

Shock waves in Lifshitz space-times

A.A. Golubtsova^{1a} based on work with I.Ya. Aref'eva^b
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(a) PFUR IGC, (b) MIAN RAS

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¹siedhe@gmail.com

Outline

1 Introduction

- Lifshitz scaling
- Shock-waves
- For what?

2 Lifshitz shock waves

- $\nu = 2$, Lif_4 -shock wave
- Shock waves and Lifshitz black holes

3 Conclusions

Introduction

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Lifshitz shock waves

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Conclusions

Lifshitz fixed points: from condensed matter systems

Anisotropy between the space and temporal directions.

$$t \rightarrow \lambda^\nu t, \quad \vec{x} \rightarrow \lambda \vec{x}. \quad (1)$$

where ν is called the Lifshitz (dynamical) exponent.

Ex.: A Landau-Ginzburg description, $\nu = 2$

$$S = \int dt d^d x ((\partial_t \varphi)^2 - k(\nabla^2 \varphi)^2).$$

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to gravity

$$ds^2 = L^2 \left(-r^{2\nu} dt^2 + \frac{dr^2}{r^2} + r^2 d\vec{x}_{d-1}^2 \right), \quad \nu = 1 : AdS. \quad (2)$$

It is invariant under anisotropic scaling, solutions to

$$S_{GR} + \Lambda + S_{matter}, \quad \Lambda (< 0),$$

$$S = \frac{1}{2} \int d^4 x (R - 2\Lambda) - \frac{1}{2} \int (F_{(2)} \wedge \star F_{(2)} + F_{(3)} \wedge \star F_{(3)}) - c \int B_{(2)} \wedge F_{(2)}.$$



S. Kachru, et al., *Phys. Rev. D* **78**, (2008) 106005; arXiv:0808.1725.

Embedding Lifshitz space-time into supergravity

Lif_4 in massive Type IIA SUGRA, Lif_3 in massive Type IIB SUGRA:

-  R. Gregory et al., Lifshitz solutions in supergravity and string theory, *JHEP* **1012**, (2010) 047; arXiv:1009.3445.

$\nu = 2$, Lif_4 , Lif_3 in Type IIB SUGRA + T-dualized solutions in Type IIA and 11d SUGRAs:

-  A. Donos and G. Gauntlett, Lifshitz Solutions of D=10 and D=11 supergravity, *JHEP* **1012**, (2010) 002; arXiv:1008.2062 .

Intersecting $D3$ - $D7$ -branes on Lifshitz spaces with $\nu = 3/2$

-  T. Azeyanagi et al., On String Theory Duals of Lifshitz-like Fixed Points, *JHEP* **0906**, (2009) 084; arXiv:0905.0688.

$\nu = 2$ Lifshitz spacetimes as deformations of $AdS_5 \times S^5$ in 10d SUGRA.

-  K. Balasubramanian and K. Narayan, Lifshitz spacetimes from AdS null and cosmological solutions, *JHEP* **1008**, (2010) 014; arXiv:1005.3291.

-  W. Chemissany and J. Hartong, From D3-Branes to Lifshitz Space-Times, *Class.Quant.Grav* **28**, (2011) 195011; arXiv:1105.0612.

More Lifshitz spacetimes

Black hole (brane) solutions

$$ds^2 = -r^{2\nu} b(r) dt^2 + \frac{dr^2}{r^2 b(r)} + r^2 d\vec{x}^2, \quad (3)$$

where the metric (blackening) function reads

$$b(r) = 1 - mr^{-(2+\nu)}, \quad (4)$$

$$F_{rt} = \sqrt{2(\nu-1)(2+\nu)} \mu^{1/\sqrt{(\nu-1)}} r^{\nu+1}, \quad (5)$$

$$e^\phi = \mu r^{2\sqrt{\nu-1}}, \quad \phi = \ln(\mu r^{2\sqrt{\nu-1}}). \quad (6)$$

The parameters are

$$\Lambda = -\frac{(2+\nu)(1+\nu)}{2}, \quad \lambda = -\frac{2}{\sqrt{\nu-1}}. \quad (7)$$

-  J. Tarrio and S. Vandoren, *JHEP* **1109**, (2011) 017; arXiv:1105.6335
-  M. Taylor; arXiv:0812.0530.

The gravitational field generated by a massless particle, which moves with the velocity of light.

The Aichelburg-Sexl shock-wave solution

$$ds^2 = -dUdV + (dx^i)^2 + F(x^i)\delta(U)dU^2, \quad (8)$$

$$F(x^i) = \frac{c_k}{|\sum(x^i - x_{k_0}^i)|^{(D-4)/2}}, \quad U = x. \quad (9)$$

$$\Delta_2 F(x^i) = -16\pi G_4 p \delta(x^i)$$



P.C. Aichelburg and R.U. Sexl, *Gen. Rel. Grav.* **2**, (1971) 303-312.

Construction

- A particle at rest is described by the Schwarzschild solution.
- A Lorenz transformation is applied to the metric

Detailed analysis:



I. Ya. Aref'eva et al. *St. Petersburg Math. J.* **22**, (2011) 337.

Motivation

- Heavy ion collisions and QGP formation
- Thermalization process
- Estimation of multiplicity

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The holographic approach

- the strong coupling quantum field in d -dimensional Minkowski space \Leftrightarrow $d + 1$ -dimensional supergravity
 - thermalization process \Leftrightarrow a process of formation of a black hole.
-  G. Policastro et al. *Phys. Rev. Lett.* **87**, (2005) 081601.
- Black hole formation \Leftrightarrow shock waves collision

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Earlier in Lifshitz spacetimes:

Black hole formation \Leftrightarrow a shell falling at the speed of light described by a Lifshitz-Vaidya solution

 V. Keranen et al., *Phys. Rev.* **D85**, (2012) 026005; arXiv:1110.5035

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- Q: How can we obtain shock waves if Lorentz invariance is broken?

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- Q: How can we obtain shock waves if Lorentz invariance is broken?
- A: Consider the Lifshitz metric coupled to a scalar field.

$\nu = 2$ Lifshitz spacetime from *AdS*-solutions, **Balasubramanian, Narayan**

$$ds^2 = -\phi \frac{du^2}{\gamma z^4} + \frac{dx_i^2}{z^2} + \frac{dz^2}{z^2} \quad (10)$$

The Lifshitz shock wave metric

$$ds^2 = \frac{1}{z^2} \left(-dudv + \frac{\phi(x_\perp, z)\delta(u)}{z^{2\nu-2}} du^2 + dx_\perp^2 + dz^2 \right), \quad (11)$$

$$S = \frac{1}{16\pi G_4} \int d^4x \sqrt{-g} \left[R - 2\Lambda - \frac{1}{2} (\partial\phi)^2 - \frac{1}{4} e^{\lambda\phi} F^2 \right]$$

The shock wave obey the equation

$$\left(\square_{\mathbb{H}_2} - \frac{2}{L^2} \right) \Phi(x_\perp, z) = -16\pi G_4 \frac{z}{L} J_{uu}(z, x_\perp), \quad (12)$$

$$\Phi(x_\perp, z) = \frac{\phi(x_\perp, z)\delta(u)}{z^{2\nu-1}}, \quad (13)$$

The d'Alembert operator $\mathbb{H}_2 = z^2 \left(\frac{\partial^2}{\partial x_\perp^2} + \frac{\partial^2}{\partial z^2} \right)$ acts on

$$ds^2 = \frac{1}{z^2} (d\vec{x}_\perp^2 + dz^2). \quad (14)$$

$$S = \int d^4x \sqrt{-g} (R - 2\Lambda) - \frac{1}{2} \int \star F_{(2)} \wedge F_{(2)} - \frac{1}{2} \int \star H_{(3)} \wedge H_{(3)} - c \int B_{(2)} \wedge F_{(2)},$$
$$F_{(2)} = dA_{(1)}, H_{(3)} = dB_{(2)}, \Lambda = -\frac{\nu^2 + \nu + 4}{2L^2}, c = \frac{2\nu}{L}.$$

Lifshitz black holes

$$ds^2 = L^2 \left(-r^{2\nu} f(r)^2 dt^2 + \frac{g(r)^2}{r^2} dr^2 + r^2 (d\theta^2 + \chi(\theta)^2 d\phi^2) \right), \quad (15)$$

$$\chi(\theta) = \begin{cases} \sin(\theta), & \text{if } k = +1; \\ \theta, & \text{if } k = 0; \\ \sinh(\theta), & \text{if } k = -1. \end{cases} \quad (16)$$

where $k = +1, 0, -1$ corresponds to a spherical, flat, and hyperbolic horizon respectively. $\nu = 2$, $\mathbf{k} = -\mathbf{1}$

$$f = \frac{1}{g(r)} = \sqrt{1 - \frac{1}{2r^2}}.$$



Da-Wei Pang, *JHEP* **1008**, (2010) 014; arXiv:1005.3291.



E. J. Brynjolfsson et al., *J.Phys.A* **43**, (2010) 065401; arXiv:0908.2611.

Global coordinates

A tortoise coordinate

$$r = \frac{1}{\sqrt{2}\sqrt{1 - \exp[r_* - r_*^\infty]}}$$

Null combinations $u = t - r_*$, $v = t + r_*$. A conformal reparametrization

$$V = \exp[\frac{1}{2}(v - r_*^\infty)], \quad U = -\exp[-\frac{1}{2}(u + r_*^\infty)]$$

$$ds^2 = -\frac{L^2}{(1+UV)^2}dUdV + \frac{L^2}{1+UV}\left(d\theta^2 + (\sinh(\theta))^2 d\varphi^2\right). \quad (17)$$

A coordinate shift

$$ds^2 = 2A(u, v)dudv + g(u, v)h_{ij}(x)dx^i dx^j.$$

is kept for $u < 0$.

$$u > 0, \quad v \rightarrow v + f(x)$$

$$ds^2 = 2A(u, v)du(dv - f(x^i)\delta(u)du) + g(u, v)h_{ij}(x)dx^i dx^j. \quad (18)$$



T. Dray and G. t Hooft, *Nucl. Phys. B* **253**, (1985) 173.

Shock wave on the black hole horizon

The generalized shock wave

$$ds^2 = \frac{L^2}{(1+UV)^2} \phi(\theta, \varphi) \delta(u) dU^2 - \frac{L^2}{(1+UV)^2} dU dV + \frac{L^2}{1+UV} (d\theta^2 + (\sinh(\theta))^2 d\varphi^2). \quad (19)$$

$$A = \frac{L^2}{2(1+UV)^2}, \quad g = \frac{L^2}{2(1+UV)}, \quad h_{11} = 1, \quad h_{22} = (\sinh(\theta))^2 \quad (20)$$

The equation of motion

$$\nu = 2, U = 0$$

$$\left(\square_2 - \frac{16}{L^2} - 2\Lambda \right) \phi(\theta, \varphi) \delta(u) = -\frac{2}{L^2} (T_{uu} + T_{matter}). \quad (21)$$

A massless particle located at $u = 0$ and moving with the speed of light in the v -direction.

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Open questions

- Collision of the shock waves: trapped surfaces
- Estimation of multiplicity: is there any dependence on ν ?

Conclusions

- The shock wave for the Lifshitz spacetimes $\nu = 2$;
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Open questions

- Collision of the shock waves: trapped surfaces
- Estimation of multiplicity: is there any dependence on ν ?
- A particular supergravity model ? Experimental data?

THANK YOU FOR YOUR ATTENTION!