strong energy condition $\rho + 3p \geq 0$) in GR implies the opposite sign. In fact this is a reflection of the Hawking-Penrose theorem.

Thus, first of all, we need the gravity modification.

The bouncing solutions above may be analyzed and shown to be compatible with the presence of normal matter.

Scalar reformulation of the non-local gravity

The previous action is equivalent to the following one

$$S=\int d^4x\sqrt{-g}\left(\frac{M_P^2R}{2}(1+\psi)-\frac{M_P^4}{8\lambda}\psi\frac{1}{\mathcal{F}(\Box)}\psi+\dots\right)$$

The conformal transform $(1+\psi)^2 g_{\mu\nu}=\bar{g}_{\mu\nu}$ **allows us to decouple the gravity and the scalar field**

$$S=\int d^4x\sqrt{-\bar{g}}\left(\frac{M_P^2}{2}\bar{R}-\frac{M_P^2}{4}\frac{3}{(1+\psi)^2}\bar{g}^{\mu\nu}\partial_\mu\psi\partial_\nu\psi-\frac{M_P^4}{8\lambda(1+\psi)^2}\psi\mathcal{G}(\mathcal{P})\psi\right)$$

Here

$$\mathcal{G}(\mathcal{P})=\frac{1}{\mathcal{F}(\mathcal{P})}\ \textbf{and}\ \mathcal{P}=(1+\psi)\bar{\Box}-\bar{g}^{\mu\nu}\partial_\mu\psi\partial_\nu$$

The ghost-free condition on ψ **implies** $\mathcal{G}(\mathcal{P})=\sum\limits_{n\geq 0}g_n\mathcal{P}^n$**, i.e. it is an analytic function.**

Limits: KGB, Galileons, p-adic string, ...

The limit of weak field

$$S = \int d^4x\sqrt{-\bar{g}}\left(\frac{M_P^2}{2}\bar{R} + \left(\frac{3}{2} - \frac{g_1 M_P^2}{4\lambda}\right)\psi\bar{\Box}\psi - \frac{g_0 M_P^2}{4\lambda}\psi^2 \right.$$
$$\left. - \sum_{n>1}\frac{g_n M_P^2}{4\lambda}\left(\bar{\Box}\psi + (\partial\psi)^2\right)\left(\bar{\Box} - \partial^\rho\psi\partial_\rho\right)^{n-1}\psi\right)$$

Here we recognize KGB models and Galileon field theories.

The limit of large field

$$S = \int d^4x\sqrt{-\bar{g}}\left(\frac{M_P^2}{2}\bar{R} - \frac{3}{2}\frac{(\partial\psi)^2}{\psi^2} - \frac{M_P^2}{4\lambda}\frac{1}{\psi}\mathcal{G}\left(\psi\bar{\Box} - \partial^\rho\psi\partial_\rho\right)\psi\right)$$

As a special limit we restore the p-adic string theory

$$S = \int d^4x\sqrt{-\bar{g}}\left(\frac{M_P^2}{2}\bar{R} + \frac{\kappa}{2}\psi e^{-\beta\bar{\Box}}\psi\right)$$

Almost finished work in progress

Thus, non-local theories can be considered as *generating functionals* for other models on the market.

Conclusions and open questions

- **Non-local generalization of Einstein's gravity is presented with the aim at resolving the initial singularity problem**

- **We were able to find out explicit analytic solutions representing the non-singular bounce**

- **Scalar reformulation of the non-local gravity is presented and important limits are discussed**

- **The current work in progress is to find explicitly the power spectrum via quantization of perturbations in non-local models**

  Ben Craps, Tim De Jonckheere, AK

- **Moreover there is a hope the latter formulation of the non-local gravity will allow us to find explicitly new black objects**

Thank you for listening!