Resonant electron-positron pairs production in magnetar polar cap

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Abstract

The scattering of the surface thermal X-ray photons on ultrarelativistic electrons with electron-positron pair production, $\gamma e^\pm \to e^\pm e^+ e^-$, in the vicinity of magnetar polar cap is considered. The amplitude of the pair production process is calculated in the limit, where the initial and final electrons are on the ground Landau level but the virtual electron occupies the arbitrary Landau level. It is shown that the cyclotron resonances are responsible for the main contribution to the amplitude. The simple analytical expression for the electron absorption coefficient is obtained. Possible astrophysical consequences of the resonant process $\gamma e^\pm \to e^\pm e^+ e^-$ are discussed.

1 Introduction

Nowadays, there exists a growing interest to the process of radio emission of some magnetars [1], i.e. isolated neutron stars with anomalously strong magnetic fields $B \gg B_e$ ($B_e = m^2/e \simeq 4.41 \times 10^{13}$ G), [1], namely, the magnetic field strength in magnetars can reach the values up to $\sim 10^{14} - 10^{15}$ G [2, 3, 4].

According to a generally accepted model, an effective generation of electron-positron plasma in the radio pulsar magnetosphere is necessary for the radio emission formation [5], and mechanisms of e^+e^- pairs production in radio pulsars are well known (see eg. [6, 7]). In the model of the magnetar magnetosphere, the e^+e^- pairs production occurs in two stages [8]:

(i) the hard X-ray production by Compton mechanism $\gamma e \to e\gamma$ (the so called the inverse Compton effect);

(ii) the increase of the angle between the photon momentum and the magnetic field direction (the so called pitch angle), and the e^+e^- pair production, $\gamma \to e^+e^-$.

However, in our view, this mechanism has a major drawback. Namely, the dispersion properties of a photon become significant in a strong magnetic field. This fact leads to an effect of the photon capture by the field [9], i.e. gamma quantum with energies greater than $2m$ will move along magnetic field line, without increases of the pitch angle. Therefore, the electron-positron pair cold not be produced by such a photon.

Thus, it is interesting to consider alternative mechanisms for the generation of e^+e^- pairs in the magnetosphere of a magnetar. The reaction, which could solve of this problem, is the Compton like process, $\gamma e \to ee^+e^-$. [2].

The main advantage of such a reaction, compared with the adopted model is that the production of a pair occurs almost instantaneously at the point of interaction of an initial

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[1] We use natural units $c = \hbar = k = 1$, m is the electron mass, $e > 0$ is the elementary charge.

[2] A symbol e in the future means an electron or a positron

photon and an electron (in fact, this scale is of the order of the Compton wavelength of an electron). With this approach, the effect of the photon capture by the magnetic field becomes negligible. On the other hand, it is possible to fill a small area by the dense e^+e^- plasmas with help of the reaction $\gamma e \to ee^+e^-$ in a short time, such as in the model of giant flare of the Soft Gamma Repeaters (SGR) [2].

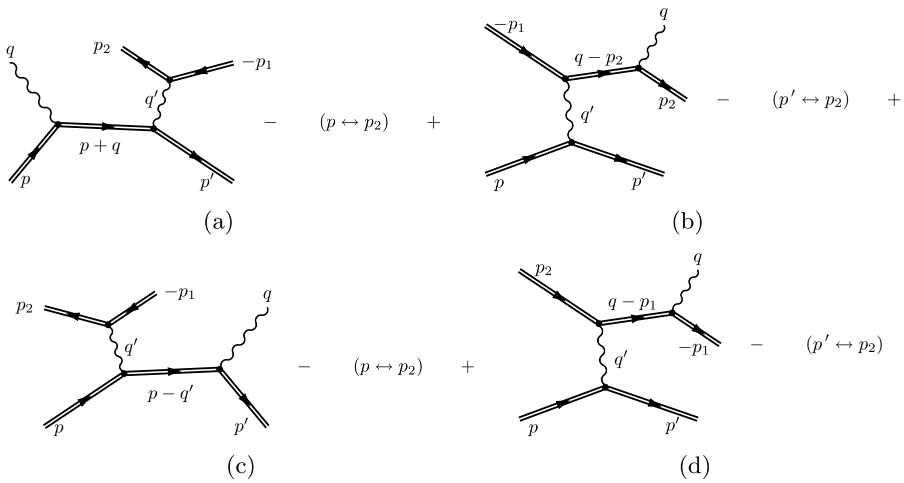

Figure 1: Feynman diagram for the process $\gamma e \to ee^+e^-$. Double lines mean that the effect of the external field on the fermions is exactly taken into account.

2 Transition amplitude $\gamma e \to ee^+e^-$

The process of the electron-positron pair production in the reaction $\gamma e \to ee^+e^-$ is described by the eight Feynman diagrams (see Fig. 1). The kinematical analysis shows, that the s-channel diagrams (Fig. 1a and the corresponding diagram with the electron momenta permutation, $p' \leftrightarrow p_2$) provide a leading contribution in to the resonance amplitude only. Nevertheless, even with taking into account of the resonance, the problem will have enough cumbersome, since the charged fermions can occupy arbitrary Landau levels. However, this problem can be essentially simplified in the application to magnetars. Indeed, we consider the situation where electron, accelerated in the electric gap of the magnetar polar cap, collides with the gamma quantum from the equilibrium thermal bath formed by radiation of X-rays from the surface of a neutron star. We will have the following parameters hierarchy in this problem formulation: $T^2 \ll m^2 \ll eB \ll E^2$.

In addition, the initial electron occupies the zero Landau level ($\ell = 0$) until the acceleration. Then electron moved along magnetic field line and it remains all the time on the ground level. (We consider the small vicinity of the polar cap, where the electric $\vec{\mathcal{E}}$ and magnetic $\vec{B}$ fields are collinear vectors and $|\vec{\mathcal{E}}| \ll |\vec{B}|$ [8].) And we will consider that the scattered electron and electron and positron of the pair are on the ground Landau level with $\ell' = n_1 = n_2 = 0$ in the first approximation.

After this remark, the $\mathcal{S}_{if}$ – matrix element of the process $\gamma e \to ee^+e^-$ can be written in the following form:

$$\mathcal{S}_{if} = (\mathrm{ie})^3 \int d^4X d^4Y d^4Z \{ \bar{\Psi}_{p'}(Y)\gamma_\beta \hat{S}(X,Y)\hat{A}(X)\Psi_p(X)\bar{\Psi}_{p_2}(Z)\gamma_\mu \Psi_{p_1}(Z) - \bar{\Psi}_{p_2}(Y)\gamma_\beta \hat{S}(X,Y)\hat{A}(X)\Psi_p(X)\bar{\Psi}_{p'}(Z)\gamma_\mu \Psi_{p_1}(Z)\} G_{\beta\mu}(Z-Y)\,, \tag{1}$$

where $p^\mu = (E, \vec{p})$ and $p'^\mu = (E', \vec{p}\,')$ are the four-momenta of initial and final electrons correspondingly, $p_2^\mu = (E_2, \vec{p}_2)$ and $p_1^\mu = (E_1, \vec{p}_1)$ are the four-momenta of electron and positron of the pair correspondingly, $X^\mu = (X_0, X_1, X_2, X_3)$,

$$A_\mu(X) = \frac{\varepsilon_\mu(q) e^{-\mathrm{i}(qX)}}{\sqrt{2\omega V}} \tag{2}$$

is the four-potential of the quantized electromagnetic field with four-momentum $q^\mu = (\omega, \vec{k})$,

$$G_{\beta\mu}(Z) = \int \frac{\mathrm{d}^4 q'}{(2\pi)^4} \mathrm{e}^{-\mathrm{i}(q'Z)} \mathcal{G}_{\beta\mu}(q'), \tag{3}$$

$$\mathcal{G}_{\beta\mu}(q') = \sum_{\lambda=1}^{3} \frac{b_\beta^{(\lambda)} b_\mu^{(\lambda)}}{(b^{(\lambda)})^2} \frac{-i}{q'^2 - \mathcal{P}^{(\lambda)}(q')}$$

is the Fourier transform of the photon propagator in the orthogonal basis $b_\alpha^{(\lambda)}$: $b_\alpha^{(1)} = (q'\varphi)_\alpha$, $b_\alpha^{(2)} = (q'\tilde{\varphi})_\alpha$, $b_\alpha^{(3)} = q'^2 (q'\varphi\varphi)_\alpha - (q'\varphi\varphi q') q'_\alpha$, $\mathcal{P}^{(\lambda)}(q')$ are the eigenvalues of the photon polarization operator corresponding to the eigenvectors $b_\alpha^{(\lambda)}$ (see [9]), $\Psi_p(X)$ is the electron wave function in the presented of external magnetic field on the ground Landau level (see [10, 11]).

We use the electron propagator in the following form [12]

$$\hat{S}(X, X') = \sum_{n=0}^{\infty} \frac{\mathrm{i}}{2^n\, n!} \sqrt{\frac{eB}{\pi}} \exp\left\{ -\, eB\, \frac{X_1^2 + X_1'^2}{2} \right\} \int \frac{\mathrm{d}p_0\, \mathrm{d}p_y\, \mathrm{d}p_z}{(2\pi)^3} \times$$

$$\frac{\mathrm{e}^{-\,\mathrm{i}\,(p\,(X-X'))_\parallel}}{p_\parallel^2 - m^2 - 2\, eB\, n + \mathrm{i}\,\varepsilon} \exp\left\{ -\, \frac{p_y^2}{eB} - p_y \left[X_1 + X_1' - \mathrm{i}\,(X_2 - X_2') \right] \right\}$$

$$\times \Big\{ \left[(p\gamma)_\parallel + m \right] \left[\mathit{\Pi}_-\, H_n(\xi)\, H_n(\xi') + \mathit{\Pi}_+\, 2n\, H_{n-1}(\xi)\, H_{n-1}(\xi') \right] +$$

$$\mathrm{i}\, 2n \sqrt{|e_f| B}\, \gamma^1 \left[\mathit{\Pi}_-\, H_{n-1}(\xi)\, H_n(\xi') - \mathit{\Pi}_+\, H_n(\xi)\, H_{n-1}(\xi') \right] \Big\}, \tag{4}$$

where $H_n(\xi)$ is the Hermitian polynomial [13].

Here it is necessary to make the following remark. When we consider the resonance on the virtual electron, it should take into account of the imaginary part of the electron propagator. On the other hand, the imaginary part of the electron propagator associated with the total width of electron absorption. However, the electron absorption width depends on the polarization of the electron, but in the strong-field limit, $B \gg B_e$, this dependence becomes insignificant. This fact allows to use of the propagator in the form (4).

In addition, we use the following definitions: the four-vectors with indices $\perp$ and $\parallel$ belong to the Euclidean $\{1,\, 2\}$-subspace and the Minkowski $\{0,\, 3\}$-subspace correspondingly in the frame were the magnetic field is directed along z (third) axis; $(ab)_\perp = (a\Lambda b) = a_\alpha \Lambda_{\alpha\beta} b_\beta$, $(ab)_\parallel = (a\widetilde{\Lambda} b) = a_\alpha \widetilde{\Lambda}_{\alpha\beta} b_\beta$, where the tensors $\Lambda_{\alpha\beta} = (\varphi\varphi)_{\alpha\beta}$, $\widetilde{\Lambda}_{\alpha\beta} = (\tilde{\varphi}\tilde{\varphi})_{\alpha\beta}$, with equation $\widetilde{\Lambda}_{\alpha\beta} - \Lambda_{\alpha\beta} = g_{\alpha\beta} = diag(1, -1, -1, -1)$ are introduced. $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$ and $\tilde{\varphi}_{\alpha\beta} = \frac{1}{2} \varepsilon_{\alpha\beta\mu\nu} \varphi_{\mu\nu}$ are the dimensionless field tensor and dual field tensor correspondingly.

After integration (1) over d^4X, d^4Y we obtain

$$\mathcal{S}_{if} = \frac{\mathrm{i}(2\pi)^3 \delta^3(\ldots) \mathcal{M}}{\sqrt{2\omega V 2E L_y L_z 2E' L_y L_z 2E_1 L_y L_z 2E_2 L_y L_z}}, \tag{5}$$

where $\delta^3(\ldots) \equiv \delta(P_0 - E' - E_1 - E_2)\delta(P_y - p_y' - p_{1y} - p_{2y})\delta(P_z - p_z' - p_{1z} - p_{2z})$, $P_\alpha \equiv (p+q)_\alpha$, $\alpha = 0, 2, 3$.

The amplitude $\mathcal{M}$ of the process $\gamma e \to e e^+ e^-$ can be presented in the following form

$$\mathcal{M} \simeq -\mathrm{i}\frac{2\sqrt{2}e^3m^2}{\pi} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{\mathrm{d}q'_x}{q'^2 - \mathcal{P}^{(2)}(q')} \exp\left[-\frac{\mathrm{i}(q\varphi q')}{2eB}\right] \exp\left[-\frac{\mathrm{i}(q_x - q'_x)(p_y + p'_y)}{2eB}\right] \times$$
$$\times \exp\left[\frac{\mathrm{i}q'_x(p_{1y} - p_{2y})}{2eB}\right] \exp\left[-\frac{2q'^2_\perp + q^2_\perp}{4eB}\right] \frac{1}{n!} \left(\frac{(q\Lambda q') - \mathrm{i}(q\varphi q')}{2eB}\right)^n \times \tag{6}$$
$$\times \frac{(pq')_\parallel [(pq)_\parallel + (p'q)_\parallel]}{\left(P^2_\parallel - m^2 - 2eBn + \mathrm{i}P_0\Gamma_n\right)\sqrt{q^2_\parallel q'^2_\parallel [(pp')_\parallel + m^2]}}\Bigg|_{\substack{q'^\alpha_\parallel = p^\alpha_{1\parallel} + p^\alpha_{2\parallel} \\ q'_y = p_{1y} + p_{2y}}} - (p' \leftrightarrow p_2)\,.$$

In the amplitude (6) Γ_n is total width of electron absorption process. The analysis shows, that the leading contribution in Γ_n gives the process $e_n \to \gamma + e_{n'}$ and the value $P_0\Gamma_n$ for ultrarelativistic electrons and in strong field limit can be presented in the following form (see [11, 14])

$$P_0\Gamma_n \simeq \alpha eB \sum_{n'=0}^{n-1} \int_0^{(\sqrt{n}-\sqrt{n'})^2} \frac{\mathrm{d}x}{\sqrt{(n+n'-x)^2 - 4nn'}} \times \tag{7}$$
$$\times \{(n+n'-x)[\mathcal{I}^2_{n,n'-1}(x) + \mathcal{I}^2_{n-1,n'}(x)] - 4\sqrt{nn'}\mathcal{I}_{n,n'}(x)\mathcal{I}_{n-1,n'-1}(x)\}\,.$$

To analyse the efficiency of the process under consideration and to compare it with other competitive reactions we calculate the electron absorption rate in the equilibrium photon gas with the temperature T:

$$W = \int \frac{\delta^3(\ldots)|\mathcal{M}|^2}{2^5(2\pi)^6\omega E E' E_1 E_2} \frac{\mathrm{d}^3 q}{\mathrm{e}^{\omega/T} - 1} \mathrm{d}p'_y \mathrm{d}p'_z \mathrm{d}p_{1y} \mathrm{d}p_{1z} \mathrm{d}p_{2y} \mathrm{d}p_{2z}\,. \tag{8}$$

As already mentioned, the main contribution to the amplitude will give the resonance region, so that we can replace the part of the integrand in (8) by δ - function

$$\frac{1}{(P^2_\parallel - m^2 - 2eBn)^2 + P^2_0\Gamma^2_n} \simeq \frac{\pi}{P_0\Gamma_n}\delta(P^2_\parallel - m^2 - 2eBn)\,. \tag{9}$$

Introducing new variables $y = q'^2_\perp/eB$ and $z = q'_z/E$, we obtain in our approximation, $q'^2_\parallel \simeq 2eBz$, $(q-q')^2_\parallel \simeq -m^2z^2/(1-z)$. In addition, the leading contribution in absorption rate (8) from the virtual electron Landau levels will be determined by $n = 1$ only, while the contributions of the higher levels are suppressed by the temperature. In this case the electron absorption width has a simple form: $P_0\Gamma_1 \simeq \alpha eB(1 - \mathrm{e}^{-1})$. With this in mind, after integration of the expression (8) with δ - function, we obtain

$$W \simeq \frac{\alpha^2 T}{2\pi(1 - \mathrm{e}^{-1})} \left(\frac{m}{E}\right)^2 \ln\left(1 - \mathrm{e}^{-\frac{eB}{2ET}}\right)^{-1} \int_{2B_e/B}^{1} \mathrm{d}z \times \tag{10}$$
$$\times \int_0^{\infty} \frac{\mathrm{d}y y \mathrm{e}^{-y}}{z^2(2z-y)^2 + 4\alpha^2(B_e/B)^2\mathrm{e}^{-y}}\,.$$

Near the second resonance (now on the virtual photon), the part of the integrand in Eq. (10) can be interpolated by δ - function also

$$\frac{1}{z^2(2z-y)^2 + 4\alpha^2(B_e/B)^2\mathrm{e}^{-y}} \simeq \frac{\pi \mathrm{e}^{y/2}}{2\alpha}\frac{B}{B_e}\delta(2z^2 - yz)\,. \tag{11}$$

Thus in the process of $e\gamma \to ee^+e^-$ in presence of the strong magnetic field and ultrarelativistic particles, the both resonances on the virtual electron and the virtual photon become possible. After substituting (11) in (10) and simple integration we obtain

$$W \simeq \frac{\alpha}{2} T \frac{B}{B_e} \left(\frac{m}{E}\right)^2 \ln\left(1 - \mathrm{e}^{-\frac{eB}{2ET}}\right)^{-1}. \tag{12}$$

The dependence of the electron absorption rate on the energy of initial electron at $B = 100B_e$ and $T = 1$ keV is presented on the Fig. 2.

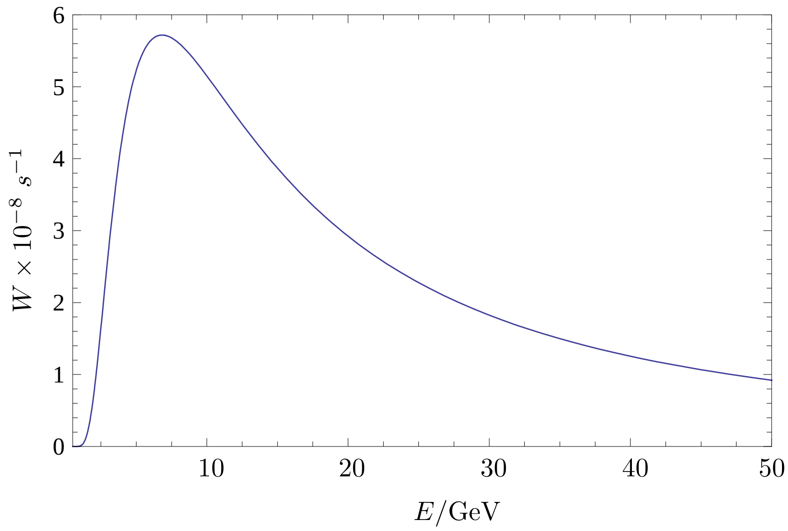

Figure 2: The electron absorption rate as a function of electron energy in a strong magnetic field ($B/B_e = 100$) at $T = 1$ keV.

In addition, for $E \simeq 10$ GeV and the same value of parameters n_e, B and T as on the Fig. 2, we obtain that the electron free path is $\ell \simeq 57$ cm. This value is very small in comparison with the electric gap width ($h \sim 10^4$ cm). On the other hand, the change of electron number in the stream can be expressed through optical thickness τ in the following way

$$N = N_0 \exp\left[-\tau\right] \simeq N_0 \exp\left[-\int_0^h \mathrm{d}x W\right], \tag{13}$$

where N_0 is the initial electron number in the stream.

We obtain the following estimation for $N/N_0 \simeq 0.99$ at $h \sim 10^4$ cm, $E \sim 10^7 m$. Thus, the considering process can increases the number of density of e^+e^- plasma in the polar cap area. However, a detailed quantitative analysis of a development cascade of e^+e^- pairs requires solution of the kinetic equation, which is beyond the scope of our problem.

Finally, we have make still one remark. The resonances on the virtual electron and virtual photon correspond to processes with the real particles. Thus the considering process, $\gamma e \to ee^+e^-$, can be presented as a group of three subprocesses:

(i) the absorption of the photon by electron with the electron production on the first Landau level,
$e_0 + \gamma \to e_1$;

(ii) the synchrotron radiation process, $e_1 \to e_0 + \gamma$;

(iii) the e^+e^- pair production by hard photon, $\gamma \to e^+e^-$.

The branching fractions of the reactions $e_1 \to e_0 + \gamma$ and $\gamma \to e^+e^-$ are equal approximately of 1 and 1/2 correspondingly. (The factor 1/2 appears, because the photon only one polarisation takes part in the reaction $\gamma \to e^+e^-$.) Therefore, the electron absorption coefficient in the process $\gamma e \to ee^+e^-$ can be obtained from the probability of the reaction $\gamma + e_0 \to e_1$ in the following way $W = W_{\gamma+e_0\to e_1}/2$. The expression for the transition probability $\gamma + e_0 \to e_n$ can be presented as

$$W_{\gamma+e_0\to e_n} \simeq \frac{\alpha e B}{E}\left(\frac{Tn}{E}\right)^n Li_n\left(\mathrm{e}^{-\frac{eBn}{2ET}}\right), \tag{14}$$

where $Li_n(x)$ is the polylogarithm of order n.

In particular, for $n = 1$ we obtain

$$W_{\gamma+e_0\to e_1} = 2W \simeq \alpha T \frac{B}{B_e}\left(\frac{m}{E}\right)^2 \ln\left(1 - \mathrm{e}^{-\frac{eB}{2ET}}\right)^{-1}. \tag{15}$$

This result coincides with that obtained previously (see (12)).

3 Conclusion

In conclusion, let us summarize some of our results. We have considered the resonant Compton like process of electron-positron pair production near polar cap of magnetar magnetosphere. It has been shown, that the leading contribution in the process amplitude $\gamma e^\pm \to e^\pm e^+e^-$ occur from cyclotron resonances. It has been found, that intensive pair production will on the initial stage of electrons acceleration in the internal gap. It has been shown, that this mechanism can be efficient for the e^+e^- plasma production in magnetar magnetosphere.

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