Inflation and reheating in the Starobinsky model with conformal higgs field

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Abstract

This is a talk presented by A.A. Tokareva at Quarks-2012. We studied the reheating after the Starobinsky inflation and have found that the main process is the inflaton decay to SM gauge fields due to the conformal anomaly. The reheating temperature is low leading to the possibility to detect the gravity wave signal from inflation and evaporation of structures formed after inflation in DECIGO and BBO experiments. Also we give predictions for the parameters of scalar perturbation spectrum at the next-to-leading order of slow roll and obtain a bound on the Higgs mass.

1 Starobinsky model

Starobinsky inflation is one of the minimal models which naturally explains inflationary stage and reheating exploiting only gravity. The action of the Starobinsky model in the Jordan frame is [4],[1]

$$S = -\frac{M_P^2}{2}\int\sqrt{-g}\,d^4x\,\left(R - \frac{R^2}{6\,\mu^2}\right) + S_{matter}. \tag{1}$$

Here $M_P = M_{Pl}/\sqrt{8\pi} = 2.4\times10^{18}$ GeV, S_{matter} means the Standard Model action. This model allows an inflationary stage in a slow roll regime that can provide a flat power spectrum of perturbations. An additional scalar degree of freedom (scalaron) plays a role of inflaton. A parameter μ is fixed by the normalization of scalar perturbation amplitude:

$$\mu = 1.3\times10^{-5}\,M_P. \tag{2}$$

After inflation the Universe reheats via the scalaron decay to the SM Higgs bosons to the temperature of

$$T_{reh} = 3.1\times10^9\ \text{GeV}. \tag{3}$$

We consider the action with additional conformal coupling between the SM Higgs boson $\mathcal{H}$ and the scalar curvature:

$$S_{\mathcal{H}} = \int d^4x\sqrt{-g}\left(\frac{1}{6}R\,(\mathcal{H}^\dagger\mathcal{H}) + |D^\mu\mathcal{H}|^2 - \frac{\lambda}{4}(\mathcal{H}^\dagger\mathcal{H} - v^2)^2\right). \tag{4}$$

In this model scalaron-to-Higgs decay is very suppressed so the Universe reheats in another way.

After the conformal transformation to the Einstein frame $g_{\mu\nu} \to e^{\sqrt{2/3}\phi/M_P}g_{\mu\nu}$ action rewrites as [2]

$$S = \int d^4x\sqrt{-g}\left(-\frac{M_P^2}{2}R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right) + \tilde{S}_{matter}, \tag{5}$$

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$$V(\phi) = \frac{3\mu^2 M_P^2}{4}\left(1 - e^{-\sqrt{2/3}\phi/M_P}\right)^2. \tag{6}$$

Here $\tilde{S}_{matter}$ is the conformally transformed action of matter fields. We see that any conformal non-invariance in the matter sector produces coupling between scalaron ϕ and SM particles.

2 Reheating via the gauge conformal anomaly

The leading interaction comes from the gauge conformal anomaly. The Yang-Mills Lagrangian is

$$L = -\frac{1}{4g_s^2}(F^a_{\mu\nu})^2, \tag{7}$$

where g_s is a gauge coupling. It's small conformal transformation (when $\Delta g_{\mu\nu} = \sqrt{\frac{2}{3}}\frac{\phi}{M_P} g_{\mu\nu}$) leads to the interaction between scalaron ϕ and trace of energy-momentum tensor T^μ_μ:

$$L_{int} = \Delta g^{\mu\nu}\frac{\delta(\sqrt{-g}L)}{\delta g^{\mu\nu}} = \sqrt{\frac{2}{3}}\frac{\phi}{M_P} g^{\mu\nu}\frac{\delta(\sqrt{-g}L)}{\delta g^{\mu\nu}} = \frac{1}{\sqrt{6}}\frac{\phi}{M_P}\sqrt{-g}T^\mu_\mu. \tag{8}$$

In a gauge theory T^μ_μ is proportional to the beta-function [3]:

$$T^\mu_\mu = \frac{1}{2}\frac{\beta(g_s)}{g_s^3}(F^a_{\mu\nu})^2,\ \beta(g_s) = \frac{\partial g_s}{\partial(\ln\mu)} = \frac{b g_s^3}{16\pi^2}, \tag{9}$$

where b is a coefficient in beta-function which depends on a gauge group and a number of interacting fermions. It's values at one loop order are $\frac{41}{6},\ -\frac{19}{6},\ -7$ for U(1), SU(2), SU(3) gauge groups of the SM respectively.

The scalaron's decay rate is

$$\Gamma_{\phi\to 2\,\text{bosons}} = \frac{b^2\,\alpha^2 N_{adj}}{768\,\pi^3}\frac{\mu^3}{M_p^2}. \tag{10}$$

Here N_{adj} is the dimension of the adjoint representation of the considering gauge group. Values of $\alpha = g_s^2/(4\pi)$ obtained by extrapolating the SM up to the scale of $\mu/2$ are 0.01430, 0.02361, 0.02649 for the U(1), SU(2), SU(3) gauge groups respectively [17]. We can obtain the reheating temperature of the Universe after inflation as a temperature at the moment of equality between the scalaron condensate and the relativistic matter [4].

$$T_{reh} = 1.11 \times g_*^{-1/4}\sqrt{\Gamma M_P} = 1.38 \times 10^8\ \text{GeV}. \tag{11}$$

Here g_* has been taken equal to 106.75 as for high temperatures in SM.

3 Parameters of primordial perturbations in the next-to-leading order of slow roll

The number of e-folds which corresponds to the time when observed by WMAP mode ($k/a_0 = 0.002$ Mpc^{-1}) crosses horizon depends on the reheating temperature [7]. It is more convenient to define $\tilde{N}_e = \ln(a\,H(k)/a_e\,H_e)$ as a measure for the moment of crossing horizon [6]:

$$\tilde{N}_e = 62 - \ln\left(\frac{k}{a_0\,H_0}\right) - \ln\left(\frac{10^{16}\ \text{GeV}}{V_e^{1/4}}\right) - \frac{1}{3}\ln\left(\frac{V_e^{1/4}}{\rho_{reh}^{1/4}}\right) = \tag{12}$$

$$= 53.80 - \frac{1}{3}\ln\left(\frac{1.38\times 10^8\ \text{GeV}}{T_{reh}}\right). \tag{13}$$

Here we define a moment when $\ddot{a} = 0$ as the end of inflation. We numerically obtained that this happens when $\chi_e \equiv \exp(\sqrt{2/3}\,\phi_e/M_P) = 4.63$, $V_e = V(\phi_e)$. In order to obtain the spectral index we need to go beyond the slow roll approximation. Deriving the slow roll parameters through the $\tilde{N}_e$ we get ($N \equiv 4\tilde{N}_e/3 + \chi_e - 1$):

$$\epsilon = \frac{4}{3}\frac{1}{N^2} + O\left(\frac{\ln(N)}{N^3}\right), \ \eta = -\frac{4}{3}\frac{1}{N} + \frac{4}{3}\frac{1}{N^2}, \ \zeta \equiv M_P^2\sqrt{\frac{V'V'''}{V^2}} = \frac{4}{3}\frac{1}{N}. \tag{14}$$

The spectral index in the next-to-leading order is given by [5, 6]

$$1 - n_s = 6\epsilon - 2\eta - \frac{2}{3}\eta^2 + 0.374\zeta^2 = \frac{8}{3}\frac{1}{N} + \frac{4.813}{N^2} + O(\frac{\ln(N)}{N^3}), \tag{15}$$

$$r = 16\epsilon = \frac{64}{3}\frac{1}{N^2} + O\left(\frac{\ln(N)}{N^3}\right), \tag{16}$$

$$n_T = -2\epsilon = -\frac{8}{3}\frac{1}{N^2} + O\left(\frac{\ln(N)}{N^3}\right). \tag{17}$$

The numerical values are:

$$n_s = 0.9638 \pm 0.00001, \quad r = 0.0038 \pm 0.00001, \quad n_T = -0.00047 \pm 0.00001. \tag{18}$$

The error of such an approximation is of order $\frac{\ln(N)}{N^3} = 10^{-5}$. The numerical coefficient in this term is expected to be of order 1. These values of n_s and r are in the center of allowed by WMAP data region.

4 Gravity wave signal

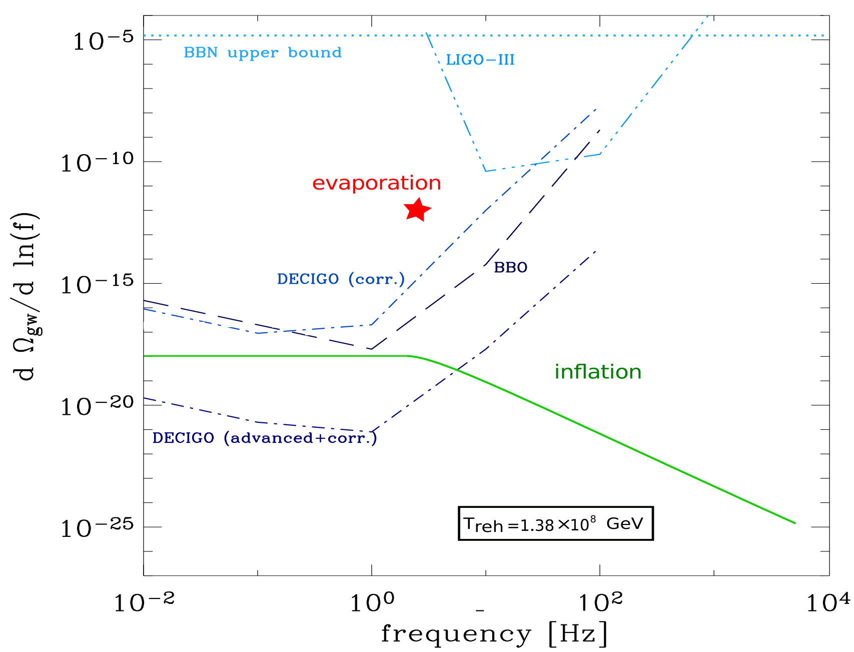

Figure 1: Energy density in gravity waves (in units of the present day critical density) as a function of frequency and the projected sensitivities of next generation gravitational wave detectors: DECIGO [10], BBO [11], LIGO [12]. The picture shows the gravity wave signal from inflation (orange line) and from structure evaporation at the moment of reheating (red star).

The long matter-dominated (MD) stage after inflation leads to the falling ($1/f^2$) spectrum of gravity waves for $f > f^*$. The reason is that sub-horizon modes fall with the scale factor as $1/a^4$ so at the MD stage their impact to the full energy density decreases as $1/a$. The frequency f^* where the amplitude of tensor perturbations starts falling corresponds to the Hubble parameter at the moment of reheating and depends on the reheating temperature T_{reh}:

$$f_* = 2.8\,\mathrm{Hz}\left(\frac{T_{reh}}{1.38\cdot 10^8\ \mathrm{GeV}}\right) \tag{19}$$

The signal and opportunities of the future experiments are shown in Fig. 1.

Another signal could be expected from the process of the structure evaporation at the moment of scalaron decay. Non-equilibrium process leads to the appearance of transverse and traceless part of the energy-momentum tensor which gives rise to a gravity wave signal. The typical frequency also corresponds to the redshifted H_{reh} and is close to f^*.

The luminosity does not depend on the reheating temperature [9]. The estimation in [8] gives a number $\Omega_{gw} \sim 4\times 10^{-13}\,\varepsilon$, where $\varepsilon < 1$ is an efficiency factor which represents a measure of the aspherisity of structure evaporation.

5 Possible dangers for Higgs potential

The effective potential of the Higgs field h for $h \gg v = 246.2$ GeV can be written as [13]

$$V(h) = \frac{\lambda(h)}{4}h^4 - \frac{1}{12}R\,h^2. \tag{20}$$

Here $\lambda(h)$ represents the solution of the SM renormgroup equations [16]. At the inflationary stage $R = -12\,H^2 - 6\,\dot{H}$. Note that a coefficient $1/12$ in the $R\,\mathcal{H}^\dagger\mathcal{H}$ term does not run (without graviton loops). Then

$$V(h) = \frac{\lambda(h)}{4}h^4 + \left(H^2 + \frac{\dot{H}}{2}\right)\,h^2. \tag{21}$$

If the Higgs mass is too small the potential is metastable because λ runs to negative values at large h. The condition when the self-coupling never reachs negative values leads to a bound on Higgs mass. Three loop renormgroup equations [16, 15] give a value needed for absolute stability up to the Planck energy scale:

$$M_{\min} = \left[128.95 + \frac{M_t - 172.9\ \mathrm{GeV}}{1.1\ \mathrm{GeV}} \times 2.2 - \frac{\alpha_s - 0.1184}{0.0007} \times 0.56\right]\mathrm{GeV}. \tag{22}$$

Here $M_t = 173.2 \pm 0.9$ GeV is top quark mass [18].

If our vacuum is metastable we need to check that in our model after all the cosmological evolution we find the Higgs field in the electroweak vacuum. Possible danger for conformal Higgs comes from the instability of it's potential during the short time after the inflation and before the reheating when the curvature is negative. We supposed that the initial value of the Higgs field is zero and following [14] estimated the maximal quantum fluctuation during inflation:

$$\sqrt{\langle h^2\rangle_{max}} = \frac{\sqrt{3}}{4\pi}\,H \tag{23}$$

We numerically calculated the classical evolution of this fluctuation after inflation. After any critical value of Higgs mass such a fluctuation rolls out to the wrong vacuum at post inflationary stage. In order to describe the situation one can write an equation on the Higgs field:

$$\ddot{h} + 3H\,\dot{h} + \left(2H^2 + \dot{H} + \lambda(h)\,h^2\right)\,h = 0. \tag{24}$$

At the scalaron-dominated stage

$$2H^2 + \dot{H} \simeq \frac{2}{9t^2}\left(1 - 3\cos(2\mu t)\right). \tag{25}$$

At small times the term $\lambda(h)\, h^2$ is negligible and the field falls as $h \sim t^{-1/3} \sim 1/\sqrt{a}$. But the moment when the potential starts to dominate exists ($\lambda\, h^2 \sim t^{-2/3}$ falls slower then $2/9t^2$), and if the corresponding value of h lies behind the maximum of the effective potential (with negative $R\, h^2$-term) then it rolls down to wrong minimum.

The critical value obtained by using 3-loop RG equations [15, 17] is

$$M_{throw} = \left[126.2 + \frac{M_t - 173.2\ \text{GeV}}{0.9\ \text{GeV}} \times 1.55 - \frac{\alpha_s - 0.1184}{0.0007} \times 0.3\right] \text{GeV}. \tag{26}$$

If the Higgs mass is larger than this value we can be sure that the Higgs field wouldn't be thrown from the EW vacuum in a process of evolution of the quantum fluctuations after inflation.

Conclusions

We studied the reheating after the Starobinsky inflation in case of Higgs conformally coupled to gravity and obtained that the leading process was the inflaton decay to SM gauge fields due to the conformal anomaly. The reheating temperature is lower than in the case of minimal Higgs coupling to gravity leading to an attractive possibility to detect the gravity wave signal from inflation and structure evaporation in DECIGO experiment. Also we obtained predictions for the parameters of scalar perturbation spectrum at the next-to-leading order of slow roll to be tested with the future Plank and CMBPol data.

The model is not valid for too light Higgs boson because of the instability at the postinflationary matter dominated stage. The critical value is in the allowed by LHC region so this model is not closed by the last LHC data.

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