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The RG inspired β-expansion approach, its uniquiness, its link with the conformal symmetry limit in QCD and phenomenological applications

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The analytical $O(\alpha_s^4)$ results for the NS contributions to the Adler function of the EM quark currents and to to the Bjorken sum rule of the polarized lepton-nucleon scattering Baikov, Chetyrkin and Kuhn (2010) are considered within β-expansion BLM extension approach with the multiple β-function generalization of the Crewther relation Kataev and Mikhailov Quarks 2010 - Theor.Math.Phys. 2012. The doubts of Brodsky, Wu, Mojaza 2011-2014 on the uniq1uiness of this approach are revealed

Studies of different ways of resummation and fixation of scale-scheme uncertainties of HO QCD PT predictions **plus** understanding of basic features and symmetries beyond these representations are theoretical and phenomenologically important. We consider the following quantities in $\overline{MS}$

$$D^{\rm EM}(Q^2/\mu^2, a_s(\mu^2)) = \left(\sum_i q_i^2\right) d_R D^{\rm NS}(Q^2/\mu^2, a_s(\mu^2)) + \left(\sum_i q_i\right)^2 d_R D^{\rm S}(Q^2/\mu^2, a_s(\mu^2))$$

$$R_{e^+e^-}(s) \equiv R(s, \mu^2 = s) = \frac{1}{2\pi i}\int_{-s-i\epsilon}^{-s+i\epsilon} \frac{D^{\rm EM}(\sigma/\mu^2; a_s(\mu^2))}{\sigma} d\sigma \Bigg|_{\mu^2=s}$$

$$D^{\rm NS}(Q^2/\mu^2, a_s(\mu^2)) \overset{\mu^2\to Q^2}{=} D^{\rm NS}(a_s(Q^2)) = 1 + \sum_{l\geq 1} d_l^{\rm NS} a_s^l(Q^2) \qquad (1)$$

$$S_{Bjp}(Q^2) = \int_0^1 [g_1^{lp}(x, Q^2) - g_1^{ln}(x, Q^2)]dx = \frac{g_A}{6} C_{Bjp}(Q^2/\mu^2, a_s(\mu^2))$$

$$C_{Bjp}(a_s) = C_{Bjp}^{\rm NS}(a_s) + \left(\sum_i q_i\right) C_{Bjp}^{\rm S}(a_s)$$ *Larin(2013) not yet confirmed analytically*

$$C_{Bjp}^{\rm NS}(Q^2/\mu^2, a_s(\mu^2)) \overset{\mu^2\to Q^2}{=} 1 + \sum_{l\geq 1} c_l^{\rm NS} a_s^l(Q^2) \qquad (2)$$

$$\left(\mu^2\frac{\partial}{\partial\mu^2} + \beta(a_s)\frac{\partial}{\partial a_s}\right) D^{\rm NS}(Q^2/\mu^2, a_s(\mu^2))\left[C_{Bjp}^{\rm NS}(Q^2/\mu^2, a_s(\mu^2))\right] = 0$$

$$\beta(a_a) = \mu^2\partial a_s(\mu^2)/\partial\mu^2 = -a_s(\mu^2)^2 \sum_{l\geq 0} \beta_l a_s^l(\mu^2) \qquad (3)$$

β-expansions approach is the $\overline{MS}$-scheme generalization of the BLM method (1983) Mikhailov, Quarks-2004, JHEP(2007) for all orders after NNLO generalization of Grunberg and Kataev (1992) and related works by Beneke and V.Braun (1995) Basis - Instead of Scalar Representation to use Matrix Representation

$$D^{NS} = 1 + \sum_{n \geq 1} a_s(Q^2)^n d_n(N_F) = 1 + (\overline{a_s}\overline{d(N_F)}) \quad (4)$$

$$D^{NS}(Q^2) = 1 + \sum_{n \geq 1} \sum_{l} a_s(Q^2)^n D_{nl} B^l(N_F) \quad (5)$$

where $B^l(N_F)$-products of β-function coefficients and $d_n(N_F) = D_{nl} B^l(N_F)$ where terrms D_{nl} do not depend from the numbers of flavours N_F and have the following form

$$d_1 = d_1[0] = \frac{3}{4} C_F \quad , \quad d_2 = \beta_0(N_F)^2 d_2[1] + d_2[0]$$

$$d_3 = \beta_0(N_F)^3 d_3[2] + \beta_1(N_F) d_3[0,1] + \boxed{\beta_0(N_F) d_3[1]} + d_3[0]$$

$$d_4 = \beta_0^3 d_4[3] + \beta_1 \beta_0(N_F) d_4[1,1] + \beta_2(N_F) d_4[0,0,1] + \beta_0^2 d_4[2] + \boxed{\beta_1 d_4[0,1]} + \boxed{\beta_0 d_4[1]} + d_4[0]$$

The result for d_3 was obtained by Mikhailov (2007) using QCD + n_{gl} multiplet of massless gluiono, contributing to $d_3(N_F, n_{gl}$ from the result of Chetyrkin (1997). Terms $\boxed{\beta_0(N_F)d_3[1]}$, $\boxed{\beta_1(N_F)d_4[0,1]}$ $\boxed{\beta_0(N_F)d_4[1]}$ were neglected by *Brodsky et al*. So ambiguity ? NO! Terms $d_n[0]$ are related to CS limit

Why $\boxed{\beta_0(N_F)d_3[1]}$, $\boxed{\beta_1(N_F)d_4[0,1]}$ $\boxed{\beta_0(N_F)d_4[1]}$ were neglected by Brodsky et ? Without neglecting these terms it was impossible to get used by them **approximate β-expansion**

$$d_1 = d_1[0] = \frac{3}{4}C_F \quad , \quad d_2 = \beta_0(N_F)^2 d_2[1] + d_2[0]$$

$$d_3 = \beta_0(N_F)^3 \tilde{d}_3[2] + \beta_1(N_F)\tilde{d}_3[0,1] + \tilde{d}_3[0]$$

$$d_4 = \beta_0^3 \tilde{d}_4[3] + \beta_1\beta_0(N_F)\tilde{d}_4[1,1] + \beta_2(N_F)\tilde{d}_4[0,0,1] + \beta_0^2\tilde{d}_4[2] + \tilde{d}_4[0]$$

Using complete N_F dependence of the $\beta_i(N_F)$ and $d_i(N_F)$

$$d_2(N_F) = N_F d_{21} + d_{20} \ , \ d_3(N_F) = N_F^2 d_{32} + N_F d_{31} + d_{30} \tag{6}$$

$$d_4(N_F) = N_F^3 d_{43} + N_F^2 d_{42} + N_F d_{41} + d_{40} \tag{7}$$

The β-expanded representation is true in the case of NS contribution to the Bjorken sum rule of the polarisez lepton-nucleon DIS Kataev, Mikhailov (2010-2012). The β-expanded form for C^{NS}_{Bjp} was obtained from the $O(\alpha_s^4)$-generalization of the matrix representation for the $\overline{MS}$-scheme generalization of the Crewther relation Kataev, Mikhailov Quarks-2010, Theor.Mat.Fiz (2012)

$$D^{NS}(a_s)C^{NS}_{Bjp} = 1 + \Delta_{csb}(a_s) = 1+ \quad (8)$$

$$c_1 = c_1[0] = -\frac{3}{4}C_F \quad , \quad c_2 = \beta_0(N_F)^2 c_2[1] + c_2[0]$$

$$c_3 = \beta_0(N_F)^3 c_3[2] + \beta_1(N_F)c_3[0,1] + \boxed{\beta_0(N_F)c_3[1]} + c_3[0]$$

$$c_4 = \beta_0^3 c_4[3] + \beta_1\beta_0(N_F)c_4[1,1] + \beta_2(N_F)c_4[0,0,1] + \beta_0^2 c_4[2] + \boxed{\beta_1 c_4[0,1]} + \boxed{\beta_0 c_4[1]} + c_4[0]$$

Note that the terms in **boxes** can not be eliminated. Without them in KM(2010)-(2012) results the powers of β-function will be spoiled

Indeed the polynmials of conformal symmetry breaking powers of β-function contain these terms and they can not be neglectred in the process of constructing "Principle of Minimal Conformality"by Brodsky et al

$$\Delta_{csb}(a_s) = \sum_{n\geq 1} \left(\frac{\beta(a_s)}{a_s}\right)^n P_n(a_s) \tag{9}$$

$$P_1(a_s) = -a_s\Big(c_2[1] + d_2[1]\Big) - a_s^2\Big(\boxed{c_3[1]} + \boxed{d_3[1]} + d_1(c_2[1] - d_2[1])\Big) + \delta_1(a_s)$$

$$\delta_1(a_s) = -a_s^3\Big(c_4[1] + d_4[1] + d_1(\boxed{c_3[1]} - \boxed{d_3[1]}) + d_2[0]c_2[1] + d_2[1]c_2[0]\Big)$$

$$P_2(a_s) = a_s\Big(c_3[2] + d_3[2] + a_s^2\Big(\boxed{c_4[2]} + \boxed{d_4[2]} - d_1(c_3[2] - d_3[2])\Big)$$

To explain what is PMC consider first β expansion for $D^{NS}(a_s(t = Q^2/\mu^2))$ and to find $a_s(t_1, t)$ to cancel a part of β-expansion step by step in every order and to **accumulate it** in new log-scale t_1, $a_s(t_1) = a_1$. It is possible to do, introducing coupling constant dependent definition of scale and absorbing all β-function dependence into the scale. The final expansion should have the form with the coefficients, respecting conformal symmetry

$$D^{NS}(Q_1^2) = 1 + d_1[0]a_s(Q_1^2) + d_2[0]a_s(Q_1^2)^2 + d_3[0]a_s(Q_1^2)^3 + d_4[0]a_s(Q_1^2)^4 \quad (10)$$

However, as we noted, in the papers by Brodsky et al the scale is defined in some approximation and definite terms are missed - they arwe using NOT COMPLETE β-expansion procedure. We have the corrected result at a_s^2 and a_s^3 level- the shifts of scales are serious. However they will be presented after additional cross-checks- Stay Tuned!