NLO corrections to chromomagnetic operator contribution to total semileptonic B meson width

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Standard first page in years 2013-...

Hurrrra!!!

We have found HIGGS!

Indeed,...

Higgs boson has been discovered at LHC

SM has formally been completed

LHC project – Atlas, CMS, LHCb –

is a big success

but... no new physics yet

SM is perfectly fine

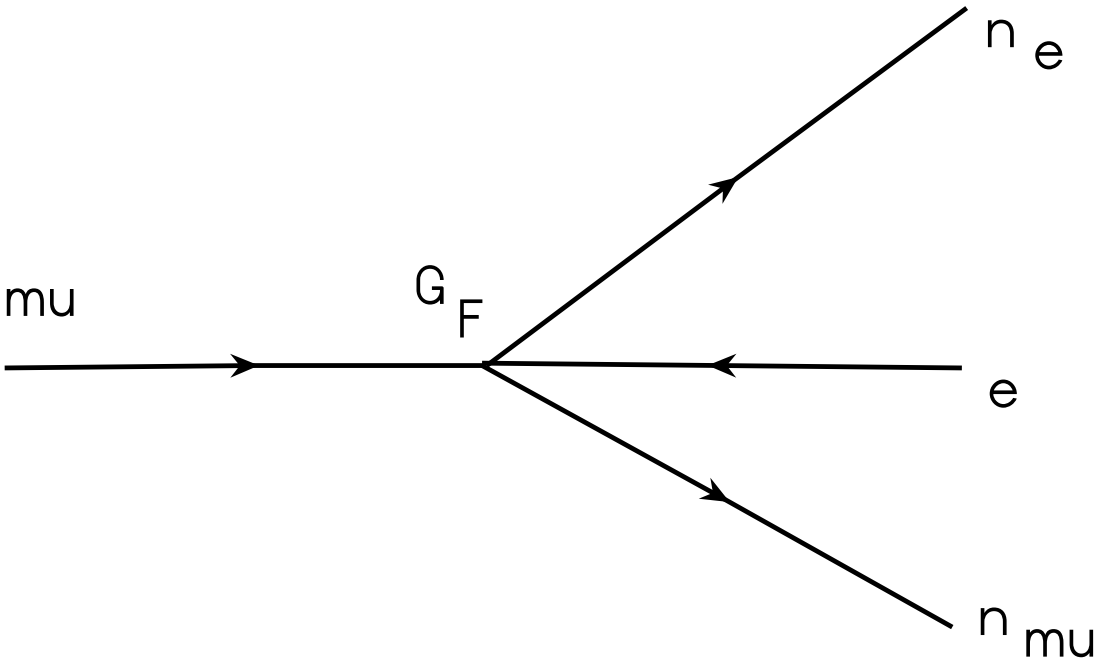
A (possible) prospect:
no explicit new modes (particles) anymore
but only implicit traces of NP through
high precision measurements

Outline

- Motivation: theory precision in SM
- HQE for inclusive B-meson decays
- NLO correction to μ_G: three-loop on-shell diagrams
- Results
- m_c corrections
- Phenomenology of $|V_{qQ}|$

Theory precision in SM

Fermi constant G_F
Leptonic sector - no mixing
Muon decay - tree level



Theory precision in SM

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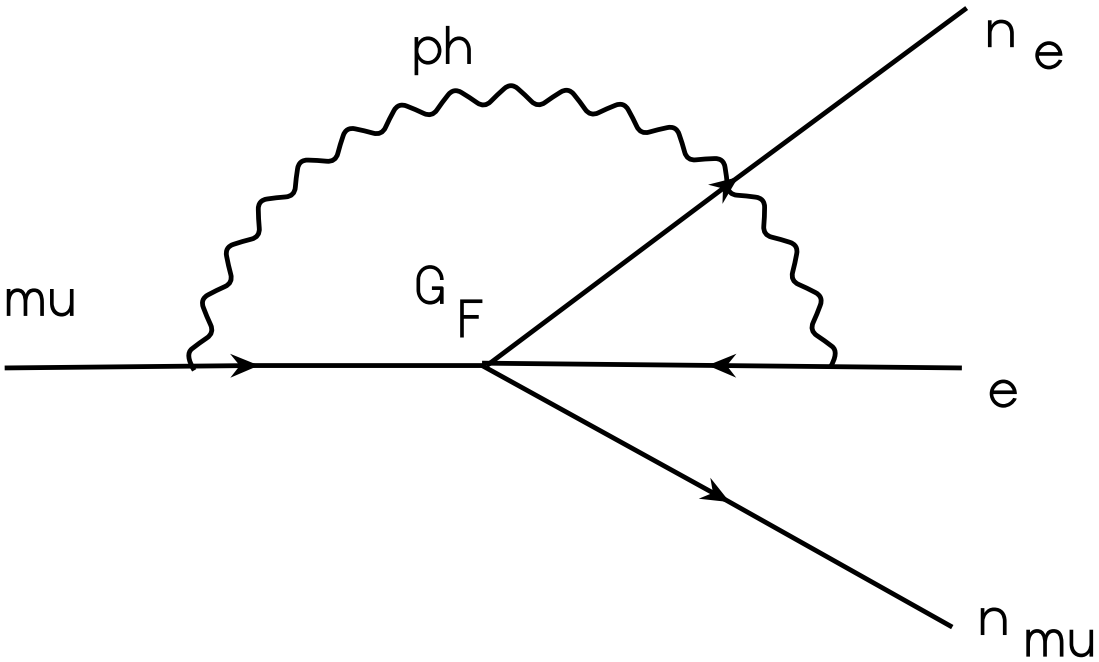
Muon decay - EM corr NLO level



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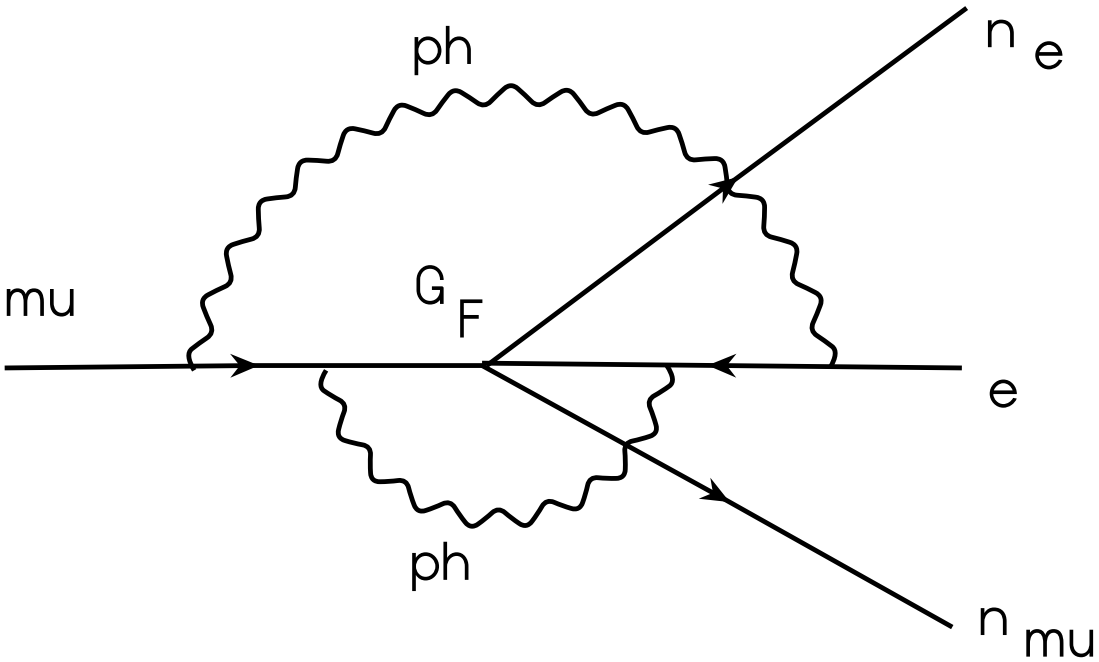


Theory precision in SM

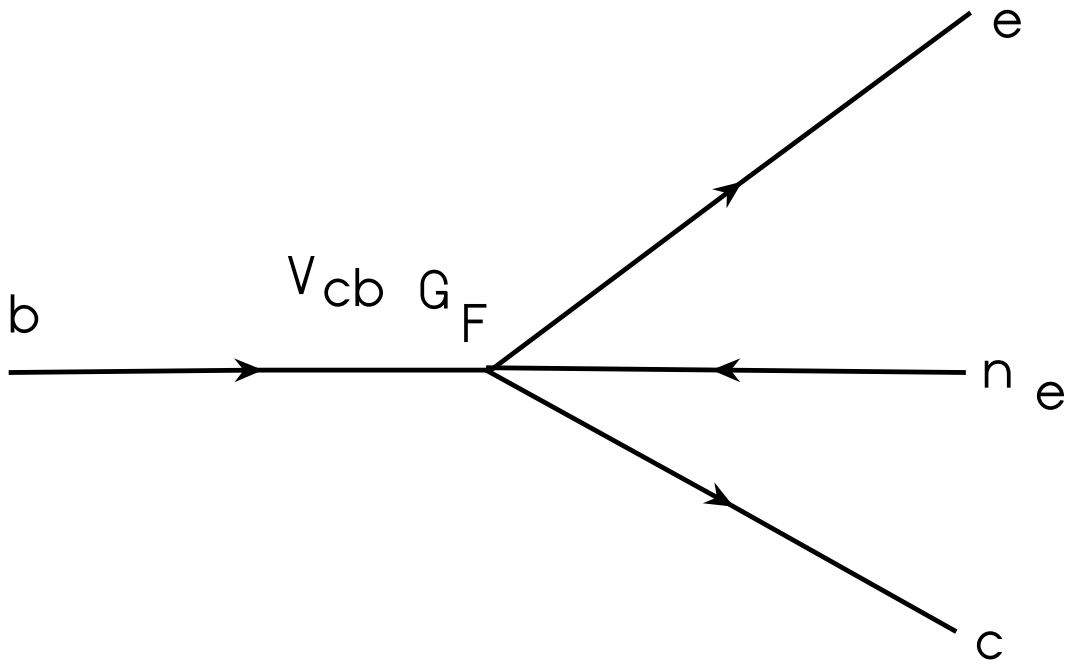
Fermi constant G_F
Leptonic sector - no mixing
Muon decay - final (1998) $\Gamma^0_\mu = G_F^2 m_\mu^5 / 192\pi^3$

$$\begin{aligned}\Gamma(\mu \to \nu_\mu \ell \bar{\nu}_\ell)/\Gamma^0_\mu &= 1 + \left(\frac{25}{8} - \frac{\pi^2}{2}\right)\frac{\alpha}{\pi} + 6.74\left(\frac{\alpha}{\pi}\right)^2 \\ &= 1 - 0.004204 + 0.000036\end{aligned}$$

$\alpha^{-1} = 137.035999074(44)$(10 digits-1ppb)
$m_\mu = 105.6583715(35)\ \mathrm{MeV}$(8 digits-1ppm)
(if we know what MeV is)

Theory precision in SM: hadrons

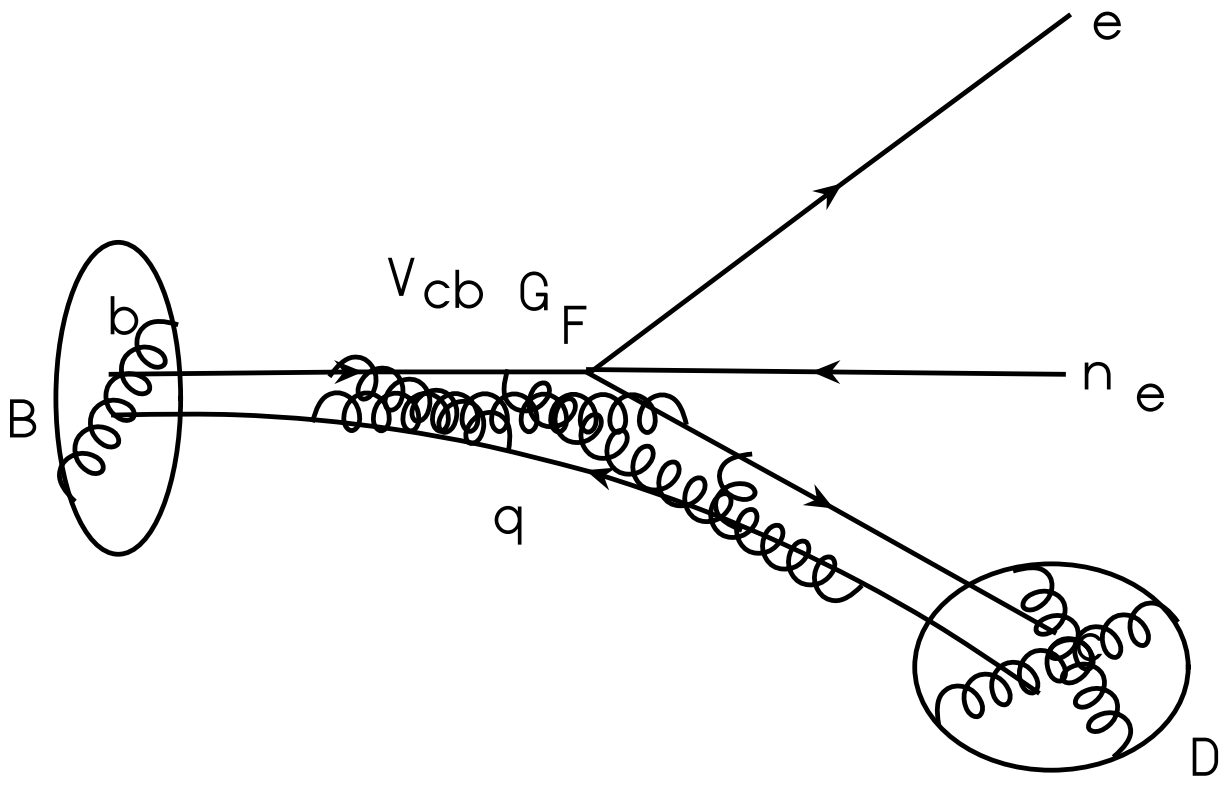
An analogue in theory - weak decay of a quark
A coupling - V_{qQ}
“Easy” to compute – impossible to measure



Theory precision in SM: hadrons

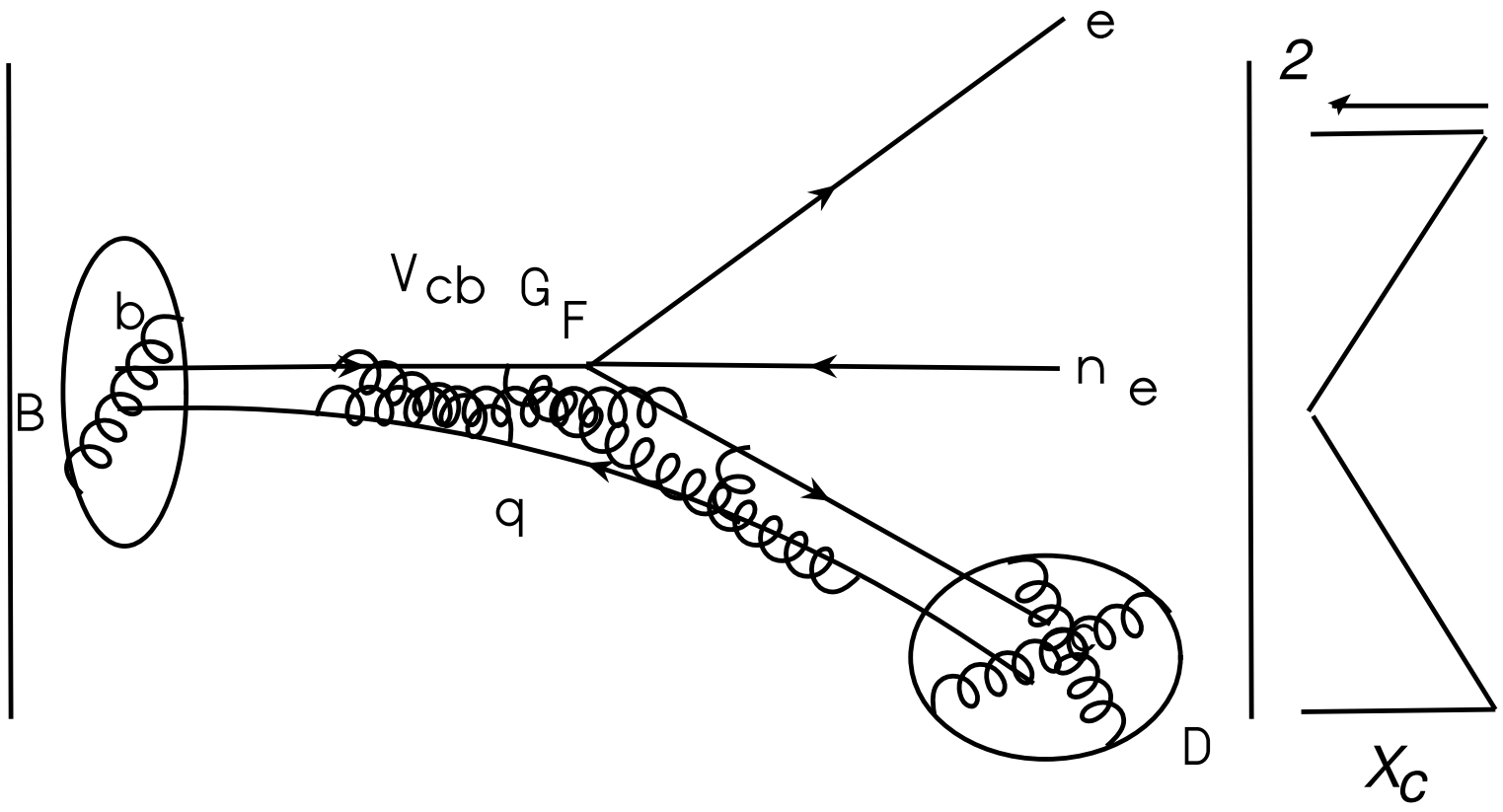
In experiment - *B*-meson decay

"Easy" to measure - difficult (impossible?) to compute



Theory precision in SM: hadrons

Simplification for theory - inclusive setup



Hadron weak decays: inclusive setup

Simplification: Unitarity + completeness of states

$$\sum_{X_c} |X_c\rangle\langle X_c| = 1$$

and $SS^+ = 1$ where

$$S = T\exp\left(i\int L_{int}dx\right), \qquad S = 1 + iT$$

$$TT^+ = -i(T - T^+) = 2\Im T$$

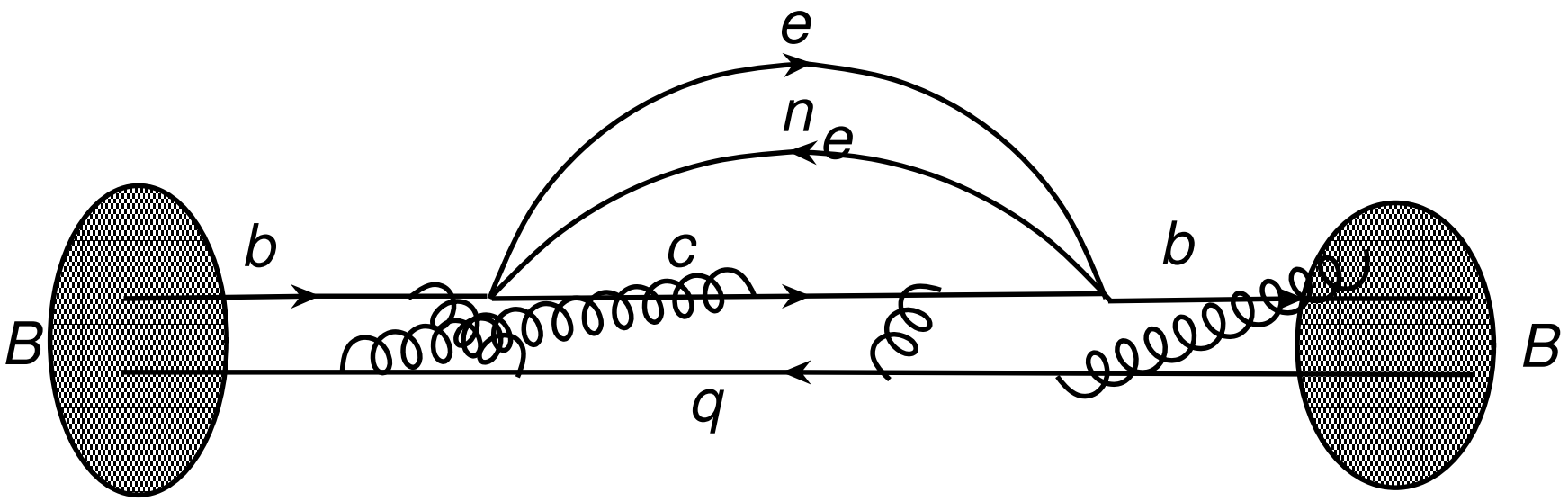
thus

$$T = \int L_{int}(x)dx + \frac{i}{2!}\int T\{L_{int}(x)L_{int}(y)\}dxdy$$

$$\left(\int L_{int}dx\right)^2 = \Im[i\int T\{L_{int}(x)L_{int}(y)\}dxdy]$$

Hadron weak decay story

The problem of dealing with final hadronic states is solved (by-passed). Initial state: forward matrix element



Theory tool - HQE

Further simplification for large m_b:
$m_b \gg \Lambda$ one can use HQE to make the dependence on m_b explicit: separation of scales m_b and Λ
Indeed, one needs

$$\mathrm{Im} \int dx \langle B(p)|iT\bar{b}(x)...\{\text{light stuff}\}...b(0)|B(p)\rangle$$

using $b(x) \to e^{imvx} h_v(x)$ ($\langle 0|b(x)|b(\vec{p})\rangle = e^{-ipx} u(p)$)
one gets large phase as in usual OPE

$$\mathrm{Im} \int dx e^{-imvx} iT\{\bar{h}_v(x)...h_v(0)\}$$

Theory tool - HQE

Theory expression is then

$$\Gamma(B \to X_c \ell \bar{\nu})/\Gamma_b^0 = |V_{cb}|^2 \left[a_0 (1 + \frac{\mu_\pi^2}{2m_b^2}) + a_2 \frac{\mu_G^2}{m_b^2} + a_3 \frac{\bar{\rho}^3}{m_b^3} + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^4}{m_b^4} \right) \right]$$

μ_π - kinetic, μ_G - chromo-magnetic, $\bar{\rho}^3$ - Darwin term
For $\Lambda_{\rm QCD} \sim 1$ GeV, $m_b \sim 5$ GeV and $\alpha_s/\pi \sim 0.1$

$$\frac{\alpha_s}{\pi} \sim \frac{\Lambda_{\rm QCD}}{m_b}$$

Theory tool - HQE - matching to HQET

$$T = i \int dx T\left[H_{\rm eff}(x) H_{\rm eff}(0)\right]$$

where $H_{\rm eff} = 2\sqrt{2} G_F V_{cb} (\bar{b}_L \gamma_\mu c_L)(\bar{\nu}_L \gamma^\mu \ell_L)$

$$\mathrm{Im}\, T / T_0 = C_0 \mathcal{O}_0 + C_v \frac{\mathcal{O}_v}{m_b} + C_\pi \frac{\mathcal{O}_\pi}{2m_b^2} + C_G \frac{\mathcal{O}_G}{2m_b^2}$$

where $T_0 = \pi \Gamma_b |V_{cb}|^2$. The local operators in the expansion are ordered by their dimensionality

$$dim\, 3: \ \mathcal{O}_0 = \bar{h}_v h_v, \quad dim\, 4: \ \mathcal{O}_v = \bar{h}_v (ivD) h_v,$$

$$dim\, 5: \mathcal{O}_\pi = \bar{h}_v (iD_\perp)^2 h_v, \quad \mathcal{O}_G = \bar{h}_v \frac{1}{2} [i\not{D}_\perp, i\not{D}_\perp] h_v$$

HQET

The modes h_v are explicitly determined by their Lagrangian

$$\mathcal{L} = \bar{h}_v(ivD)h_v + \frac{1}{2m_b}(\mathcal{O}_\pi + C_m(\mu)\mathcal{O}_G) + ...$$

$$C_m(\mu) = 1 + \frac{\alpha_s(\mu)}{2\pi}\left\{C_F + C_A\left(1 + \ln\frac{\mu}{m_b}\right)\right\}$$

being a coefficient of chromo-magnetic operator $\mathcal{O}_G$ with radiative correction.

Local operator: b-quark number

General structure of HQET matching

$$\mathrm{Im} i \int dx T \bar{b}(x) ... b(0) = C_0 \bar{h}_v h_v + C_v \frac{\mathcal{O}_v}{m_b} + C_\pi \frac{\mathcal{O}_\pi}{2m_b^2} + C_G \frac{\mathcal{O}_G}{2m_b^2}$$

It is convenient to use b-quark number operator

$$\bar{b} \not{v} b = \bar{h}_v h_v - \frac{\mathcal{O}_\pi}{2m_b^2} + \tilde{C}_G \frac{\mathcal{O}_G}{2m_b^2} + O(1/m_b^3)$$

that is absolutely normalized

$$\langle B(p) | \bar{b} \not{v} b | B(p) \rangle = 1 \times pv$$

that leads to a partonic picture

$$\Gamma = \Gamma_0 (\langle B(p) | \bar{b} \not{v} b | B(p) \rangle + ...) = \Gamma_0 (1 + ...)$$

HQE

Thus,

$$
\begin{aligned}
T/T_0 &= C_0 \left\{ \bar{b}\not{v} b - \frac{\mathcal{O}_\pi}{2m_b^2} \right\} \\
&\quad + \left\{ -C_v C_m + C_G - \tilde{C}_G C_0 \right\} \frac{\mathcal{O}_G}{2m_b^2}
\end{aligned}
$$

$C_m(\mu)\mathcal{O}_G(\mu)$ is RG invariant (μ independent) and $\langle B(p_B)|C_m(\mu)\mathcal{O}_G|B(p_B)\rangle$ generates mass splitting between the pseudo scalar and vector mesons

$$
m_{B^*}^2 - m_B^2 = \frac{4}{3} C_m(\mu) \langle B(p_B)|\mathcal{O}_G(\mu)|B(p_B)\rangle
$$

Experiment: $m_{B^*}^2 - m_B^2 = \Delta m_B^2 = 0.49\ \mathrm{GeV}^2$

Width representation

The final representation is

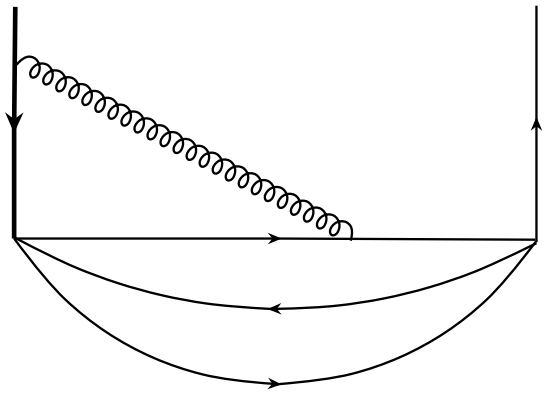
$$\Gamma(B \to X_c \nu \ell) = \Gamma_b |V_{cb}|^2 \left\{ C_0 \left(1 + \frac{\mu_\pi^2}{2m_b^2} \right) + \left(-C_v + \frac{C_G - \tilde{C}_G C_0}{C_m} \right) \frac{3\Delta m_B^2}{8m_b^2} \right\}$$

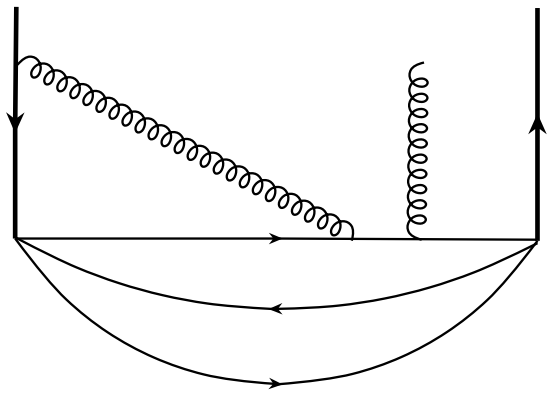
To compare with

$$\begin{aligned} T/T_0 &= C_0 \left\{ \bar{b} \not{v} b - \frac{\mathcal{O}_\pi}{2m_b^2} \right\} \\ &\quad + \left\{ -C_v C_m + C_G - \tilde{C}_G C_0 \right\} \frac{\mathcal{O}_G}{2m_b^2} \end{aligned}$$

Technology – 3-loop integrals

To find coeffs C_0, C_v, C_π, C_G one takes ME between PT states. It suffices to compute ME between $\langle b(m_b v) g(l) | ... | b(mv) \rangle$ quarks on-shell and/or gluons that reduces to 3loop on-shell integrals.





We used: dim.reg.(*'t Hooft*), REDUCE(*A.C.Hearn*), Mathematica(*Wolfram*), FeynCalc(*feyncalc.org*), IBP reduction(*K.G.Chetyrkin, F.V.Tkachov*), packages "LiteRed"(*R.N.Lee*), "HypExp"(*T.Huber and D.Maitre*)..

Results

By using these methods we have

$$C_0 = 1 + \Delta_0^{(0)}(\rho) + C_F \frac{\alpha_s}{\pi}\{\left(\frac{25}{8} - \frac{\pi^2}{2}\right) + \Delta_0^{(1)}(\rho)\}$$

$\Delta_0^{(0)}(\rho)$, $\Delta_0^{(1)}(\rho)$ are corrections due to c-quark mass known analytically, $\rho = m_c^2/m_b^2$, $\Delta_0^{(0)}(0) = \Delta_0^{(1)}(0) = 0$. Coefficient C_v (operator $\bar{h}_v ivDh_v$) reads

$$C_v = 5 + C_F \frac{\alpha_s}{\pi}\left\{-\frac{25}{24} - \frac{\pi^2}{2}\right\}$$

No μ dependence. No C_A color structure - can be obtained using small momentum expansion near the quark mass shell, $p = mv(1 + kv)$ (gauge inv).

Final number

Final combination

$$C_{fin} = -C_v + (C_G - \tilde{C}_G C_0)/C_m$$

reads

$$C_{fin} = -3 + \Delta_G^{(0)}(m_c) + \frac{\alpha_s}{\pi}\Delta_G^{(1)}(m_c)$$
$$+\frac{\alpha_s}{\pi}\left\{C_A\left(\frac{31}{18} - \frac{\pi^2}{9}\right) + C_F\left(\frac{43}{144} - \frac{19\pi^2}{36}\right)\right\}$$

$\Delta_G^{(0)}(\rho)$ is known analytically while $\Delta_G^{(1)}(\rho)$ enters the numerical computation (*Gambino et al.*)
Here $\Delta_G^{(0)}(0) = \Delta_G^{(1)}(0) = 0$

Numerics

$$C_{fin} = -3 + \frac{\alpha_s}{\pi}(0.63 C_A - 4.91 C_F)$$
$$= -3 + \frac{\alpha_s}{\pi}(-4.67) = -3(1 + 1.56\frac{\alpha_s}{\pi})$$

Impact of correction.

Accounting for c-quark mass at tree approximation

$\Delta_0^{(0)}(\rho) = -8\rho - 12\rho^2 \ln \rho + 8\rho^3 - \rho^4$

$$V_{cb}^{new} = V_{cb}^{old}(1 + 4.67\frac{\alpha_s}{\pi}\frac{3\Delta m_B^2}{8m_b^2}\frac{1}{2(1+\Delta_0^{(0)}(\rho))})$$

for $m_c = 1.25\ \mathrm{GeV}$, $m_b = 4.6\ \mathrm{GeV}$

$$|V_{new}/V_{old}| = 1 + 0.004$$

with $\alpha_s/\pi = 0.1$

m_c corrections

c-quark mass corrections
$\Delta_G^{(0)}(\rho) = 8\rho + ...$ and $\Delta_G^{(1)}(\rho) = \rho(A + 32\ln(\mu/m_b)) + ...$
For $m_c(3\ \mathrm{GeV}) = 0.986(10)\mathrm{GeV}$,$m_b = 4.8\ \mathrm{GeV}$ one finds $\rho = 0.04$ and

$$C_{fin} = -3 + \frac{\alpha_s}{\pi}(-4.67 + 0.04(A - 15.0))$$

For $|A| \leq 50$ the massless approximation dominates the corrections though the sign of the constant term can be important.

Phenomenology

For $b \to u$, or $c \to d$ our result is exact $m_u = 0$, $m_d = 0$

$|V_{ub}| = (4.41 \pm 0.15 \pm 0.16) \times 10^{-3}$ (inclusive)
$|V_{ub}| = (3.23 \pm 0.31) \times 10^{-3}$ (exclusive)

Correction to μ_G is well below the errors. The very fact that it is known and under control is important though.

Inclusive semileptonic B-meson decays to charm

$|V_{cb}| = (42.4 \pm 0.9) \times 10^{-3}$ from inclusive
$|V_{cb}| = (39.5 \pm 0.8) \times 10^{-3}$ from exclusive ones.

The results are only marginally consistent and the accurate theoretical formulas are of crucial importance

Shift is $\Delta|V_{cb}| = 0.17 \times 10^{-3}$ (42.4+0.17)

Outlook

- More accurate analysis of phenomenology
- Moments of differential distributions $\frac{d\Gamma}{d(qv)}$, $\frac{d\Gamma}{d(q^2)}$,...
- m_c corrections