Multipariclе Processes and Space-Time Dimension

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Three Blocks

Fun with Graphs

I define an integer valued function on graphs, R(g), and make a claim about its values. (Surprising, checked on examples, unproven.)

Multi-particle Processes

A new representation for multi-particle amplitudes, involving the above R(G)

A Speculation

Space time is an attribute of a set of graphs

Recursive definition

Sum over nontrivial partitions

$$R(g) = \chi(g) \sum_{p \in P'(g)} f(p) \prod_{g' \in p} R'(g')$$

Recursion base

$$R'(v) = 1, \; R'(g \neq v) = R(g)$$

The characteristic function

- $\chi(g) = 1$ if $|E(g)| \leq 2|V(g)| - 3$
- and there are no tadpoles (loops)
- and number of adjacent edges is less than 4 for each vertex
- $\chi(g) = 0$ otherwise

The double sum

Reduced graph

g/p is graph obtained from g by contraction of all the edges starting and ending inside the same element of the partition p. The elements of the partition become the vertexes of the reduced graph

The coefficient

$$f(p) = \sum_{p' \in P(g/p)} \chi(p')(-1)^{|p'|}(|p'| - 1)!$$

Which p′-s do contribute?

$\chi(p') = 1$ if there is no edges of g/p starting and ending inside one and the same element of p′; otherwise $\chi(p') = 0$

The double sum (continued)

Putting it all in a single formula

$$R(g) = \chi(g) \sum_{p \in P'(g)} \sum_{p' \in P(g/p)} \chi(p')(-1)^{|p'|}(|p'| - 1)! \prod_{g' \in p} R'(g')$$

Graphical rules

The contributions to the sum in R(g) will be depicted as the graph g with the elements of p encircled in red, and the elements of p′, in green. The signed factor of the contribution follows the graph.

R(g) for |V(g)| < 3

$R(\cdot) = \emptyset$ (no subgraphs)

$R(\cdot\ \cdot) =$  $+$  (-1)

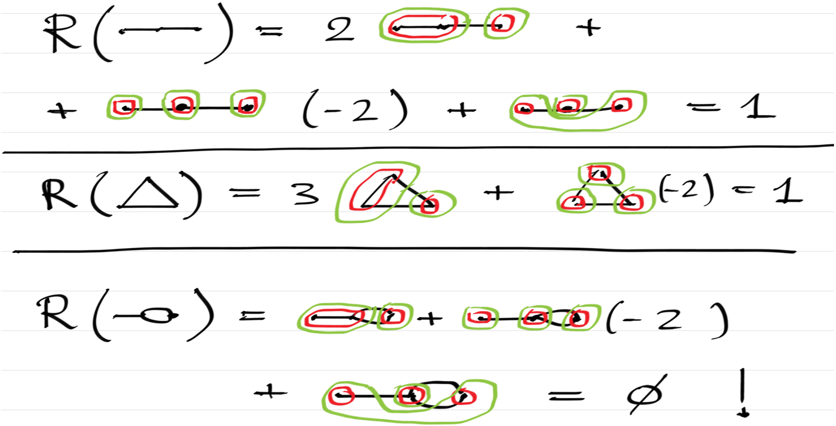
$= \emptyset$

(holds for all disconnect. graphs)

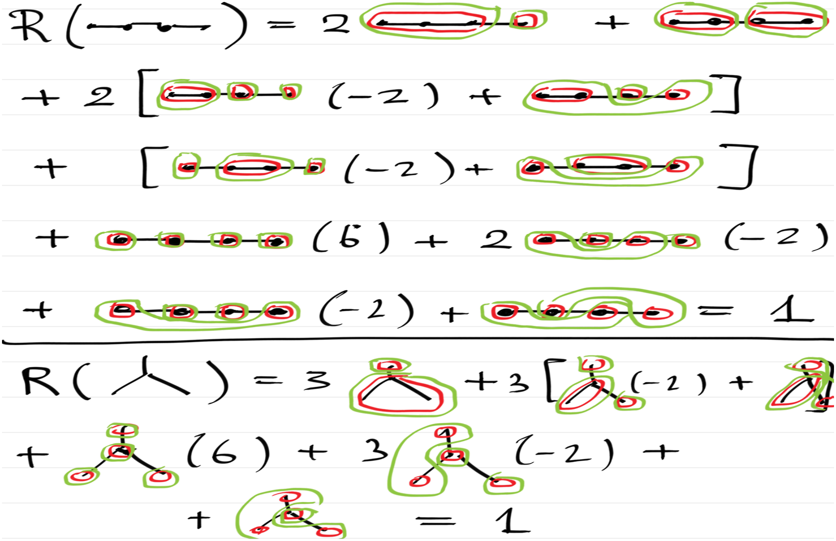
$R(-) =$  $= 1$

[No other graphs with $|V| = 2$ because $|E| \leq 2|V| - 3$]

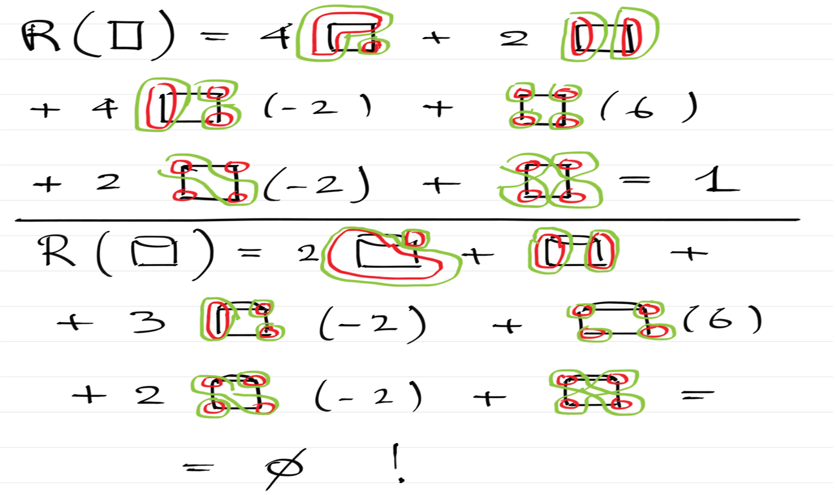
$R(g)$ for $|V(g)| = 3$



R(g) for |V(g)| = 4



R(g) for |V(g)| = 4 (continued)



Conjectures on R(g)

- R(g) is a characteristic function of a subset of graphs
- This set is the set of connected graphs corresponding to the Feynman amplitudes convergent in the ultraviolet at physical space-time dimension

Separation

Generating functional of Connected Green functions

$$\mathrm{W(J)} = -\frac{1}{2}\mathrm{JDJ} + \mathrm{A(iDJ)}$$

$\mathrm{A}(\phi)$ is the generating functional of connected amplitudes

Inaction and vertex functionals

$$\mathrm{A}(\phi) = \mathrm{I}(\phi) + \mathrm{V}(\phi)$$

$$\mathrm{I}(\phi) = \mathrm{I}_3 * \phi^3 + \mathrm{I}_4 * \phi^4$$

$\mathrm{V}(\phi)$ is the generating functional of multi-particle amplitudes

Effective Feynman amplitudes

Graph $\leftrightarrow$ effective amplitude

$$A(g,\phi) = \Big(\prod_{e \in E(g)} \frac{\delta}{\delta \phi_{v_{e1}}} D \frac{\delta}{\delta \phi_{v_{e2}}} \prod_{v \in V(g)} I(\phi_v) \Big)|_{\phi_v = \phi}$$

Representation for the vertex functional

$$V(\phi) = \sum_g \frac{R(g)}{|Aut(g)|} A(g,\phi)$$

Inaction equation

The above representation is implied by the inaction equation:

$$\mathrm{PT}^{-1}\log \mathrm{T}\exp[\mathrm{I}(\phi)+\mathrm{V}(\phi)] = 0$$

See derivation in G. P. arXiv:1312.7526
It follows from the property of the action:

$$\mathrm{PS}(\phi) = 0$$

A fork

first branch

R(g) is not zero on graphs with ultraviolet divergences $\rightarrow$ A form of asymptotic freedom

second branch

R(g) behaves as I expect $\rightarrow$ graph combinatorics defines the space time dimension!

Current tasks

- Check conjecture on R(g) for larger graphs (put it on computer)
- Introduce various types of vertexes and lines and generalize the defintion of R(g) accordingly
- Try to prove the conjecture (using Hopf algebras?)