

Leptogenesis via hypermagnetic fields and baryon asymmetry

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Abstract

We study lepton asymmetry evolution in plasma of the early Universe before the electroweak phase transition (EWPT) accounting for chirality flip processes via inverse Higgs decays entering equilibrium at temperatures below $T_{\text{RL}} \simeq 10$ TeV, $T_{\text{EW}} < T < T_{\text{RL}}$. We solve appropriate kinetic equations for leptons taking into account the lepton number violation due to Abelian anomalies for right- and left electrons (neutrinos) in the self-consistent hypercharge field. This field obeys Maxwell equations modified in Standard Model of electroweak interactions due to parity violation. Assuming the Chern-Simons (CS) wave configuration of the seed hypercharge field, we get the estimates of baryon and lepton asymmetries evolved from the primordial right electron asymmetry existing alone as partial asymmetry at $T \geq T_{\text{RL}}$. One finds a strong dependence of the asymmetries on the CS wave number.

1 Introduction. Two scenarios of leptogenesis in hypermagnetic fields

The nature of the initial fields that seed subsequent dynamo for observed galactic magnetic fields is largely unknown [1, 2]. It might be that seed fields are produced during the epoch of the galaxy formation, or ejected by first supernovae or active galactic nuclei. Alternatively to this astrophysical scenario the seed fields might originate from much earlier epochs of the Universe expansion, down to the cosmological inflation phase transition epoch [3]. There are first observational indications of the presence of cosmological magnetic fields (CMF) in the inter-galactic medium which may survive even till the present epoch [4, 5].

Note that Maxwellian CMF might arise during electroweak phase transition (EWPT) from massless (long-range) hypercharge fields Y_μ existing in primordial plasma before EWPT. Indeed, it is well known that in the Standard Model (SM), at the high-temperature symmetric phase of universe expansion all gauge bosons acquire a "magnetic" mass gap $\sim g^2 T$, except for the Abelian gauge field associated to weak hypercharge. Such massless hypercharge field Y_μ in hot plasma occurs a progenitor of the Maxwellian field which evolves after EWPT. The hypercharge field itself may arise from phase transitions in the very early universe, before EWPT, such as during the inflationary epoch.

In the absence of hypermagnetic fields *the baryon asymmetry of universe (BAU) can be produced through leptogenesis*. In particular, such case was considered in [6] assuming that an initial BAU, say, a negligible value in our case, is preserved being stored in right electrons e_R . Then violation of the lepton number due to Abelian anomaly in a strong external hypercharge field provides the growth of lepton number from that small value, meanwhile, lepton- and baryon number evolution proceeds preserving $B - L = \text{const}$. Thereby accounting for hypermagnetic

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fields one can enhance BAU too. This allows to assert [7] that fermion number "sits" in hypermagnetic field.

One can easily understand why authors [6] considered the scenario of BAU generation with one lepton generation chosen as e_R . They found that the tiny Yukawa coupling to the Higgs for right electrons, $h_e = \sqrt{2}m_e/v$, provides for such leptons the latest entering the equilibrium with left particles through Higgs decays and inverse decays. This means that any primordially-generated lepton number that occurred as e_R may not be transformed into e_L soon enough switching on sphaleron interactions which wipe out the remaining BAU.

On the other hand, in the presence of a nonzero right electron chemical potential, $\mu_{eR} \neq 0$, there arises the Chern-Simons (CS) term $\sim \mu_{eR} \mathbf{B}_Y \mathbf{Y}$ in the effective Lagrangian for the hypercharge field Y_μ [7, 8, 9] that modifies Maxwell equation in SM with parity violation producing additional pseudovector current $\mathbf{J}_5 \sim \mu_{eR} \mathbf{B}_Y$ in plasma. Namely this current leads to the important α_Y -helicity parameter in modified Faraday equation,

$$\alpha_Y(T) = \frac{g'^2 \mu_{eR}(T)}{4\pi^2 \sigma_{\text{cond}}(T)}, \quad (1)$$

that is scalar instead of the pseudoscalar $\alpha_{MHD} \sim - \langle \mathbf{v} \cdot (\nabla \times \mathbf{v}) \rangle / 3$ in standard MHD originated by vortices in plasma. Obviously in isotropic early universe such vortices are absent, at least, at large scales we consider here. In Eq. (1) $\sigma_{\text{cond}} = 100T$ is the relativistic plasma conductivity; g' is the $U_Y(1)$ gauge coupling.

The dynamo amplification of a large-scale hypermagnetic field [10, 11] for changing chemical potential $\mu_{eR}(t)$ as well as its growth $\partial_t \mu_{eR} > 0$ due to the Abelian anomaly in the self-consistent hypermagnetic field \mathbf{B}_Y were never explored before in literature due to the difficulty to solve the corresponding non-linear integro-differential equations. In the recent work [12] we have solved that problem in the two scenarios.

In the first one we followed scenario with the single partial chemical potential $\mu_{eR}(t) \neq 0$ which evolves during the universe cooling down to the EWPT temperature $T_{EW} \simeq 100 \text{ GeV}$ accounting both the Abelian anomaly for right electrons in the self-consistent hypermagnetic field (see below Eq. (3)) and inverse Higgs decays. This approach differs for BAU estimates from the case based on the adiabatic approximation $\partial_t \mu_{eR} = 0$ adopted in the similar scenario in [7].

For temperatures $T < T_{RL} \simeq 10 \text{ TeV}$ chirality flip reactions enter equilibrium since the rate of chirality flip processes, $\Gamma_{RL} \sim T$, becomes faster than the Hubble expansion $H \sim T^2$, $\Gamma_{RL} > H$. This motivates us to consider in the second scenario the extended equilibrium state at $T < T_{RL}$ when left leptons enter equilibrium with e_R through inverse Higgs decays and acquire non-zero asymmetries $\sim \mu_{eL}(T) \equiv \mu_{\nu_{eL}}(T) \neq 0$.

The left lepton asymmetries can grow from a negligible (even zero) value at $T < T_{RL}$ due to the corresponding Abelian anomaly which has the opposite sign relatively to the sign of anomaly for e_R and because left leptons have different coupling constant $g' Y_L / 2$ with hypercharge field. Such a difference guarantees the presence of leptogenesis in hypermagnetic fields even below T_{RL} all the way down to T_{EW} , hence it supports generation of the BAU.

Note that for $T > T_{RL}$, before left leptons enter equilibrium with right electrons, the anomaly for them was not efficient since the left electron (neutrino) asymmetry was zero, $\mu_{eL} = \mu_{\nu_{eL}} = 0$, while a non-zero primordial right electron asymmetry, $\mu_{eR} \neq 0$, kept the baryon asymmetry at the necessary level. In other words, for $T > T_{RL}$ the Abelian anomaly for left particles was present at the stochastic level, with $\langle \delta j_L^\mu \rangle = 0 = \langle \mathbf{E}_Y \mathbf{B}_Y \rangle$ valid only on large scales.

Since $\mu_{eL} \equiv \mu_{\nu_{eL}} \neq 0$ at temperatures $T < T_{RL}$ there appear additional macroscopic pseudovector currents in plasma $\mathbf{J}_5 \sim \mu_{eL} \mathbf{B}_Y$ which modify α_Y -helicity parameter governing Maxwell equations for hypermagnetic (hyperclectric) fields. Indeed, such helicity parameter

$$\alpha_Y^{\text{mod}} = \frac{g'^2 (2\mu_{eR} + \mu_{eL})}{8\pi^2 \sigma_{\text{cond}}}. \quad (2)$$

differs from (1) due to inclusion of left leptons and the difference of their gauge coupling from the right one, $g'Y_L/2$ from $g'Y_R/2$, where $Y_L = -1$ and $Y_R = -2$ are the hypercharges of the left electron (neutrino) and right electron correspondingly. This occurs due to the same polarization effect described for e_R in [9] which led to the appearance of CS term in the effective SM Lagrangian.

Note that in the first scenario five (=5) chemical potentials describe equilibrium in hot plasma before EWPT: three μ_i for the three global charges $B/3 - L_i = \text{const}$, where $i = 1, 2, 3$ enumerates generations in SM, μ_Y for the conserved hypercharge (global $\langle Y \rangle = 0$) and the single partial μ_{e_R} for right electrons e_R with the conservation of their lepton number, $\partial_\mu j_{e_R}^\mu = 0$, unless $T > T_{\text{RL}}$ [7]. Then, if one assumes the presence of large-scale hypercharge fields Y_μ in the symmetric phase the number of right electrons is not conserved because of the Abelian anomaly:¹

$$\partial_\mu j_{e_R}^\mu = \frac{g'^2 Y_R^2}{64\pi^2} Y_{\mu\nu} \tilde{Y}^{\mu\nu}, \quad (3)$$

where $Y_{\mu\nu}$ and $\tilde{Y}_{\mu\nu}$ are, respectively, the $U_Y(1)$ hypercharge field strengths and their duals.

There are no asymmetries of left leptons and Higgs bosons in this scenario, $\mu_{e_L} = \mu_0 = 0$.

In a broadened (second) scenario with nonzero asymmetries, $\xi_{e_L} = \xi_{\nu_e^L} \neq 0$, where $\xi_a = \mu_a/T$, appropriate for the stage $T < T_{\text{RL}}$ [9, 12], we somehow violate the equilibrium described in ref. [7] by five chemical potentials for five globally conserved charges. Nevertheless, it can lead only to an additional factor of the order one, $c_\Delta \sim 1$, that describes the dependence of $n_L = (n_{e_L} - n_{\bar{e}_L}) = \xi_{e_L} T^3/6 \neq 0$ on five global charges in primordial plasma. For instance, rewriting the canonical Abelian anomaly for the left doublet $L_e^T = (\nu_e^L, e_L)$,

$$\partial_\mu j_{e_L}^\mu = -\frac{g'^2 Y_L^2}{64\pi^2} Y_{\mu\nu} \tilde{Y}^{\mu\nu}, \quad Y_L = -1, \quad (4)$$

in the form $d\xi_{e_L}/dt = -c_\Delta(6g'^2/16\pi^2 T^3)(\mathbf{E}_Y \cdot \mathbf{B}_Y)$, we put below $c_\Delta = 1$ simplifying solution of our kinetic equations for the lepton asymmetries.

2 Kinetics of leptons in hypermagnetic fields

In ref. [12] we forced for simplicity the presence of zero Higgs asymmetry, $n_{\varphi^{(0)}} - n_{\tilde{\varphi}^{(0)}} = T^2 \mu_0/3 = 0$, $\mu_0 = 0$, considering leptogenesis with the inverse decays only, $e_R \bar{e}_L \rightarrow \tilde{\varphi}^{(0)}$, $e_R \nu_e^L \rightarrow \varphi^{(-)}$, etc. In the second broadened scenario the system of kinetic equations for leptons accounting for Abelian anomalies (3) and (4) takes the form,

$$\begin{aligned} \frac{dL_{e_R}}{dt} &= \frac{g'^2}{4\pi^2 s} (\mathbf{E}_Y \cdot \mathbf{B}_Y) + 2\Gamma_{\text{RL}} \{L_{e_L} - L_{e_R}\}, \\ &\quad \text{for inverse decays } e_R \bar{e}_L \leftrightarrow \tilde{\varphi}^{(0)} \quad \text{and} \quad e_R \nu_e^L \leftrightarrow \varphi^{(-)}, \\ \frac{dL_{e_L}}{dt} &= -\frac{g'^2}{16\pi^2 s} (\mathbf{E}_Y \cdot \mathbf{B}_Y) + \Gamma_{\text{RL}} \{L_{e_R} - L_{e_L}\}, \\ &\quad \text{for } \bar{e}_R e_L \leftrightarrow \varphi^{(0)}, \quad \text{as well as} \\ \frac{dL_{\nu_e^L}}{dt} &= -\frac{g'^2}{16\pi^2 s} (\mathbf{E}_Y \cdot \mathbf{B}_Y) + \Gamma_{\text{RL}} \{L_{e_R} - L_{e_L}\}, \\ &\quad \text{for } \bar{e}_R \nu_e^L \leftrightarrow \varphi^{(+)}. \end{aligned} \quad (5)$$

Here $L_a = (n_a - n_{\bar{a}})/s$ is the lepton number, $a = e_R, e_L, \nu_e^L$, $s = 2\pi^2 g^* T^3/45$ is the entropy density, and $g^* = 106.75$ is the number of relativistic degrees of freedom. The factor=2 in the first line takes into account the equivalent reaction branches. Of course, for the left doublet

¹We use opposite sign for the Abelian anomaly comparing with the sign in ref. [7] relying on the definition of right states $\Psi_R = (1 + \gamma_5)\Psi/2$ in the book [13].

$L_{eL} = L_{\nu_e^L}$. In the scenario with the single partial chemical potential $\mu_{eR} \neq 0$ that is similar to the case studied in [7] we put $L_{eL} = 0$ considering the first kinetic equation in (5) only. From the modified Maxwell equation,

$$-\frac{\partial \mathbf{E}_Y}{\partial t} + \nabla \times \mathbf{B}_Y = \left(\mathbf{J} + \frac{g'^2 \mu_{eR}}{4\pi^2} \mathbf{B}_Y \right) = \sigma_{\text{cond}} \left[\mathbf{E}_Y + \mathbf{V} \times \mathbf{B}_Y + \alpha_Y^{\text{mod}} \mathbf{B}_Y \right], \quad (6)$$

where in the second scenario the helicity coefficient α_Y^{mod} is given by Eq. (2), omitting in MHD approach the displacement current, $\partial \mathbf{E}_Y / \partial t = 0$, one finds the hyperelectric field \mathbf{E}_Y to be substituted in kinetic equations (5),

$$\mathbf{E}_Y = -\mathbf{V} \times \mathbf{B}_Y + \frac{\nabla \times \mathbf{B}_Y}{\sigma_{\text{cond}}} - \alpha_Y^{\text{mod}} \mathbf{B}_Y. \quad (7)$$

Below we simplify Abelian anomaly contribution $\sim (\mathbf{E}_Y \cdot \mathbf{B}_Y)$, considering, as in ref. [12], the simplest configuration of hypermagnetic field – CS wave $Y_x = Y(t) \sin k_0 z$, $Y_y = Y(t) \cos k_0 z$, $Y_z = Y_0 = 0$. Substituting the hyperelectric field (7) we get the pseudoscalar $(\mathbf{E}_Y \cdot \mathbf{B}_Y)$ entering the Abelian anomaly as

$$(\mathbf{E}_Y \cdot \mathbf{B}_Y) = \frac{1}{\sigma_{\text{cond}}} (\nabla \times \mathbf{B}_Y) \cdot \mathbf{B}_Y - \alpha_Y^{\text{mod}} \mathbf{B}_Y^2 = \frac{B_Y^2}{100} \left[\frac{k_0}{T} - \frac{g'^2}{4\pi^2} \left(\xi_R + \frac{\xi_L}{2} \right) \right]. \quad (8)$$

Here we substituted $(\nabla \times \mathbf{B}_Y) \cdot \mathbf{B}_Y = k_0 B_Y^2(t)$ for the CS wave, where $B_Y(t) = k_0 Y(t)$ is the hypermagnetic field amplitude.

The rate of all inverse Higgs decay processes Γ_{RL} [6],

$$\Gamma_{\text{RL}} = 5.3 \times 10^{-3} h_e^2 \left(\frac{m_0}{T} \right)^2 T = \left(\frac{\Gamma_0}{2t_{\text{EW}}} \right) \left(\frac{1-x}{\sqrt{x}} \right), \quad (9)$$

vanishes just at EWPT time, $x = 1$. Here $h_e = 2.94 \times 10^{-6}$ is the Yukawa coupling for electrons, $\Gamma_0 = 121$, variable $x = t/t_{\text{EW}} = (T_{\text{EW}}/T)^2$ is given by Friedman law, $m_0^2(T) = 2DT^2(1 - T_{\text{EW}}^2/T^2)$ is the temperature dependent effective Higgs mass at zero momentum and zero Higgs vev. The coefficient $2D \approx 0.377$ for $m_0^2(T)$ is given by the known masses of gauge bosons m_Z , m_W , the top quark mass m_t and a still problematic zero-temperature Higgs mass estimated as $m_H \sim 125 \text{ GeV}$ [14].

Let us rewrite eqs. (5) using the asymmetries, $L_{eR} = \xi_{eR} T^3 / 6s$, $L_{eL} = \xi_{eL} T^3 / 6s$, as

$$\begin{aligned} \frac{d\xi_{eR}}{dt} &= \frac{6g'^2}{4\pi^2 T^3} \mathbf{E}_Y \cdot \mathbf{B}_Y + 2\Gamma_{\text{RL}} (-\xi_{eR} + \xi_{eL}), \\ \frac{d\xi_{eL}}{dt} &= -\frac{6g'^2}{16\pi^2 T^3} \mathbf{E}_Y \cdot \mathbf{B}_Y + \Gamma_{\text{RL}} (\xi_{eR} - \xi_{eL}), \\ \frac{d\xi_{\nu_e^L}}{dt} &= -\frac{6g'^2}{16\pi^2 T^3} \mathbf{E}_Y \cdot \mathbf{B}_Y + \Gamma_{\text{RL}} (\xi_{eR} - \xi_{eL}). \end{aligned} \quad (10)$$

The third equation for neutrinos is excess since $\xi_{\nu_e^L} = \xi_{eL}$. Thus, we have two equations for the two lepton asymmetries ξ_{eL} , ξ_{eR} .

Using the notations $y_R(x) = 10^4 \xi_{eR}(x)$, $y_L(x) = 10^4 \xi_{eL}(x)$, and $y_0(x) = 10^4 \xi_0(x)$, as well as accounting for eq. (8), the system (10) can be rewritten in the form (without contribution of

neutrinos which is identical to that for left electrons):

$$\begin{aligned}
\frac{dy_R}{dx} &= \left[B_0 x^{1/2} - A_0 \left(y_R + \frac{y_L}{2} \right) \right] \left(\frac{B_Y^{(0)}}{10^{20} \text{ G}} \right)^2 x^{3/2} e^{\varphi(x)} - \\
&\quad - \Gamma_0 \frac{(1-x)}{\sqrt{x}} (y_R - y_L), \\
\frac{dy_L}{dx} &= -\frac{1}{4} \left[B_0 x^{1/2} - A_0 \left(y_R + \frac{y_L}{2} \right) \right] \left(\frac{B_Y^{(0)}}{10^{20} \text{ G}} \right)^2 x^{3/2} e^{\varphi(x)} - \\
&\quad - \Gamma_0 \frac{(1-x)}{2\sqrt{x}} (y_L - y_R).
\end{aligned} \tag{11}$$

Here

$$B_0 = 6.4 \left(\frac{k_0}{10^{-7} T_{\text{EW}}} \right), \quad A_0 = 19.4, \tag{12}$$

are constants chosen for hypermagnetic fields normalized. The function $e^{\varphi(x)}$ is given by the hypermagnetic field squared,

$$e^{\varphi(x)} = \left[\frac{B_Y(x)}{B_Y^{(0)}} \right]^2. \tag{13}$$

We also substituted the hypermagnetic field, $B_Y(t) = k_0 Y(t)$, found as the solution of modified Faraday equation [10, 11] for the CS wave, ²

$$\begin{aligned}
B_Y(t) &= B_Y^{(0)} \exp \left\{ \int_{t_0}^t \left[\alpha_Y^{\text{mod}}(t') k_0 - \frac{k_0^2}{\sigma_{\text{cond}}(t')} \right] dt' \right\} = \\
&= B_Y^{(0)} \exp \left\{ 3.5 \left(\frac{k_0}{10^{-7} T_{\text{EW}}} \right) \right. \\
&\quad \left. \times \int_{x_0}^x \left[\frac{(y_R + y_L/2)}{\pi} - 0.1 \left(\frac{k_0}{10^{-7} T_{\text{EW}}} \right) \sqrt{x'} \right] dx' \right\}.
\end{aligned} \tag{14}$$

We choose initial conditions at $x_0 = 10^{-4}$, or at $T_0 = T_{\text{RL}} \simeq 10 \text{ TeV}$, when inverse Higgs decay becomes faster than the Hubble expansion, $\Gamma_{\text{RL}} > H$,

$$y_R(x_0) = 10^{-6}, \quad y_L(x_0) = y_0(x_0) = 0. \tag{15}$$

Such conditions correspond to the right electron asymmetry $\xi_{e_R}(x_0) = 10^{-10}$ chosen at the level of baryon asymmetry.

One can see from eqs. (14) that dynamo amplification occurs negligible even for the maximum wave number $k_0 = 10^{-7} T_{\text{EW}}$ for which hypermagnetic field survives against ohmic dissipation.

The solution of the system (11) obtained in paper [12] is presented here in Fig. 1 and Fig. 2 (as given in Erratum to [12]) for the two scenarios above: a) a growth of right electron asymmetry $y_R(t)$ shown in Fig. 1a when $y_L = 0$ all the way down to T_{EW} , and b) when both asymmetries evolve, $y_R(t) \neq 0$, $y_L(t) \neq 0$ as plotted in Fig2a,b, with the same initial condition (15) in both cases.

²In numerical estimates we substitute either the parameter $k_0/(10^{-7} T_{\text{EW}}) = 1$ that is the upper limit for the CS wave number, $k_0 \leq 10^{-7} T_{\text{EW}}$, to avoid ohmic dissipation of hypermagnetic field, or $k_0/(10^{-7} T_{\text{EW}}) = 10^{-4}$ to get observable baryon asymmetry $B = 0.87 \times 10^{-10}$ at the EWPT time $x = 1$. Dynamo amplification is negligible in both cases.

3 Baryon asymmetry evolution in hypermagnetic field

One can see from kinetic eqs. (5) that in the absence of hypercharge fields the total lepton number is conserved, $dL_e/dt = \dot{L}_{e_R} + \dot{L}_{e_L} + \dot{L}_{\nu_e^L} = 0$. The baryogenesis arises through the leptogenesis due to the conservation law $B/3 - L_e = \text{const}$, where $B = (n_B - n_{\bar{B}})/s$ is denoted in Figs. 1,2 as $B(t) = \eta_B(t)$. Accounting for Abelian anomalies in system (5) such baryogenesis occurs possible, $\dot{B} \neq 0$, since in the presence of hypermagnetic fields $dL_e/dt \neq 0$ [12].

Three global charges are conserved ($\delta_i = \text{const}$):

$$\frac{B}{3} - L_e = \delta_1, \quad \frac{B}{3} - L_\mu = \delta_2, \quad \frac{B}{3} - L_\tau = \delta_3, \quad (16)$$

as well as $L_{e_R} = \delta_R$ well above T_{RL} , $T \gg T_{RL}$. If the initial BAU differs from zero, $B(t_0) \neq 0$, and if we assume the absence of lepton asymmetries for the second and third generations all the way down to T_{EW} , $L_\mu = L_\tau = 0$, we get that the relation, $\delta_2 = \delta_3 = B(x_0)/3$, is valid only for the initial time. From the first conservation law in eq. (16) one finds the change of BAU, $B(t)$, at temperatures $T < T_{RL}$ due to chirality flip processes and the presence of hypermagnetic fields inducing Abelian anomalies hence changing lepton number $L_e(t)$. This change obeys the relations,

$$\frac{B(t)}{3} - L_e(t) = \frac{B(t_0)}{3} - L_{e_R}(t_0) = \delta_{2,3} - \delta_R = \delta_1.$$

If for simplicity we assume the zero initial BAU, $B(t_0) = 0$, or $\delta_{2,3} = 0$, then finally we get the conservation law $B(t)/3 - L_e(t) = -L_{e_R}(t)$.

Thus, in the second scenario when both lepton asymmetries $L_{e_L}(t) \equiv L_{\nu_e^L}(t)$, $L_{e_R}(t)$ evolve as given by kinetic equations (5), BAU sits in hypercharge fields, as follows from the sum of kinetic eqs. (5):

$$B(t) = 3 \int_{t_0}^t \left[\frac{dL_{e_R}(t')}{dt'} + \frac{dL_{e_L}(t')}{dt'} + \frac{dL_{\nu_e^L}(t')}{dt'} \right] dt' = \left(\frac{3g'^2}{8\pi^2} \right) \int_{t_0}^t (\mathbf{E}_Y \cdot \mathbf{B}_Y) dt'. \quad (17)$$

Using the first equation in the system (11), where hypermagnetic term comes from the Abelian anomaly $\sim (\mathbf{E}_Y \cdot \mathbf{B}_Y)$, one obtains from eq. (17) the baryon asymmetry in the following form:

$$B(x) = 5.34 \times 10^{-7} \int_{x_0}^x dx' \left\{ \frac{dy_R(x')}{dx'} + \Gamma_0 \frac{(1-x')}{\sqrt{x'}} [y_R(x') - y_L(x')] \right\}. \quad (18)$$

In the first scenario when $y_L(t) = 0$ all the way down to T_{EW} we get BAU too big for the shortest CS wave length surviving ohmic losses, $k_0^{-1} = 10^7/T_{EW}$, while for larger scales, e.g. for $k_0^{-1} = 10^{11}/T_{EW}$, one gets its desirable value at EWPT, $B(t_{EW}) = \eta_B(t_{EW}) \sim 10^{-10}$.

4 Analytic estimates of growing lepton asymmetry

Let us explain qualitatively the growth of the lepton asymmetries shown in Figs. 1,2. One can simplify the kinetic equations for ξ_{e_R} and ξ_{e_L} in the system (10) decoupling them. We also omit the asymmetry of left leptons, $\xi_{e_L} = 0$, in the first line of eq. (10), and the right electron one in the second line of eq. (10), $\xi_{e_R} = 0$. For example, from the first equation in the system (10), substituting the pseudoscalar value $(\mathbf{E}_Y \cdot \mathbf{B}_Y)$ for the CS wave from eq. (8), one gets the simple differential equation for the right electron asymmetry $y_R = 10^4 \xi_{e_R}$,

$$\frac{dy_R}{dt} + (\Gamma + \Gamma_B)y_R = Q, \quad (19)$$

where $\Gamma = 2\Gamma_{RL}$ is the chirality flip rate, $\Gamma_B = 6(g'^2/4\pi^2)^2 B_Y^2/100T^3$, and $Q = 6 \times 10^4 \times g'^2 B_Y^2 k_0/400\pi^2 T^4$ come from the second (helicity) term in eq. (8) and from the first (diffusion)

term in the same eq. (8). The solution of eq. (19) obtained for strong and constant hypermagnetic fields, $\Gamma_B \gg \Gamma$ and $\mathbf{B}_Y^2 \approx \text{const}$,

$$y_R(t) = \left[y_R(t_0) - \frac{Q}{\Gamma + \Gamma_B} \right] e^{-(\Gamma + \Gamma_B)(t-t_0)} + \frac{Q}{\Gamma + \Gamma_B}, \quad (20)$$

gives the asymptotic growth of $y_R(t)$ up to $y_R(t_{EW})$ (here for its initial value, $\xi_{eR}(t_0) = 0$),

$$y_R(t_{EW}) = \frac{Q}{\Gamma_B} \left[1 - e^{-\Gamma_B(t_{EW}-t_0)} \right] = \frac{Q}{\Gamma_B} \approx \left(\frac{4\pi^2}{g'^2} \right) \left(\frac{k_0}{T_{EW}} \right) = 0.32.$$

Here we put $\Gamma_B t_{EW} \gg 1$ for strong fields as well as substituted $g'^2 = e^2 / \cos^2 \theta_W = 0.12$ and $k_0 / T_{EW} = 10^{-7}$ for the case of $B_0 = 6.4$. Note that this value ($y_R(t_{EW}) = 0.32$) does not depend on the amplitude of hypermagnetic field $B_Y(t)$ and occurs close to the numerical result shown in Fig. 1a which stems from the first kinetic equation (11) in the case $y_L = 0$. This happens owing to the real constancy $B_Y(t) \approx \text{const}$ for growing $y_R(t)$ in the self-consistent dynamo formula (14).

5 Discussion

In our calculations of BAU and lepton number evolution with the use of the CS wave as the simplest 1-D configuration of hypermagnetic field we found a sharp dependence of $\eta_B(t)$ and $y_{R,L}(t)$ on the wave number $k_0 \leq 10^{-7}T$. The bigger the wave number (maximum $k_0 = 10^{-7}T_{EW}$) the larger both lepton asymmetry and the baryon one evolve at T_{EW} (see e.g. Fig 1b and Fig 2c). In order to get the observable BAU value, $\eta_B(t_{EW}) \sim 10^{-10}$, we should assume much larger scales of hypermagnetic field, $k_0^{-1} \sim 10^{12}/T_{EW}$ in the case $y_L = 0$ seen in Fig. 1c and $k_0^{-1} \sim 10^{11}/T_{EW}$ in Fig. 2d when both asymmetries $y_{R,L}(t)$ evolve. These CS large-scale waves shurely survive against hypermagnetic field diffusion. On the other hand, while our choice of the CS wave simplifies analyses of lepton kinetics in magnetized plasma such hypermagnetic configuration is not the best one from the viewpoint of passing through EWPT of the first order. In paper [15] authors showed that any CS wave existing in symmetric phase of primeval plasma and having a nonzero helicity does not penetrate the wall of a bubble of the new (broken) phase during EWPT. As a result helicity of hypermagnetic field is not conserved. Meantime the helicity of the 3-D hypermagnetic field is conserved transforming into the magnetic helicity at EWPT [15]. The evolution of hypermagnetic helicity as an important characteristic of helical field was studied in paper [16] neglecting diffusion of hypermagnetic field all the way down to EWPT. Passing EWPT we used then at $T < T_{EW}$ the dynamo model for CMF amplification developed in [17]. The corresponding magnetic helicity evolution was studied in [18]. We think that a more correct calculation of the helicity transport accounting for hypermagnetic (magnetic) field diffusion could improve calculations of BAU and lepton asymmetries including estimates of the magnetic chiral anomaly driven by the asymmetry difference $y_R(t_{EW}) - y_L(t_{EW}) \neq 0$.

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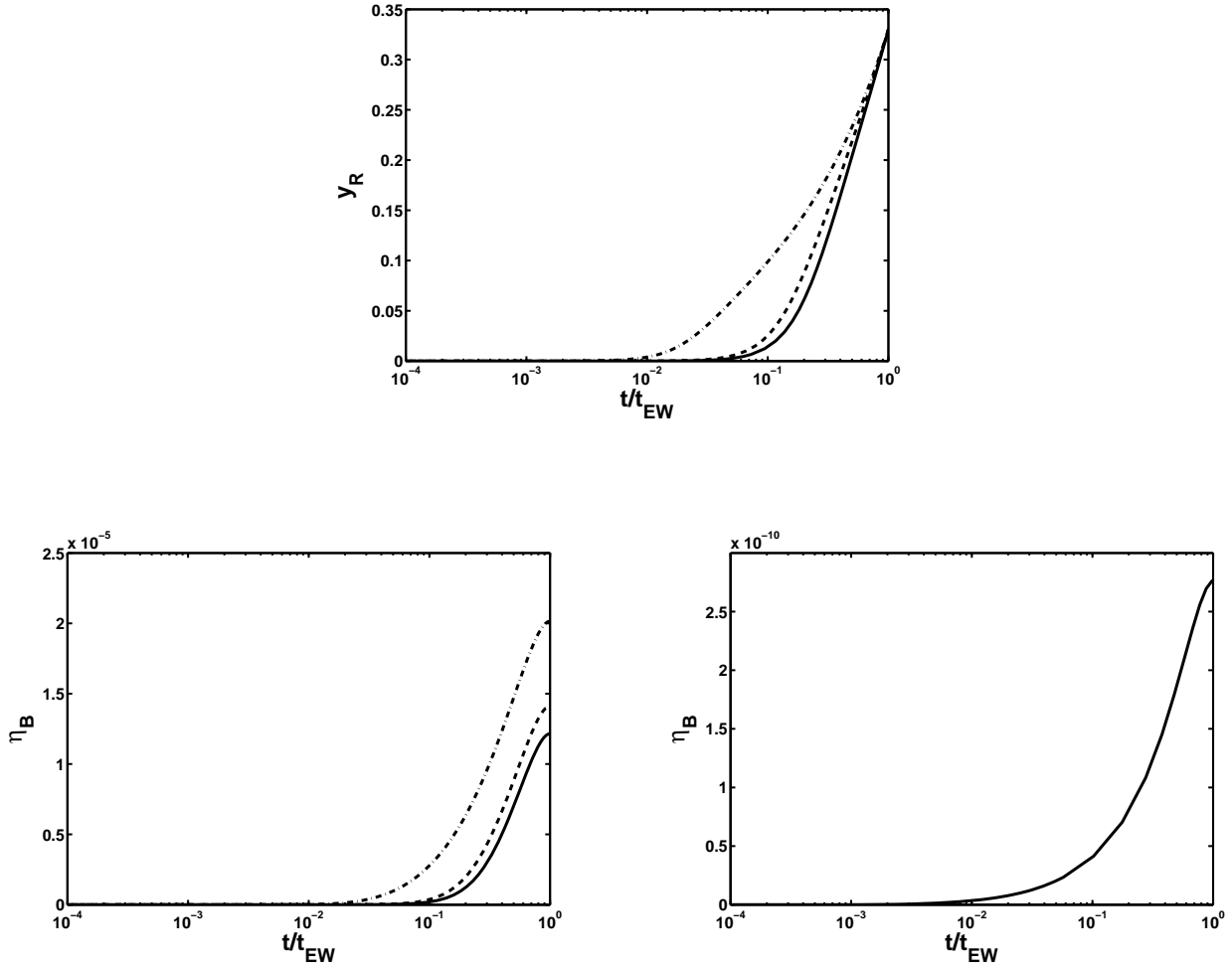


Figure 1: (a) Normalized chemical potential $y_R = 10^4 \times \xi_{eR}$ versus time. (b) Baryon asymmetry versus time. Panels (a) and (b) are built for $k_0/(10^{-7} T_{EW}) = 1$. The solid lines correspond to $B_Y^{(0)} = 10^{20}$ G, the dashed lines to $B_Y^{(0)} = 10^{21}$ G, and the dash-dotted lines to $B_Y^{(0)} = 10^{22}$ G. (c) Baryon asymmetry versus time for the small wave number $k_0/(10^{-7} T_{EW}) = 10^{-5}$ and $B_Y^{(0)} = 10^{20}$ G.

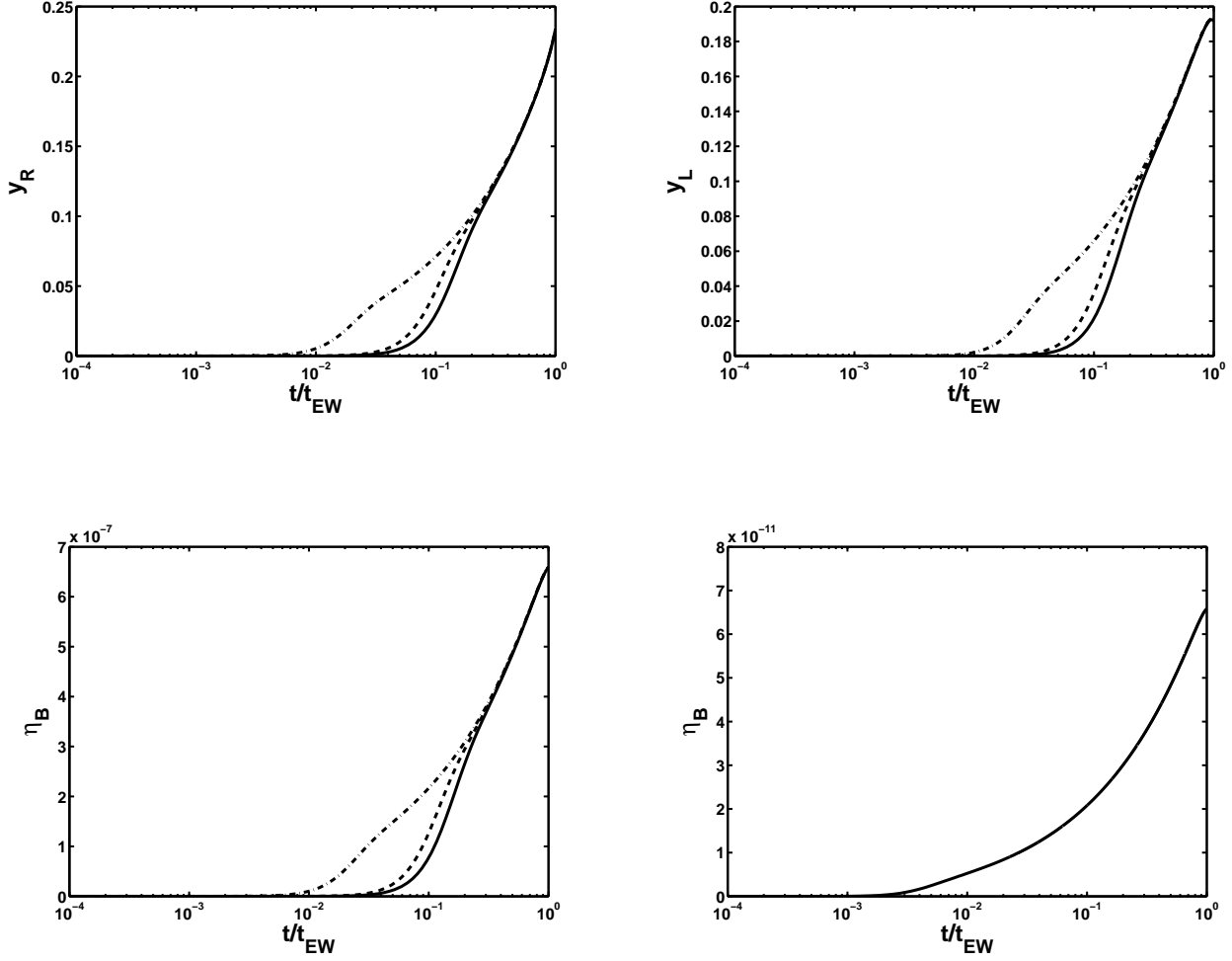


Figure 2: (a) Normalized chemical potential $y_R = 10^4 \times \xi_{eR}$ versus time. (b) Normalized chemical potential $y_L = 10^4 \times \xi_{eL}$ versus time. (c) Baryon asymmetry versus time. Panels (a)-(c) are built for $k_0/(10^{-7} T_{EW}) = 1$. The solid lines correspond to $B_Y^{(0)} = 10^{20}$ G, the dashed lines to $B_Y^{(0)} = 10^{21}$ G, and the dash-dotted lines to $B_Y^{(0)} = 10^{22}$ G. (d) Baryon asymmetry versus time for the small wave number $k_0/(10^{-7} T_{EW}) = 10^{-4}$ and $B_Y^{(0)} = 10^{20}$ G.

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