On the theory of neutrino oscillations

A. E. Lobanov

Moscow State University, Moscow, Russia

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Introduction

What we know about neutrino oscillations?
1. Neutrino oscillations do exist.
2. Neutrino oscillations are related to propagation in space-time (translations).
3. Neutrino oscillations are not related to rotations and (it seems) to Lorentz transformations.
4. The theory of neutrino oscillations is known for more then 30 years.
5. However it is a problem to construct flavor states of neutrino.

The aim of this talk is an attempt to solve this problem.

The standard approach to the problem of description of oscillations is connected to introduction of mass and flavor basis. However flavor solutions do not form the basis of irreducible representation of the Poincaré group. It is not good.
It will be of interest to construct irreducible representation which describe flavor states of neutrino. Obviously it is impossible in the framework of the Dirac spinor space. However, assuming that the effect of oscillations is Lorentz invariant, for constructing such a representation it is possible to modify only generators of translations. Since the translation subgroup is the Abelian subgroup, this modification can be reduced to multiplying by a matrix. Taking in mind that there are three type of neutrino, the matrix must have dimension 3×3.

Thus the representation space is the direct product of the Dirac spinor space and the space of finite-dimensional irreducible representation of some group. It is natural to chouse the space of fundamental representation of $SU(3)$ group.
The matrix of interest can be represented as

$$\mathcal{N}_\nu = \sum_{l=1,2,3} \frac{m}{m_l} \mathcal{P}_\nu^{(l)}, \tag{1}$$

where $\mathcal{P}_\nu^{(l)}$ are the orthogonal projectors: $\mathcal{P}_\nu^{(l)} \mathcal{P}_\nu^{(k)} = \delta_{kl} \mathcal{P}_\nu^{(l)}$, and m, m_l are real numbers. We can write

$$\mathcal{P}_\nu^{(l)} = n^{(l)} \otimes \overset{*}{n}{}^{(l)}.$$

Here $n^{(l)}$ are eigenvectors of $\mathcal{N}_\nu$. Eigenvectors $n_i^{(k)}$ are normalized by condition

$$n_i^{(k)} \overset{*}{n}{}_i^{(l)} = \delta_{kl}, \qquad \sum_{k=1}^{3} n_i^{(k)} \overset{*}{n}{}_j^{(k)} = \delta_{ij}.$$

Then the Dirac equation takes the form

$$\mathcal{N}_\nu \left(i\gamma^\mu \partial_\mu - m\right) \psi^\nu(x) = 0. \tag{2}$$

Multiplying (2) by

$$\mathcal{N}_\nu^{-1} = \sum_{l=1,2,3} \frac{m_l}{m} \mathcal{P}_\nu^{(l)},$$

we get

$$\left(i\gamma^\mu \partial_\mu - \mathcal{M}_\nu\right) \psi^\nu(x) = 0. \tag{3}$$

Here

$$\mathcal{M}_\nu = \sum_{l=1,2,3} m_l \left(n^{(l)} \otimes \overset{*}{n}{}^{(l)}\right) \tag{4}$$

is the mass matrix of neutrino.

The complete orthonormalized system of solutions of this equation takes the form

$$\psi^{\nu}_{q,\zeta_0,(l)}(x) = e^{-i(qx)[m_l/m]}\,(m_l/m)^{3/2}\,n^{(l)}(1-\zeta_0\gamma^5\hat{S}_0(q))(\hat{q}+m)\psi^{\nu}_0. \quad (5)$$

4-vector q^{μ} obeys $q^2 = m^2$; 4-vector $S^{\mu}_0(q)$ determines the direction of polarization of the particle, $\zeta_0 = \pm 1$ is the sign of spin projection on this direction, and ψ^{ν}_0 is the constant bispinor that is $\bar{\psi}^{\nu}(x)\psi^{\nu}(x) = m/q^0$.

It is possible to take a similar consideration for charged leptons. As a result we have the Dirac equation

$$\mathcal{N}_e \left(i\gamma^\mu \partial_\mu - M\right) \psi^e(x) = 0. \tag{6}$$

This equation can be transformed to

$$\left(i\gamma^\mu \partial_\mu - \mathcal{M}_e\right) \psi^e(x) = 0. \tag{7}$$

Here

$$\mathcal{M}_e = \sum_{l=1,2,3} M_\beta \left(a^{(\beta)} \otimes \overset{*}{n}{}^{(\beta)} \right) \tag{8}$$

is the mass matrix of charged leptons.

The complete orthonormalized system of solutions of this equation takes the form

$$\psi^{e}_{q,\zeta_0,(\beta)}(x) = e^{-i(qx)[M_l/M]}\,(M_\beta/M)^{3/2}\,a^{(\beta)}(1-\zeta_0\gamma^5\hat{S}_0(q))(\hat{q}+M)\psi^{e}_0. \quad (9)$$

4-vector q^μ obeys $q^2 = M^2$; 4-vector $S^\mu_0(q)$ determines the direction of polarization of the particle, $\zeta_0 = \pm 1$ is the sign of spin projection on this direction, and ψ^ν_0 is the constant bispinor that is $\bar{\psi}^e(x)\psi^e(x) = M/q^0$.

If

$$n^{(l)} = a^{(\beta)}$$

we have the Standard Model with three generations.

If

$$n^{(l)} \neq a^{(\beta)}$$

we have mixing of generations.
The elements of the mixing matrix are

$$n_i^{(l)} \overset{*}{a}{}_i^{(\beta)} = U_{\beta l}, \quad \overset{*}{n}{}_i^{(l)} a_i^{(\beta)} = U^*_{\beta l}. \tag{10}$$

Now we can construct the flavor basis for neutrino

$$\psi^{\nu}_{q,\zeta_0,(\alpha)}(x) = \\ = \left[\sum_{l=1}^{3} e^{-i(qx)[m_l/m]}\,(m_l/m)^{3/2}\,U^*_{\alpha l} n^{(l)}\right](1-\zeta_0\gamma^5\hat{S}_0(q))(\hat{q}+m)\psi^{\nu}_0. \quad (11)$$

And we can calculate the probabilities of the processes with neutrino using the common Feynman rules.

Let us consider the decay

$$\pi^+ \Rightarrow l^+ + \nu,$$

where $l_\beta^+ = \mu^+, e^+$.
Assume that 4-momentum of π^+ is equal to k^μ, $k^2 = m_\pi^2$ and 4-momentum of l^+ is equal to p^μ, $p^2 = M_\beta^2$.

Suppose that the (large) distance from source of neutrino is equal to L, and the linear size of the source is equal to L_0.

Then

$$W^L_{\alpha\beta} = \frac{G_{\mathrm{F}}^2 f_\pi^2}{4(2\pi)^6 k^0} \int d^4x d^4y \int d^4q d^4p \, \delta(p^2 - M_\beta^2)\delta(q^2 - m^2)\times$$

$$\times \left[\sum_{k,l=1}^{3} e^{i(qx)[m_k/m] - i(qy)[m_l/m] + i((p-k)(x-y)) + 2\pi i L|\mathbf{q}|/m(m_k - m_l)} \times \right.$$

$$\left. \times\, (m_l/m)^{3/2} (m_k/m)^{3/2}\, U_{\beta k} U^*_{\beta l} U_{\alpha l} U^*_{\alpha k} \right] \times$$

$$\times\, \mathrm{Sp}\left[(\hat{p} - M_\beta)\hat{k}(1+\gamma^5)(\hat{q}+m)\hat{k}(1+\gamma^5) \right]. \quad (12)$$

Let us take for simplicity $\mathbf{k} = 0$, i.e. π^+ is at rest. Using the standard ansatz of scattering theory

$$x^\mu, y^\mu \Rightarrow (x^\mu - y^\mu), (x^\mu + y^\mu)/2$$

we have up to terms $\sim m_{k,l}^2/m_\pi \mathcal{E}_\nu$ ($\mathcal{E}_\nu$ is neutrino energy)

$$W_{\alpha\beta}^L = \frac{G_{\mathrm{F}}^2 f_\pi^2}{8\pi} M_\beta^2 \frac{\left(m_\pi^2 - M_\beta^2\right)^2}{m_\pi^3} \times$$
$$\times \left[\sum_{k,l=1}^{3} \frac{U_{\beta k} U_{\beta l}^* U_{\alpha l} U_{\alpha k}^*}{\Delta_{kl}^3} \frac{\sin(\pi L_0 / L_{osc}^{(kl)})}{\pi L_0 / L_{osc}^{(kl)}} e^{2\pi i L / L_{osc}^{(kl)}} \right], \quad (13)$$

where

$$L_{osc}^{(kl)} = \frac{4\pi \mathcal{E}_\nu}{m_k^2 - m_l^2}, \quad \mathcal{E}_\nu = \frac{m_\pi^2 - M_\beta^2}{2 m_\pi}, \quad \Delta_{kl} = (m_k + m_l)/(2\sqrt{m_k m_l}) \geqslant 1.$$

.

In the standard theory of neutrino oscillations we have

$$W^L_{\alpha\beta} = \frac{G_{\mathrm{F}}^2 f_\pi^2}{8\pi} M_\beta^2 \frac{\left(m_\pi^2 - M_\beta^2\right)^2}{m_\pi^3} \times$$
$$\times \left[\sum_{k,l=1}^{3} U_{\beta k} U^*_{\beta l} U_{\alpha l} U^*_{\alpha k} \frac{\sin(\pi L_0/L^{(kl)}_{osc})}{\pi L_0/L^{(kl)}_{osc}} e^{2\pi i L/L^{(kl)}_{osc}} \right]. \quad (14)$$

Discussion

Using these results it is possible to draw a conclusion (if the point of view discussed above is reasonable), that the products of reaction with neutrino contain not only neutrino, possessing the main flavor, but and admixture of neutrino of other flavors.
Formula (13) demonstrates, that investigation of neutrino oscillations can give information not only about differences of neutrino masses but also about their absolute values.

Conclusion

This talk, of course, is not a revelation, but rather a visual aid. Though it, I think, gives causes for reflections. Therefore there were no words like "reactor anomaly", "sterile neutrino" and so on.

However...

Thank you for your attention!